This project has been supported by an American Council on Education – Alfred P. Sloan Foundation Faculty Retirement Transition award to San José State University
Financial Literacy

Module 1 – Time Value of Money (TVM) and the Power of Compounding
Overview

- Time Value Of Money (TVM)
  - Definition
  - Why Do We Care?
  - Examples
  - Graphs

- Conclusion
The Power of Compounding

- “A nearby penny is worth a distant dollar”, Anonymous
- “The most powerful force in the universe is compound interest”, Albert Einstein
- “…the world’s greatest invention was 6 per cent compound interest, which goes on twenty-four hours a day, seven days a week and fifty-two weeks a year.”, Roger W. Babson
TVM - Compounding

- $1 invested today at r% per year for n years

\[ FV_n = C \times (1 + r)^n \]
TVM – Compounding – Single Cash Flow - Example 1

- $1,000 invested today at 8% per year for 10 years

\[ FV_{10} = \$1,000 \times (1.08)^{10} = \$2,158.92 \]
TVM – Compounding – Single Cash Flow - Example 2

- $1,000 invested today at 8% per year for 30 years

\[ FV_{30} = $1,000 \times (1.08)^{30} = $10,062.66 \]
$1,000 invested at 8% for 30 years
TVM – Compounding – Single Cash Flow - Example 3

- $1,000 invested today at 10% per year for 30 years

\[ FV_{30} = $1,000 \times (1.1)^{30} = $17,449.40 \]
TVM – Compounding – Ordinary Annuity - Example 1

- Series of cash flows: $300 invested at the end of each month for 10 years at 8% per year (APR)

\[
FV_{120} = C \times \left[ \frac{(1 + \frac{r}{m})^{n \times m} - 1}{\frac{r}{m}} \right] = $300 \times \left[ \frac{(1 + \frac{0.08}{12})^{12 \times 10} - 1}{\frac{0.08}{12}} \right] = $54,883.81
\]

\[m = 12 \quad i = \text{monthly rate} \quad i = \frac{8\%}{12} = 0.6667\%\]
TVM – Compounding – Ordinary Annuity - Example 2

- Series of cash flows: $300 invested at the end of each month for 30 years at 8% per year (APR)

\[
FV_{360} = C \times \left[ \frac{(1+i)^{n \times m} - 1}{i} \right] = C \times \left[ \frac{\left(1 + \frac{r}{m}\right)^{n \times m} - 1}{\frac{r}{m}} \right] = 300 \times \left[ \frac{(1.006667)^{360} - 1}{0.006667} \right] = 447,107.83
\]

\[m = 12 \quad i = \text{monthly rate} \quad i = \frac{8\%}{12} = 0.6667\%\]
$300 Monthly Ordinary Annuity Invested at 8% p.a. for 30 Years
TVM – Compounding – Ordinary Annuity - Example 3

• Series of cash flows: .....similar to example 2, but NO contributions during years 6-10.

\[ FV_{360} = \left\{ \$300 \times \left[ \frac{(1.006667)^{60} - 1}{0.006667} \right] \times (1.006667)^{300} \right\} + \left\{ \$300 \times \left[ \frac{(1.006667)^{240} - 1}{0.006667} \right] \right\} \]

= $338,506.02
TVM – Compounding – Ordinary Growing Annuity - Example 1

- Series of cash flows: monthly investment of $300 and growing 3% per year (APR) invested for 30 years at 8% per year (APR)

\[
\begin{align*}
FV_{360} &= C \times \left[ \frac{(1+i)^{n\times m} - (1+g)^{n\times m}}{i-g} \right] \\
&= $300 \times \left[ \frac{(1.006667)^{360} - (1.0025)^{360}}{0.006667 - 0.0025} \right] \\
&= $610,480
\end{align*}
\]

\[
i = \frac{8\%}{12} = 0.6667\% \\
g = \frac{3\%}{12} = 0.25\%
\]
TVM – The Pernicious Effect of Inflation

- The purchasing power of a monthly income of $2,000 over a twenty-year period when the annual inflation rate is 3% (APR)

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Purchasing Power
Conclusion

- TVM and the Power of Compounding can be Your Friends

- When is it Best to Start Saving?

- How do I Save?
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