VALUE OF PERFECT & SAMPLE INFORMATION FROM THE RISK-AVERSE POINT OF VIEW:
Risk Tolerance Parametrics using Exponential Utility

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The standard value of information concepts EVPI/EVSI defined for risk neutral evaluations have natural extensions in the risk-averse case. Instead of measuring an increase in Expected Monetary Value (EMV), one measures an increase in Cash Equivalent Value (CEV). This is simple enough to define, but some surprising things sometimes happen when you carry out the risk tolerance parametric analysis in specific cases. The example developed in this paper shows, in particular, that CEVSI can be MUCH LARGER than EVSI in certain mid-range risk tolerance intervals, meaning that the decision to buy or not buy information may be different as well. In the example below, the CEVSI/EVSI ratio rises to over 5.5, so that in many cases the information option that might be passed over by the risk neutral decision maker should in fact be purchased by the risk-averse decision maker. The risk tolerance range where CEVSI is highest is related to the policy regions in the maximum value frontier, and occurs where there is a major policy change from one rule to another. The CEVs for the two policies on either side of the policy breakpoint are very close to each other, so it is hard to choose between them unaided by the sample information. This results in a higher value for the sample information in the neighborhood of that policy breakpoint.

There is another extremely important point that can be illustrated with this same example. This point constitutes one of the principal justifications for accepting the delta-property axiom and therefore using exponential utility for risk-averse analyses. It is the consistency between the backwards induction process and the value of the information. Since the value of the information is computed as if it were free, the cost of the information does not enter into either EVSI or CEVSI. Let us suppose that the CEVSI obtained for a given risk tolerance exceeds the cost of the information, indicating that the information should be bought. If the cost of information is now deducted from all terminal payoffs on branches following the decision to buy, the backwards induction can be done again. We would like to see that the optimal policy (and value) obtained taking the cost of information into account agrees with the optimal policy (and value) indicated by the CEVSI computation. In fact, if we do not get the same policy (and same value), then we seem to have a contradiction that is hard to explain. It turns out that the only way to avoid this kind of contradiction is to require that the delta-property continue to hold in the risk-averse case as it does for the EMV case. And as we have seen, this means that the utility functions for risk-averse analysis must come from the exponential utility family.

THE EXAMPLE ANALYSIS

ACE Computer Company has been using the Be-Sure Survey Company to predict the success of new products. Over a period of years ACE has found that when a new product was successful, i.e., sales were high for that product, the survey Co. study had predicted success 30% of the time, showed inconclusive results 60% of the time and predicted failure 10% of the time. The record also indicated that when sales for a new product were low Be-Sure Survey Co. predicted success 10% of the time, showed inconclusive results 40% of the time and predicted failure 50% of the time. ACE Company has established the probability of high sales for a new product at 40% and low sales have a 60% probability.
It will cost ACE Company $1 million to introduce its new product, and if Be-Sure Survey is retained again, it will cost $100 thousand for the survey. If sales are high they expect to gross $4 million and they would expect to gross $0.5 million on low sales.

PRIOR ANALYSIS

We first construct a payoff table for the “main” decision, which is whether or not to market the product. If the product is not marketed, there is no introduction cost and no revenue, so the payoff is zero regardless of the level of potential demand. If the product is marketed, the net profit is 4 - 1 or $3 million under the High demand scenario and 0.5 -1 or a loss of ($0.5) million under the Low demand scenario.

<table>
<thead>
<tr>
<th></th>
<th>A0</th>
<th>A1</th>
<th>V*</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1 (High demand) 0.4</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>E2 (Low demand) 0.6</td>
<td>0</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>EMV</td>
<td>0</td>
<td>0.9</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Observe that the expected value is higher for the A1 decision (market the product). Hence the highest expected value that can be achieved without perfect or sample information is $0.9 million or $900,000. With perfect information available, the expected value increases to $1.2 million or $1,200,000. EVPI is the difference, or $300,000. Since Be-Sure Survey Company is only asking $100,000 for their survey, there is some possibility, at least, that it might be worthwhile to use their services. But the decision about this issue must await the outcome of the analyses described next.

POSTERIOR ANALYSIS

Since the EVPI for this situation exceeds the asking price of $100,000 for the Be-Sure Survey marketing study, it is conceivable that the market study might be worthwhile. But this depends upon the track record, which Be-Sure has had in previous studies of the same sort in the past. From the data given in the problem statement we can tabulate the following prior and conditional survey result probabilities:

| Prior Probability | State     | P(PS|Ei) | P(I|Ei) | P(PF|Ei) | SUM  |
|-------------------|-----------|--------|--------|---------|------|
| .4                | High Demand | .5     | .4     | .1      | 1.0  |
| .6                | Low Demand  | .1     | .4     | .5      | 1.0  |

Notice that the row sums are 1.0, indicating that each row is a separate probability distribution, conditioned on which demand level applies. The column sums need not be 1.0. Now, to form the joint probability table, we must multiply each conditional probability by the prior probability at the beginning of the row in which it occurs. This yields the following joint probability table:
Notice that now the row sums give the marginal probabilities for the demand levels, i.e. the prior probabilities in this case, and the column sums give the marginal probabilities for the survey results. Finally, by dividing the joint probabilities in each column by the marginal survey result probability at the base of the column, we get the “posterior” probabilities, or the conditional probabilities for the demand levels given the survey result.

PRE-POSTERIOR ANALYSIS

Our next task is to evaluate the EVSI (Expected Value of Sample Information) for the Be-Sure survey result. We wish to know what it would be worth to us in increased expected value, if the information were free. By comparing this EVSI with the price Be-Sure is asking for its prediction ($100,000) we can determine whether or not to buy the information. In particular, we can compute the ENGSI (Expected Net Gain of Sample Information) which is the difference between the two, ENGSI = EVSI - Cost of Information. If there were more than one survey company we were considering, each with a different track record and a different cost of survey, we could compute the ENGSI for each alternative information source. In this case we could pick that one which gives the largest expected net gain, if any is positive, or make the main decision without surveying if they are all negative.

In order to evaluate EVSI, we need to develop and “roll-back” the decision tree corresponding to the “BUY SURVEY” decision. In this analysis, the cost of the survey will be neglected (i.e.
treated as zero) and the marginal survey result probabilities and the posterior demand level probabilities will be employed, as shown in the tree below.

Notice that we have computed the expected revenues at the end of the tree, and then subtracted the $1 million cost of marketing only when the Market decision yields the higher net return. The net return figures are shown in each decision box based on the optimal policy from that point forward. In this case the optimal decision policy is to market the product if the Be-Sure result is PS or I, and not market the product if the Be-Sure result is PF.

The net expected values for each survey result are then weighted by the survey result probabilities. This gives an expected value of $0.93 million given (free) sample information, or $930,000. When this is compared with the best we could do using prior information, namely $900,000, we have

\[ EVSI = EV|SI - \text{Max EMV}(A_i) = $930,000 - $900,000 = $30,000 \]

The expected value of having access to the Be-Sure Survey Co. result prior to the marketing decision is only $30,000 and they are asking $100,000 for it. Thus based on expected values, the ENGSI comes out to be a large negative amount ($70,000). If the decision-maker is risk neutral (i.e., makes decisions based on expected values only), then the most that could be paid for the Be-Sure survey result would be $30,000. Paying anything more would cause the expected net gain to go negative, and hence is inferior to simply marketing the product without the benefit of the survey result.

THE RISK-averse ANALYSIS USING A RISK TOLERANCE

In reality, we know that decision-makers generally have a certain amount of risk aversion, so that they exhibit some sensitivity to the worst case result as well as the expected value of the outcome. The certainty equivalent value (or CEV, for short) of a chance outcome is therefore obtained by discounting the expected value of the outcome by a certain “risk premium” which depends upon the risk tolerance of the decision-maker. The optimal decision rule and also the
value of perfect and survey information must therefore be computed in terms of the two cash equivalent values involved, assuming the information is free, so that

\[ \text{CEVPI}_\tau = \text{CEV}_\tau|\text{PI} - \text{Max CME}_\tau (A_i) \]

and

\[ \text{CEVSI}_\tau = \text{CEV}_\tau|\text{SI} - \text{Max CME}_\tau (A_i). \]

As the risk tolerance scale factor \( \tau \) decreases from \( \infty \) towards 0, the policies, both with and without sample information, may change to reflect a progressively more and more risk-averse posture, until the risk tolerance is so small that the product would not be marketed under any conditions. At this point the CEVSI drops to 0. Our problem in this risk tolerance parametric analysis is to determine at exactly which values for risk tolerance does the policy change (with or without survey information). Also, how do \( \text{CEV}|\text{PI}, \text{CEV}|\text{SI} \) and \( \text{Max CE}_\tau (A_i) \) and thus \( \text{CEVPI}_\tau \) and \( \text{CEVSI}_\tau \) change between these “breakpoints” in the risk tolerance level. When we plot these cash equivalent values between the policy change “breakpoints” in different colors, we get what is called a “Rainbow Diagram.” EXCEL spreadsheets for accomplishing this, and the associated charts of CEVPI and CEVSI, are shown below, but first we illustrate the computational process using just one particular value for \( \tau \), namely \( \tau = 1 \) million $ (i.e. $1,000,000).

**Prior Analysis with a risk tolerance.**

Let’s calculate the CE value for the prior analysis first, without using the results of the survey. We have

\[ \text{CE}(A1) = -1*\ln[0.4\exp(-4/1)+0.6\exp(-0.5/1)] - 1 = -0.009106 = -9,106 \]

where the prior probabilities have been used to evaluate the chance node on level of demand. We would get the same value if we had evaluated the net profit figures from the payoff table,

\[ \text{CE}(A1) = -1*\ln[0.4\exp(-3/1)+0.6\exp(0.5/1)] = -0.009106 = -9,106 \]

This is due to the "Value Additivity" or delta-property of Exponential Utility mentioned before. We can compute the CE value ignoring the 1M cost and then subtract the 1M, or we can subtract the 1M cost first and then compute the CE value; we get the same answer either way. Also note that the value of the A1 alternative has dropped from its former $900,000 level all the way down to just under zero, so the former GO with the product is replaced with a NOGO preference. The Risk Premium in this case is given by

\[ \text{RP} = \text{EMV} - \text{CE}_\tau = 900,000 - (-9,106) = 909,106 \]

Notice also that the value of the V* gamble (given Perfect Information) has changed as well, since we have

\[ \text{CE}_\tau|\text{PI} = -1*\ln[0.4\exp(-3/1) + 0.6\exp(0/1)] = .478173185 = 478,173.185 \]
Consequently, in this case the value of perfect information is given by

$$\text{CEVPI}_t = \text{CEV}_t|\text{PI} - \text{Max}\{\text{CE}(A_i)\} = \$478,173.19 - \$0 = \$478,173.19$$

This is substantially greater than the EVPI obtained before, namely, $300,000. Thus the value of perfect information can be worth more to the risk-averse decision-maker than to the risk neutral decision-maker. We shall see shortly that the same is true for the value of sample information. Also by varying the risk tolerance through a range of values, we get a chart of CEVPI as a function of $\tau$. (see attached pages).

**Posterior Analysis with a risk tolerance.**

Let's turn now to the computation of the value of sample information. When the backwards induction process is carried out for a risk-averse decision maker, ALL chance nodes are regarded as gambles at which a cash equivalent value must be computed from the probabilities and the cash equivalent values for outcomes at a node. Thus all expected value computations are replaced with cash equivalent value evaluations. This must be done THROUGHOUT the ENTIRE TREE, not just at the ends of the tree. Let’s see how this works out for the Be-Sure decision with $\tau = 1$.

Consider the three demand level chance nodes using the three sets of posterior probabilities. After the “predict success” result (PS), the posterior probabilities are 10/13 and 3/13, which lead to a net value of

$$\text{CE}|\text{PS} = -1\times \ln[(10/13)\exp(-3/1)+(3/13)\exp(0.5/1)] = 0.870428936 = \$870,428.94$$

Likewise, after the indeterminate result (I) we have

$$\text{CE}|\text{I} = -1\times \ln[0.4\exp(-3/1)+0.6\exp(0.5/1)] = -0.009106 = -\$9,106 < 0$$

As previously obtained in the Prior Analysis, so the decision is NOGO in this case with a net value of 0. And after the “predict failure” result (PF) we have

$$\text{CE}|\text{PF} = -1\times \ln[(2/17)\exp(-3/1)+(15/17)\exp(0.5/1)] = -0.37885509 < 0$$

so in the last case it's also preferable not to market the product, and the net value is 0.

Now backing up to the chance node for the survey result, we again use the cash equivalent formula (NOT an expected value calculation) and obtain

$$\text{CE}|\text{SI} = -1\times \ln[0.26\exp(-0.870428936/1)+0.4+0.34]=0.163836632 = \$163,836.63$$

where the last two exponential terms reduce to 1.0 since the payoff value in that case is zero and $\exp(0)=1$. Placing these cash equivalent values on the decision tree gives us the following modified diagram:
And the corresponding tree for the prior analysis has changed to:

Finally taking the difference between CEV|SI and CEV|priors we obtain

\[ \text{CEVSI} = \text{CE|SI} - \text{Max CME}(A_i) = \$163,836.63 - \$0 = \$163,836.63. \]

Now this IS interesting! The CEVSI has gone UP dramatically from the EVSI, which was only $30,000. The information is worth MUCH MORE to the risk-averse decision-maker than to the risk neutral one. Hence we cannot assume that the most one should pay for the sample information is $30,000. We have shown that when \( \tau = 1 \), the decision-maker would be willing to pay up to $163,836.63 for the survey result, a full $130,000 more than the risk neutral decision-maker would. Hence the $100,000 asking price is seen as attractive in this case, and CENGSI = \$163,836.63 - \$100,000 = \$63,836.63. Also, note the magnitude of the increase in information value in terms of the ratio CEVSI/EVSI = 5.46.

RISK TOLERANCE PARAMETRICS
By varying the risk tolerance across a range of values, one can easily show that in fact there is a substantial range of risk tolerances that would justify the $100,000 price for the sample information. In fact, as you systematically vary $\tau$, you will find that there are three distinct “breakpoints” in the analysis where either the prior or the posterior policy changes. As you increase $\tau$ from 0, you will first notice that the Max CEV for the prior policy is zero, which means that the product is not introduced, based on prior information only. At a somewhat larger value, one finds that the Be-Sure Survey option becomes attractive, and that an I result from Be-Sure is not sufficient, and one only goes ahead with the product if the Be-Sure result is PS. In the next policy region, a PS or an I result is sufficient to justify going ahead with the product. And finally, the CEVSI drops below $100,000 again so the optimal decision is to just introduce the product with no survey, which is the risk neutral or EMV policy. From the attached graph of CEVSI, you can see that it varies nonlinearly in a smooth way between breakpoints, and that it achieves a maximum value at a unique value of $\tau$. What is the value of $\tau$ which maximizes CEVSI? What is the maximum value that CEVSI attains? What is the maximum value that CEVSI/EVSI attains? Explain the significance of this result.

To summarize the results of the parametric analysis on risk tolerance, it is convenient to create a table showing the optimal policies for each policy range, and the interval of risk tolerances over which that policy is optimal. The policies that appear in this table constitute the “MAXIMAL VALUE FRONTIER” for the problem, and all other policies are said to be CEV-dominated by these which are optimal for one risk tolerance or another. A plot of the maximum CEV for each risk tolerance in which the area under the curve is color coded according to which policy is optimal is called a Rainbow Diagram for the Analysis and depicts the range of optimality for each policy graphically. The table and chart for the Be-Sure Survey analysis are shown below. The color coding in the third Column correlates with the color coding on the Rainbow Diagram.

<table>
<thead>
<tr>
<th>POLICY</th>
<th>Risk Tolerance Range</th>
<th>Color Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Don’t Survey, Don’t Market</td>
<td>0 - $694,576</td>
<td></td>
</tr>
<tr>
<td>Survey, Market on PS only</td>
<td>$694,577 - $1,021,720</td>
<td></td>
</tr>
<tr>
<td>Survey, Market on PS or I</td>
<td>$1,021,721 - $3,199,662</td>
<td></td>
</tr>
<tr>
<td>Don’t Survey, Just Market</td>
<td>$3,199,663 - +INF</td>
<td></td>
</tr>
</tbody>
</table>
Note that the first policy is appropriate for the ultra risk-averse decision maker who evaluates gambles in terms of their worst case. And the last policy is appropriate for the risk neutral decision maker who evaluates gambles in terms of their EMV. The benefit of the risk tolerance parametric analysis is that it shows up two more “in between” policies that are optimal for mid-range risk tolerance levels, both of which indicate purchase of the Be-Sure Survey result. Note that the risk tolerance range in which the Survey is purchased extends from $694,577 all the way up to $3,199,662. This is a significant interval that might very well include the risk tolerance appropriate for the ABC Computer Company executives. Hence the information option cannot be rejected just because the $100,000 cost exceeds the EVSI of $30,000. If the decision makers are risk averse, as they usually are, then one must compute the CEVPI values, and these may well indicate purchase of the information even when EVSI does not.

**CONSISTENCY CHECK**

One of the nice features of risk neutral EMV analysis is that the EMV of the optimal policy risk profile is equal to the EMV developed by the backwards induction process which shows over the first node in the decision tree. This is insured because EMV satisfies the delta-property requirement that $\text{EMV}(X+c) = \text{EMV}(X)+c$ where $c$ is any constant. It would be nice if this remained true for risk-averse analyses as well, and, as we shall see, it does remain true if the utility function also satisfies the delta-property so that $\text{CEV}(X+c) = \text{CEV}(X) + c$. In this section we show that for other utility functions that do not have this property, contradictions may arise in which the value of information results are different from what is obtained via the backwards induction process. That is, the results obtained by keeping the cost of information on the branch preceding the sample information chance node may be different from the results obtained by netting out the cost of the sample from the payoffs at the end of the tree. If this occurs, then one is in a quandary to explain why the optimal solution from the backwards induction is not the one indicated by the value of information results. The only way to avoid this quandary is to require
the utility function to have the delta-property, which, as we know, implies that it be from the exponential utility family.
Let us first confirm equality of the results for the analysis completed in this example with risk tolerance at $1 million as before. By deducting the $100,000 survey cost from the CEVSI we obtained the value $63,836.63 for the optimal policy, which was to “Buy Survey; Market only if PS”. Now we will develop the risk profile for this policy by collapsing the tree down to a single chance node, deducting the $100,000 survey cost from the affected terminal node values. We find there is a 74% chance of just losing the $100,000 cost of the sample, a 20% chance of the “big hit” being $2.9 million in this case, and only a 6% chance of taking a major loss of $600,000. Hence in diagrammatic form, we have

```
RiskTol=$1M

CEV=$63,836.63

0.20   $2,900,000
0.74   -$100,000
0.06   -$600,000
```

Note that the CEV of the risk profile for the optimal policy is EXACTLY equal to the CEV we got by backwards induction on the decision tree with the cost of information subtracted only once at the beginning of the tree, not multiple times at the end of the tree. The policy obtained is the same as well, because the choices made at each of the decision nodes will be the same in either analysis. This equality of results will always be true for exponential utility analysis because of the delta-property that is true for this family of utility functions.

PROBLEMS ENCOUNTERED USING LOGARITHMIC UTILITY

To see the type of trouble one gets into if the delta-property is dropped, let’s carry out some analyses using the utility function $U(x) = \ln(1+x)$ where $x$ is in millions of dollars (this could also be written as $\ln(1 + x/1000000)$ if $x$ were measured in plain dollars. As for the exponential utility, we determine the certainty equivalence formula by solving the equation $\ln(1+x^\diamond) = \sum p_i \ln(1+x_i)$ for $x^\diamond$ (the CEV for the gamble). Taking exponentials on both sides and subtracting 1 yeilds the formula we want, $x^\diamond = \exp(\sum p_i \ln(1+x_i))-1$. We will now apply this CEV formula to the trees depicting the Be-Sure Survey problem shown above. Notice that utilities are NEVER shown on the decision tree analysis; we use only cash equivalent values that come from the derived CEV formula.

First of all, let us consider the dilemma that is encountered when we consider the two ways of representing the cost of introducing the product to market, the $1M cost of marketing given in the problem statement. It can either be associated with the decision node branch where the decision to market is made, in which case the terminal payoffs are $4M and $0.5M, or it can be
“carried out to the end of the tree” and deducted from the revenue projections there, so that the terminal payoffs are 3M and -0.5M. We will compute CEVSI both ways (assuming the survey is free) and see what the difference is.

First, with the cost of marketing on the decision node branch, for the posterior analysis we have

\[
\ln(1+x)
\]

<table>
<thead>
<tr>
<th>Sample</th>
<th>Market ($1 mil)</th>
<th>Not Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi</td>
<td>1.787</td>
<td>0</td>
</tr>
<tr>
<td>Lo</td>
<td>10/13</td>
<td>3/13</td>
</tr>
<tr>
<td>$4 mil</td>
<td>2/5</td>
<td>3/5</td>
</tr>
<tr>
<td>CE=$2.787090456 M</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\ln(1+x)
\]

<table>
<thead>
<tr>
<th>Sample</th>
<th>Market ($1 mil)</th>
<th>Not Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi</td>
<td>0.428</td>
<td>0</td>
</tr>
<tr>
<td>Lo</td>
<td>2/5</td>
<td>3/5</td>
</tr>
<tr>
<td>$4 mil</td>
<td>10/13</td>
<td>3/13</td>
</tr>
<tr>
<td>CE=$1.4279668784 M</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The prior analysis is equivalent to the subtree following the “I” branch above. Hence the CEVSI for this situation is $0.505312021 - $0.427966874 = 0.077345146 M or $77,345.15 and this is less than the $100,000 cost of the survey, so the indication is that the information is “too pricey” so the indicated decision is to proceed to market with benefit of the survey result.

Now, let us take the cost of marketing out to the end of the tree, and see what we get.

\[
\ln(1+x)
\]

<table>
<thead>
<tr>
<th>Sample</th>
<th>Market ($1 mil)</th>
<th>Not Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi</td>
<td>1.475</td>
<td>0</td>
</tr>
<tr>
<td>Lo</td>
<td>10/13</td>
<td>3/13</td>
</tr>
<tr>
<td>$3 mil</td>
<td>2/5</td>
<td>3/5</td>
</tr>
<tr>
<td>CE=$1.475452571 M</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Again, the prior analysis is identical to the subtree following the “I” survey outcome, so CEVSI is equal to $0.337927554 - $0.148698355 = $0.189229199 or $189,229.20 which is substantially
larger than the $100,000 cost of the information. Hence the indicated policy in this case is to acquire the survey and then market on “PS” or “I” but not market on “PF”. So here is the quandary. One analysis is showing that one should forego the survey, and the other is showing that one should buy the survey, and both analyses were done for the SAME utility function. How can that be? Something must be wrong, one feels. Where is the mistake? How can I explain this to “the BOSS”?

Well, to avoid this paradox, people have adopted a “convention” which says that to get the “right answer” with general utility functions, one “has to” carry all branch costs (or benefits) out to the end of the tree first, and then do the backwards induction. So let’s give that a try and see what we get.

The value of the “buy survey” option when cost of survey is netted out at the end of the tree has dropped to $0.196309934 so the “net gain” CENGSI is the difference between this value and the value of the prior analysis solution, which was $0.148698355 so we would have a positive net gain, and the indicated action would be to buy the information and cancel the product only if a “PF” result is obtained. So the policy is coming out the same now, but let’s carry out the arithmetic for CENGSI and find that CENGSI - $0.047611579 or $47,611.58. This would imply that the CEVSI would be $100,000 greater than this, or $147,611.58. However, when we did the computation based on the increase in cash equivalent value assuming the information was free, we found that CEVSI was $189,229.20. So the value of CEVSI is ambiguous here, since we have two different values for the same thing.

The situation becomes even more embarrassing, alas, when we consider the situation we would face if the cost of the survey were stated as $150,000 instead of $100,000. The CEVSI result computed assuming the information is free would still be $189,229.20 so the indicated action would be to buy the survey before proceeding. But if you do the backwards induction with the survey cost netted off at the end of the tree, you get the following results.
The value of this option has now dropped to $121,989.29 whereas the “Market Product without Survey” policy still has the $148,698.36 value so the indicated action is to forego the survey and just market the product. Now we have one analysis telling us to buy the survey, while the other one tells us to forego the survey. Which one is right?

These examples show the kind of inconsistent values and contradictory policy results that may occur when the delta-property is relaxed. In order to be sure of consistent values and policies for all cases, one must impose the delta-property axiom. And this is equivalent to requiring the utility functions used to be drawn from the exponential utility family. Consistency of results, and avoidance of paradoxical results, is the “real” reason why the default utility function in all software programs is the exponential utility function. And it is the main reason for adopting the delta-property as an axiom for the rational decision maker. It is only in this case that a consistent definition of CEVSI can be made.