27. Market Area for the Sale of Goods

The price at which goods leave the location of their origin usually experiences an additional charge for the cost of dispatching them from the location of origin to the location of use or consumption. Due to the increase in price relative to an increasing distance to be covered by transport, demand will decrease, and after a certain distance from the place of origin, deteriorate to zero.

If $p$ is the price payable for goods at the place of origin and $f$ the rate of freight for a given unit and size of goods, the price resulting at a distance, $z$, from the place of origin is $p + fz$. If then $z$ is the marginal utility of the first unit for the consumer who can buy, in accordance with his financial status, at a priceworthiness of $w$, then the maximum dispatch distance, $z'$, at which goods become so expensive that demand ceases is determined thus:

$$\frac{z'}{p + fz'} = w$$

and the maximum dispatch distance arrived at:

$$z' = \frac{(\alpha/w - p)/f.}{(81)}$$

The goods can therefore bear the following additional dispatch expenses:

$$v = fz' = \alpha/w - p$$

which can be called the dispatch value. This is the amount by which the price $p$ at the place of origin stays below the price $\alpha/w$, for which the first unit is still purchased by the consumer who can buy, in accordance with his financial circumstances, at a priceworthiness of $w$.

The transport charges are estimated in proportion to the distance.
the goods will travel while – except for the 'differential tariffs' which are of no concern in this context – there are still regular additional costs irrespective of costs growing with the distance, which are independent of these, like packaging and loading, unpacking, storage and transport charges. Let us suppose that these charges which are independent of the distance have been included in the price \( p \) payable at the location of origin.

The goods will be sold everywhere within the maximum feasible distance, within the *dispatch borders*, so that the *market area* for a good which can be produced in a given location in unlimited quantity will form a circle, given that the respective economic conditions are equal in all directions, with a radius which equals the furthest dispatch distance. In the opposite direction, a location of consumption of goods, which depends on the expanse of the land, can be supplied from the total area right up to the dispatch borders. The area thus determined by both an area of selling and of buying is its *market area*.

When investigating the economic conditions in a market area one must differentiate strictly whether the area under investigation is for the sale of goods which can be produced in a single spot in any quantity regardless of the expanse of land around, as occurs in mining or mass production or the import business, or whether one deals with an area to be supplied, in which the quantity of the produced goods depends on the expanse of the land, as with the supply, to an area of consumption or a place of export, of agricultural produce and products of the timber industry.

Goods produced in a certain location have, depending on their value and weight, market areas of different sizes. The size of the market area is

\[
\pi z^2 = (\pi/P)(\alpha/w - p)^2
\]

and under otherwise equal conditions is inversely proportional to the square of the freight rate.

If the utility function is based once more on the previously used formula \( \alpha x - \alpha x^2 \), which can be supposed to be approximately correct, then for a consumer who can purchase at a priceworthiness of \( w \), demand at a price of \( p + fz \) becomes:

\[
x = (\alpha - (p + fz)w)/2\alpha.
\]
If the buyers are evenly distributed over the market area so that there are \( n \) buyers per surface unit, then total sales within the market area are:

\[
\frac{n\pi}{a_1} \int_0^{z'} \{z - (p + fz)w\} zdz
\]

that is:

\[
\frac{n\pi}{a_1} \{(\alpha - pw)z'^2/2 - fzw'^3/3\}
\]

or, because the longest dispatch distance is \( z' = (\alpha / w - p) / f \), or with the introduction of the dispatch value \( v = \alpha / w - p \), amounts to \( z' = v / f \) it follows that

\[
Q = \frac{n\pi w v^3}{6a_1 f^2}.
\]  

(83)

*Sales therefore increase in proportion to the cube of the dispatch value and inversely in proportion to the square of the freight rate.*

The weight to be transported per unit distance (ton kilometres) is:

\[
V = \frac{n\pi}{a_1} \int_0^{z'} \{z - (p + fz)w\} zdz
\]

that is:

\[
\frac{n\pi w v^4}{12 a_1 f^2}.
\]  

(84)

The number of transport units (ton kilometres) to be covered within a market area increases in proportion to the fourth power of the dispatch value and inversely in proportion to the cube of the freight rate.

The last two formulae demonstrate the extraordinary influence of improvement in the means of transport to the expansion of transport. Due to the building of roads, through which the freight rate compared to that on unmade roads was reduced to a third (from 75 pfennigs per ton kilometre to 25 pfennigs), the number of ton kilo-
metres travelled for the sale of goods increased 27 times; due to railways which charge one-sixth of road transport, or in the example of coal, only one-tenth, further increases the quantity of goods transported by 36, or 100 times, and an increase of ton kilometres of 216, and even up to 1000 times had to occur.

The average dispatch distance of goods in the dispatch area is
\[ z'' = \frac{V}{Q} = \frac{v}{2f} \]
that is equal to half the furthest dispatch distance. The average price of goods corresponding to this average dispatch distance is:

\[ p' = p + z''f = p + \frac{v}{2} \]

or, as \( v = \frac{\alpha}{w} - p \),

\[ p' = \left( \frac{\alpha}{w} + p \right)/2. \tag{85} \]

This means: *The average price at which buyers within the total market area obtain goods depends on the freight rate and represents the arithmetic mean of the price paid at the place of origin and that paid on the furthest borderline of the market area.*

When adding a profit margin, \( g \), to the production price, \( p \), of goods, the furthest dispatch distance decreases to \( (\alpha/w - p - g)/f \), the dispatch value decreases to \( v - g \) and total turnover becomes:

\[ Q = \frac{n\pi w (v - g)^3}{6\alpha_1 f^2} \]

at which a total profit will be obtained by the company of:

\[ G = \frac{n\pi wg(v - g)^3}{6\alpha_1 f^2}. \tag{86} \]

If the same number \( N \) of buyers were settled in close proximity to the origin instead of being spread across the entire market area, the demand would be:

\[ Q_0 = \frac{N(\alpha - (p + g)w)}{2\alpha_1} \]

or, as \( N = n\pi z'^2 = n\pi(\alpha/w - p - g)^2/f^2 \), it amounts to:

\[ Q_0 = \frac{n\pi w(\alpha/w - p - g)^3}{2\alpha_1 f^2} \]
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\[ = n\pi w(v - g)^{3/2}x_1\]

whereby a total profit is obtained of:

\[ G_0 = n\pi wg(v - g)^{3/2}x_1. \quad (87) \]

If the buyers are settled in close proximity to the origin of the goods instead of being distributed over the market area then sales and profits will increase threefold.

The higher the profit margin per unit, the more sales will decrease. If a businessman can exploit the production of certain goods as a monopoly then he will determine the profit margin in such a way that his total profit will reach a maximum. This most favourable profit margin is arrived at for sales within the market area by differentiation of equation (86) with respect to \( g \) at:

\[ g' = v/4 \]

whereby the total profit will amount to:

\[ 9n\pi vw^4/512x_1. \quad (88) \]

The number of buyers in the market would be \( N = 9n\pi v^2/16x_1 \). If this number of buyers were settled in the market centre, then the demand would be:

\[ Q_0 = Nx_1 - (p + g)w/2x_1 = 9n\pi v^2x_1 - (p + g)w/32x_1 \]

and the profit would be

\[ G_0 = 9n\pi v^2x_1/32x_1 \]

which reaches a maximum for:

\[ g'' = (\alpha/w - p)/2 = v/2 \]

that is, a maximum profit of:

\[ G'' = 9n\pi vw^4/128x_1. \quad (89) \]
In a business which is conducted as a monopoly, profit will quadruple provided buyers are not widely dispersed in a market, but are all settled at the location of origin of the goods.

In the case where buyers are all in the vicinity of production they will have to pay a price \( p + g'' = \frac{\alpha}{w + p}/2 \). If they were dispersed over a market area, they would have to pay on average half the maximum freight rate for the furthest distance, i.e. \( 3(\alpha/w - p)/8 \) and a price of \( p + g' = \frac{\alpha}{4w + 3p}/4 \), i.e. in total an average of \( \frac{5\alpha}{8w + 3p}/8 \), that is \( (\alpha/w - p)/8 \) more than if they lived at the point of origin of the goods.

Even in a case where the production of goods can be exploited as a monopoly, buyers of the goods will benefit if they are concentrated at the place of origin of the goods.

The three last formula are of far-reaching importance for the change occurring in the pattern of human settlement through the improvement of transport facilities, and discussion will return to this aspect later on.

If the quantity of goods produced cannot be increased to a level where it can satisfy the greater demand from a market area enlarged by improved transport facilities, then the profit margin will be raised until the market area shrinks to such an extent that the remaining demand can be satisfied. A profit surcharge \( g \) would necessitate a quantity of goods:

\[
Q = \frac{n\pi w(v - g)^{3}}{6\alpha f^{2}}
\]

from which the profit margin is:

\[
g = v - \left( \frac{6\alpha fQ}{n\pi w} \right)^{1/3}.
\] (90)

It is obvious that this profit margin will increase dramatically with a decrease in the freight rate. Finally, it should be pointed out that for perishable goods like fish, fruit, vegetables, and milk a shortening of the time necessary for transport has the same effect as a fall in the freight rate. Some of the goods in this category, such as oysters and fresh fish the quantity of which cannot infinitely be extended, had to experience a considerable price rise through improved transport facilities and thus give their producers a considerably higher profit.