Polarization of Light

Thursday, 11/09/2006

Physics 158

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Class Outline

- Polarization of Light
- Polarization basis'
- Jones Calculus

Polarization

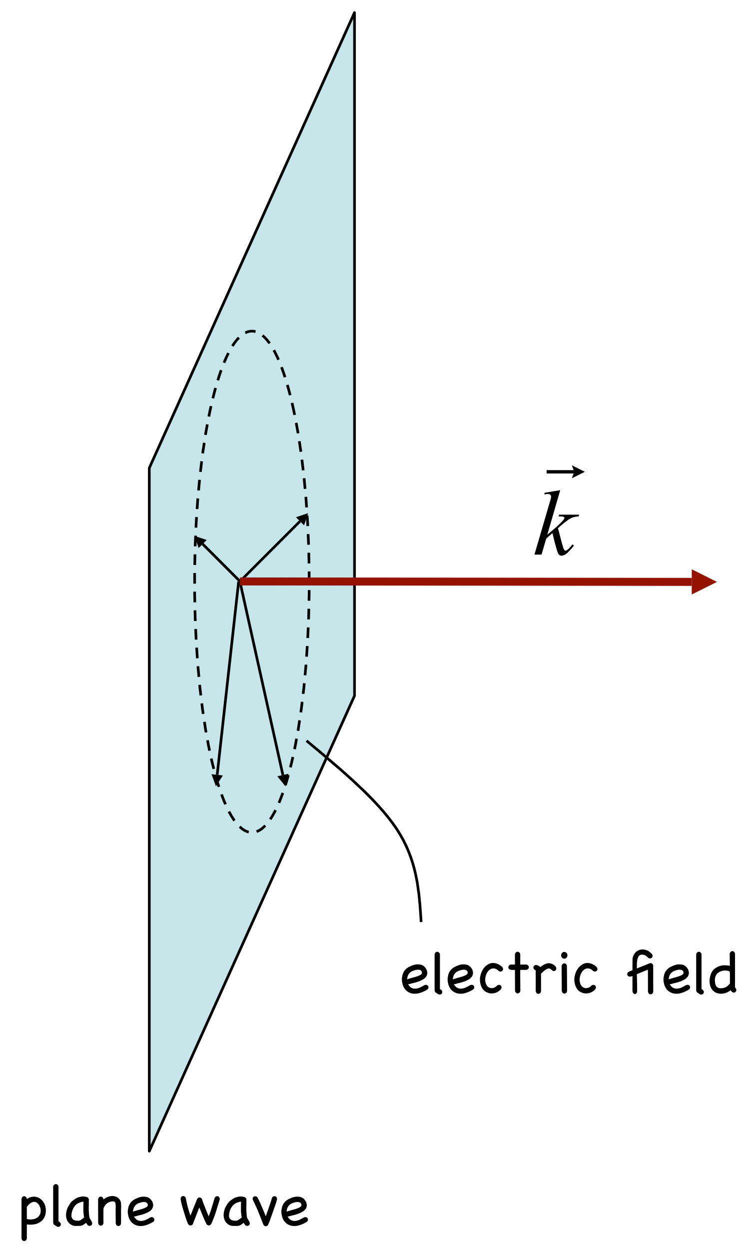
- The Electric field direction defines the polarization of light
- Since light is a transverse wave, the electric field can point in any direction transverse to the direction of propagation
- Any arbitrary polarization state can be considered as a superposition of two orthogonal polarization states (i.e. it can be described in different bases)

Electric Field Direction

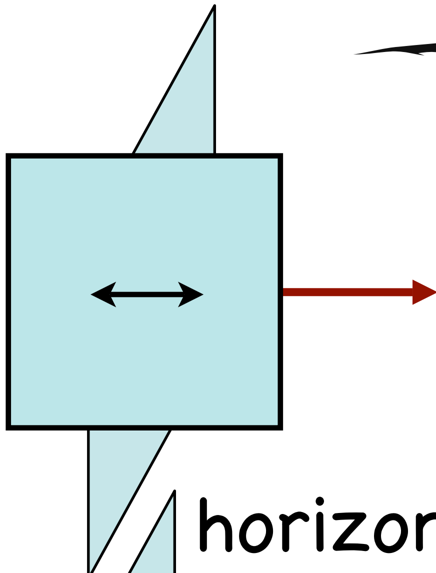
Light is a transverse electromagnetic wave so the electric (and magnetic) field oscillates in a direction transverse to the direction of propagation

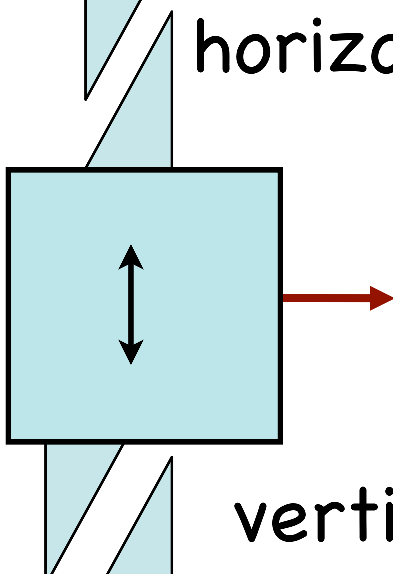
Possible states of electric field polarization are

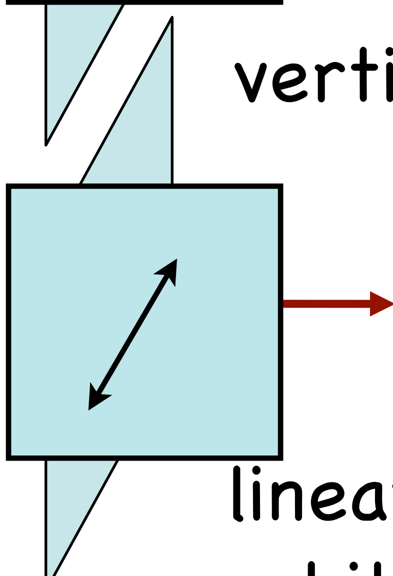
- Linear
- Circular
- Elliptical
- Random

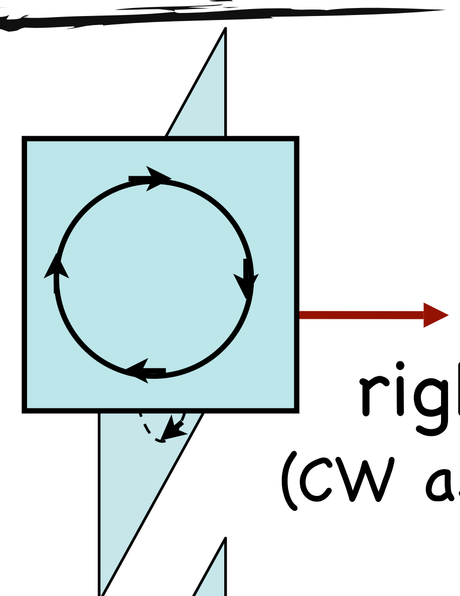


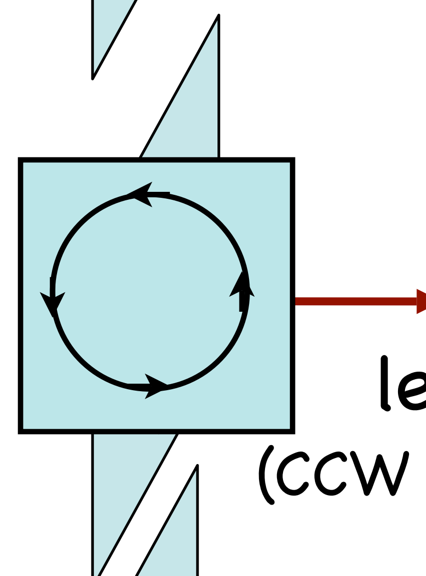
Examples of polarization states

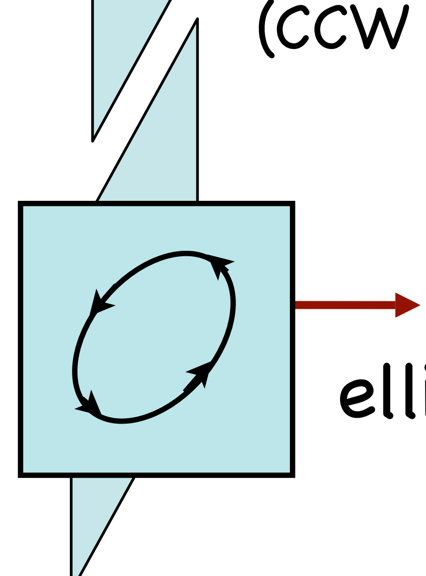

horizontal


vertical


linear polarization at an arbitrary angle


right hand circular
(CW as seen from observer)


left hand circular
(CCW as seen from observer)


elliptical

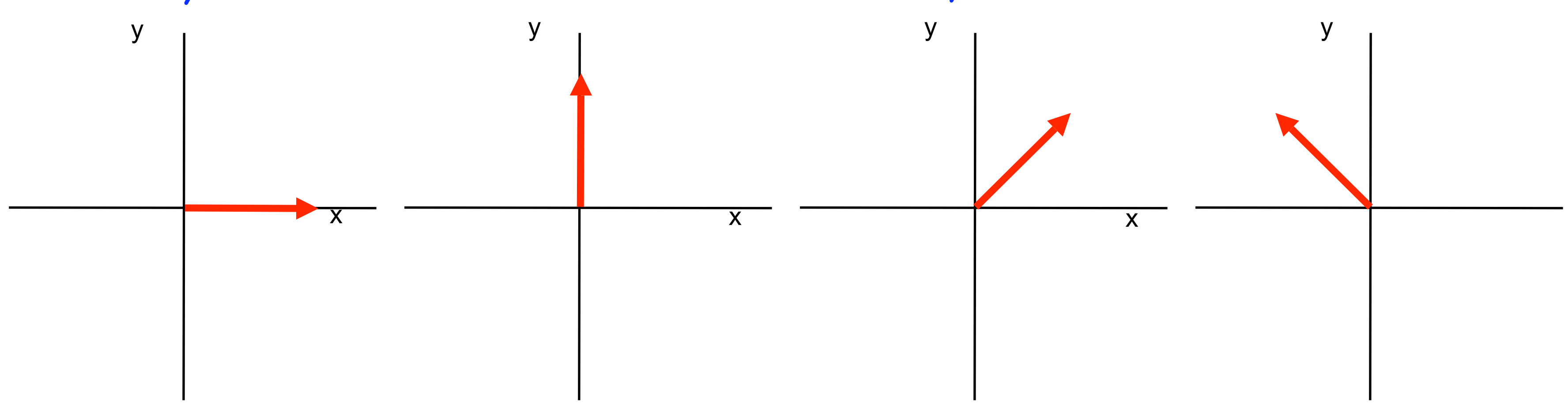
Linear Polarization Basis

Any polarization state can be described as the sum of two orthogonal linear polarization states

$$\widetilde{E}_x(z,t) = E_{0x}\hat{i}e^{i(kz-\omega t+\phi_x)}$$
$$\widetilde{E}_y(z,t) = E_{0y}\hat{j}e^{i(kz-\omega t+\phi_y)}$$

$$\widetilde{E}(z,t) = \widetilde{E}_x(z,t)+\widetilde{E}_y(z,t) = \left(E_{0x}e^{i\phi_x}\hat{i}+E_{0y}e^{i\phi_y}\hat{j}\right)e^{i(kz-\omega t)}$$

$E_{0y}=0$ | $E_{0x}=0$ | $E_{0y}=E_{0x}$ | $E_{0y}=-E_{0x}$



horizontal | vertical | diagonal $\varphi_x=\varphi_y$ | diagonal $\varphi_x=\varphi_y+\pi$

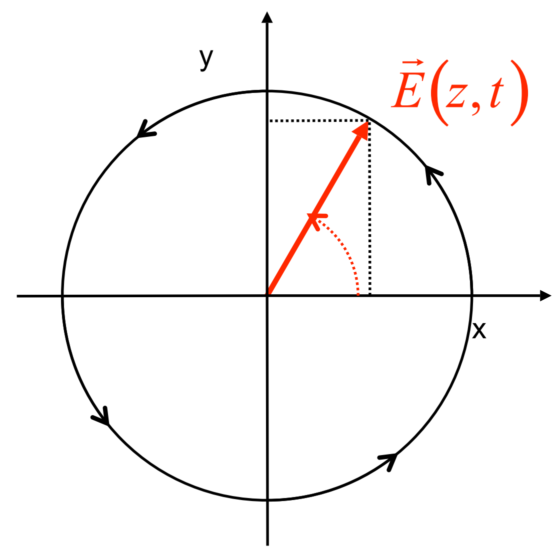
Circular Polarization

$$\widetilde{E}(z,t) = \left(E_{0x}e^{i\phi_x}\hat{i} + E_{0y}e^{i\phi_y}\hat{j}\right)e^{i(kz-\omega t)}$$

For the case $|\varphi_x-\varphi_y|=\pi/2$ the magnitude of the field doesn't change, but the direction sweeps out a circle

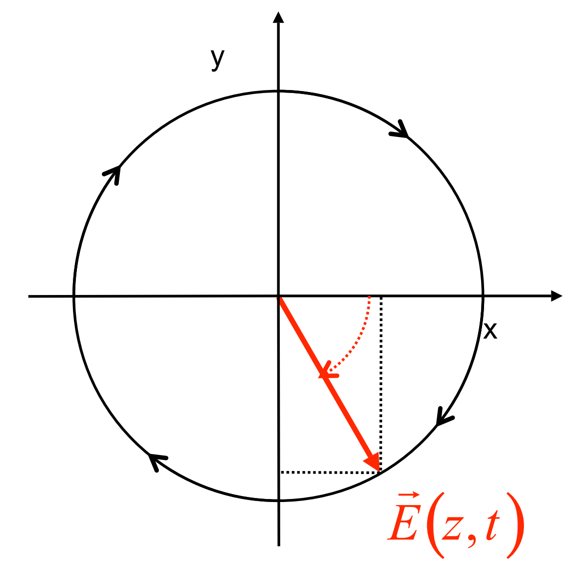
The polarization is said to be right-handed if it progresses clockwise as seen by an observer looking into the light. Left-handed if it progresses counterclockwise

Left-Handed



$\varphi_y-\varphi_x=\pi/2$

Right-Handed



$\varphi_y-\varphi_x=-\pi/2$

Circular Polarization basis

Circular (as well as any arbitrary) polarization can be described as a linear combination of orthogonal linear polarization states

$$E_{rhc} = \begin{bmatrix} iE_x \\ E_y \end{bmatrix} \qquad E_{lhc} = \begin{bmatrix} E_x \\ iE_y \end{bmatrix}$$

alternatively we can define linear (or any arbitrary) polarization as the sum of orthogonal circular polarization states

$$E_x = (E_{lhc} - iE_{rhc})/2$$

$$E_y = (E_{lhc} + iE_{rhc})/2i$$

Elliptical Polarization

Linear and Circular polarization are just special cases of elliptical polarization

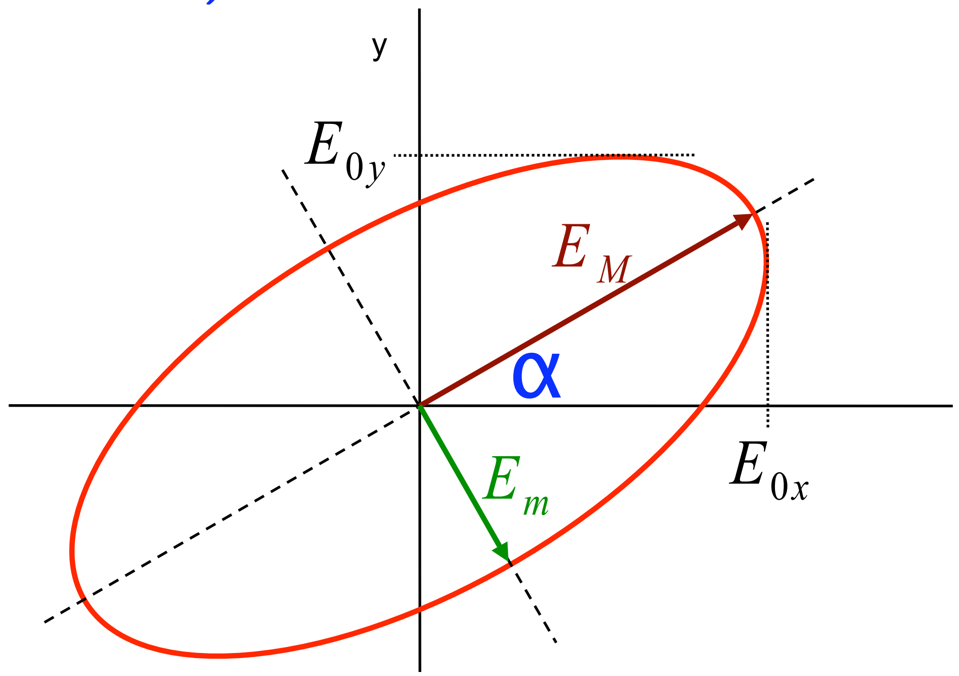
$$\widetilde{E}(z,t) = \left(E_{0x}e^{i\phi_x}\hat{i} + E_{0y}e^{i\phi_y}\hat{j}\right)e^{i(kz-\omega t)}$$

The orientation of the major axis is

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos(\Delta\phi)}{E_{0x}^2 - E_{0y}^2}$$

The ratio of the major to minor axes is

$$e = \frac{-E_{0x}\sin(\Delta\phi) + E_{0y}\cos(\Delta\phi)\cos(\Delta\phi)}{E_{0x}\cos(\Delta\phi) + E_{0y}\sin(\Delta\phi)\cos(\Delta\phi)}$$



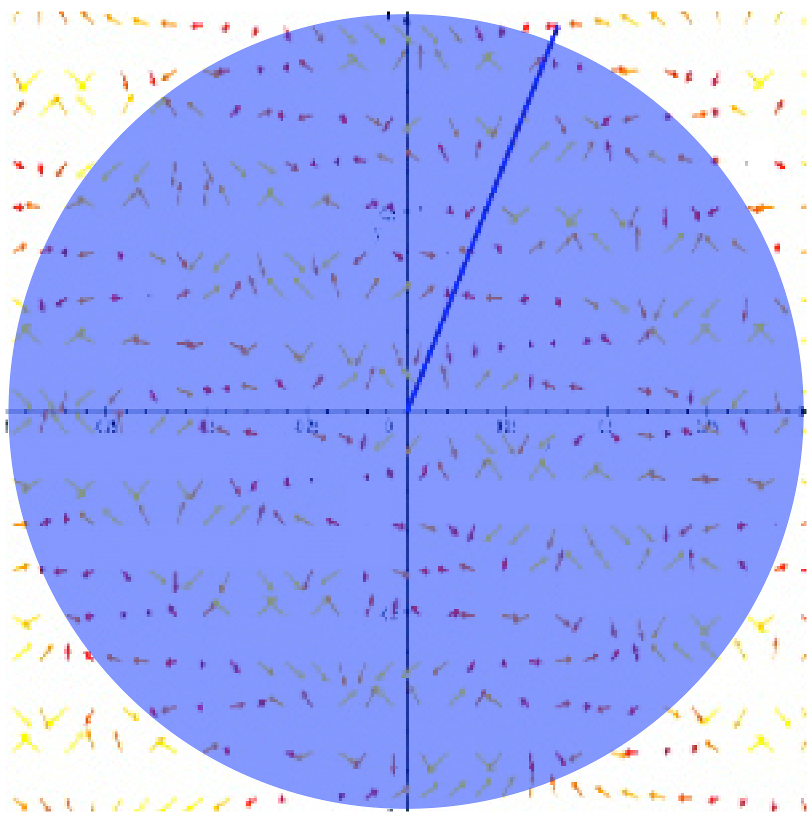
“Unpolarized” light

Most sources of natural light are “unpolarized”

Randomly oriented dipoles radiate for about 10ns at a time before collisions cause the phases to change. The sum or the radiation from all dipoles is polarized in a direction that depends on the relative phases of the dipoles, which changes every 10 ns or so (called the coherence time).

When viewed at time scales greater than the coherence time the radiation appears unpolarized

“Unpolarized” light



Describing Polarization Mathematically

- Since polarization, like vectors, can be described by the value of two orthogonal components, we use vectors to represent polarization
- Amplitude of the components can be complex to represent a time delay between the components of the waves
- Jones calculus allows us to keep track of the polarization of waves as they propagate through a system

Jones Vectors

Expressing polarization in terms of two orthogonal states with complex amplitude (i.e. amplitude and phase) of each component expressed in vector form

vertical	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
horizontal	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
linear at +45°	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
linear at θ	$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$
right circular	$\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix}$
left circular	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$

Jones Matrices

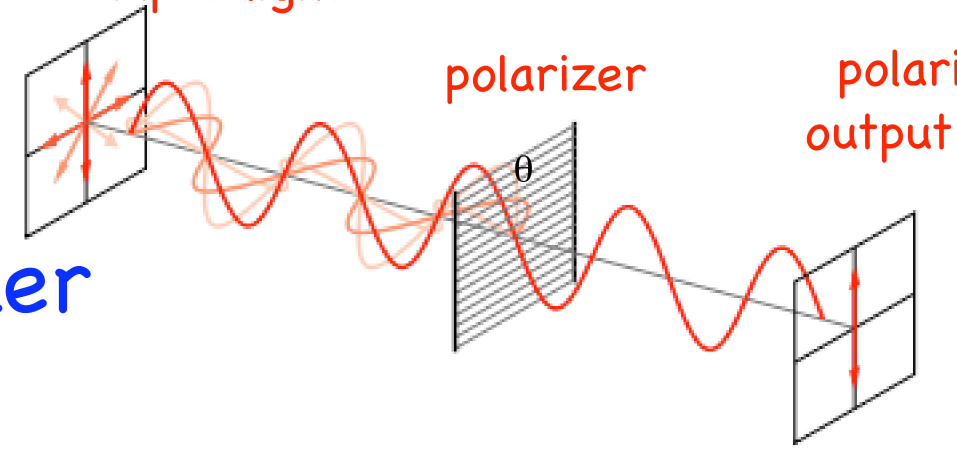
An optical element that transforms one polarization state into another can be treated as a 2x2 matrix acting on a jones vector

$$\begin{bmatrix} E_{x,out} \\ E_{y,out} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$

The Jones Matrices for a series of optical elements can be multiplied together to find how the optical system transforms the polarization of an input beam

Polarizers

Selectively attenuates one polarization

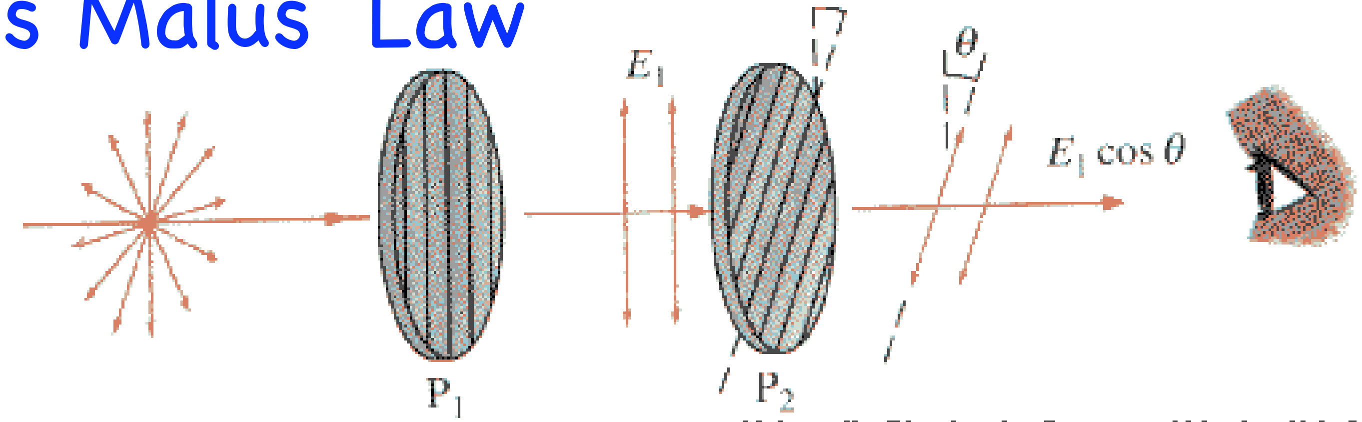


Wire Grid Polarizer

Malus discovered that an analyzer oriented at θ with respect to a polarizer would have atransmission of

$$I(\theta) = I_0 \cos^2 \theta$$

known as Malus' Law



University Physics by Sanny and Moebs, Vol. 2

Jones Matrix of a Polarizer

What is the Jones matrix for a polarizer that transmits horizontal polarization?

$$\begin{bmatrix} E_{x,out} \\ 0 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$

What if it is rotated at an angle θ?

$$R(\theta) M R(-\theta) \qquad R(\theta) \equiv \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Use this to verify Malus' Law

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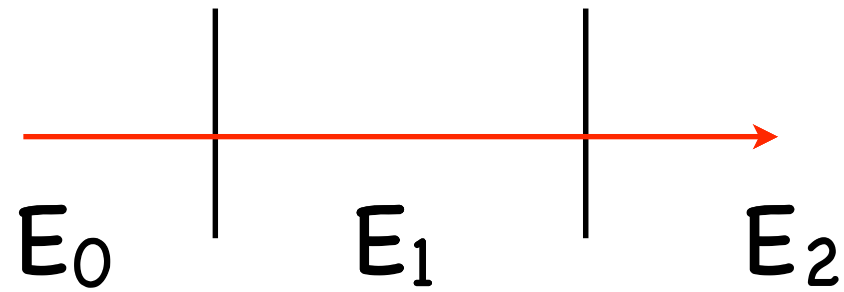
Malus' Law



$$\begin{bmatrix} E_{x,1} \\ E_{y,1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_{x,0} \\ E_{y,0} \end{bmatrix} = \begin{bmatrix} E_{x,0} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} E_{x,2} \\ E_{y,2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} E_{x,1} \\ E_{y,1} \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} E_{x,2} \\ E_{y,2} \end{bmatrix} &= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} E_{x,0} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta E_{x,0} \\ \sin\theta E_{x,0} \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta E_{x,0} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\theta E_{x,0} \\ -\sin\theta\cos\theta E_{x,0} \end{bmatrix} \end{aligned}$$

$$\frac{|E_2|}{E_{0,x}} = \cos\theta\sqrt{\cos^2\theta + \sin^2\theta} = \cos\theta \longrightarrow \frac{I_2}{I_1} = \cos^2\theta$$

Retarders

Devices which delays one polarization component with respect to the other

For a **birefringent** material of thickness d

$$\Delta\phi = (kn_y d - kn_x d) = \frac{2\pi}{\lambda}\Delta n d$$

Thus the Jones matrix is

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{\Delta\phi} \end{bmatrix}$$

Quarter Wave Plate

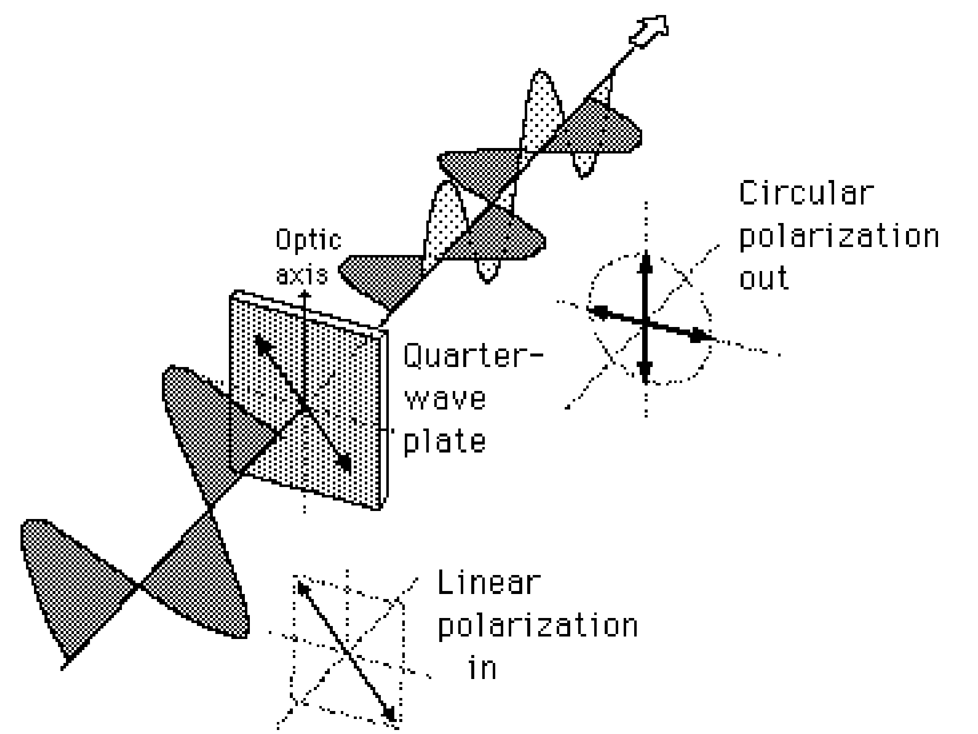
For a retarder with Δφ=π/2 (i.e. a retardation of λ/4)

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

And when a wave with linear polarization at ±45° passes through the retarder it gets converted to left (right) circular polarization

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}i}\begin{bmatrix} i \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



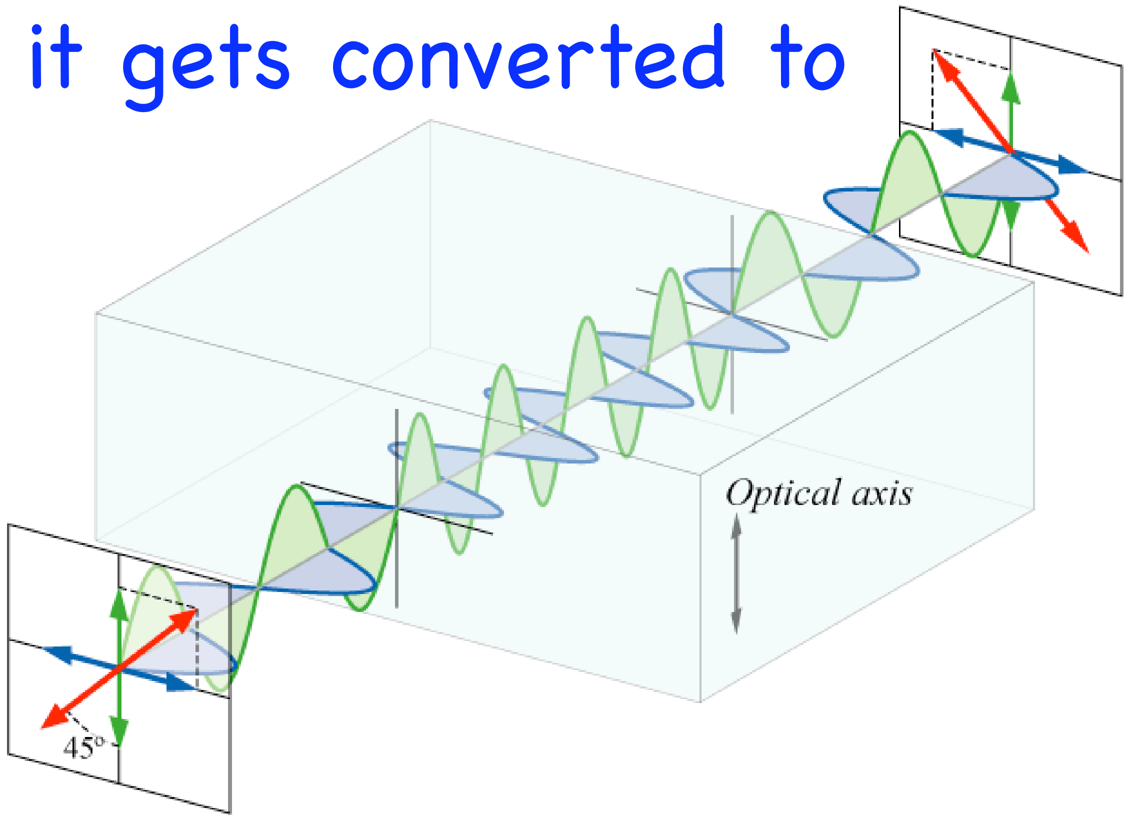
Half Wave Plate

For a retarder with Δφ=π (i.e. a retardation of λ/2)

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

And when a wave with linear polarization at θ passes through the retarder it gets converted to linear polarization at -θ

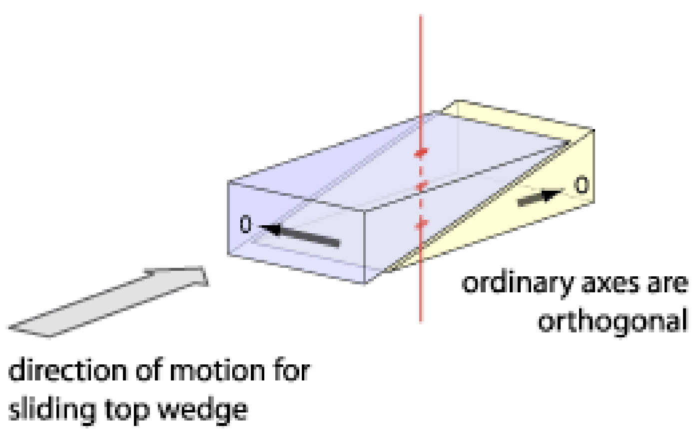
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} = \begin{bmatrix} \cos(-\theta) \\ \sin(-\theta) \end{bmatrix}$$



Compensator

Variable waveplate with Jones matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{\Delta\phi} \end{bmatrix}$$



Babinet compensator

Berek compensator

Waveplate Order

Recall that the relative delay between the two polzrization states is

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta n d$$

For a zero-order quartz waveplate if we wish $\Delta\varphi \approx \pi$ we need $d \approx 100\mu m$

For a typical multi-order waveplate $\Delta\varphi \approx 11\pi$ so $d \approx 1mm$ which is easier to manufacture and has the same retardation (modulo 2π) as a zero order waveplate

Waveplate Order

dispersion of a waveplate is the sensitivity of the retardation to wavelength

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta n d$$

$$\frac{d\Delta\phi}{d\lambda} = -\frac{2\pi}{\lambda^2}\Delta n d$$

which is minimized in a zero order waveplate.

Typical zero-order waveplates are actually made of two multiorder waveplates cemented together oriented at 90° so that $\Delta\varphi=\Delta\varphi_1-\Delta\varphi_2\approx\pi$ but the total thickness $d\approx 2$mm

Summary

- Polarization of light can be described as linear, circular, elliptical or unpolarized
- unpolarized light is really just polarized light that has the polarization changing very rapidly
- Any type of polarization can be described in terms of any other polarization basis
- Jones vectors mathematically describe polarized light, Jones matrices describe the action of optical elements on polarized light
- Various elements are used to manipulate the polarization state of light