Mathematical Description of Light

Thursday, 8/24/2006
Physics 158
Peter Beyersdorf

Class Outline

- Introductions/Announcements
- Properties of light
- Mathematical description of transverse waves

Introductions

- Instructor: Dr. Peter Beyersdorf
  - Office: Science 235
  - email: pbeyersdorf@science.sjsu.edu
  - Phone: 924-5236
- Office Hours
  - Tuesday and Thursday from 9-10:15 am in my office
  - Tuesday and Thursday from Noon-1 pm at student union (by Jamba Juice)
- Webpage:
- Textbook: Hecht, "Optics, 4th edition",

Modern Optics provides a framework for understanding and analyzing optical wave propagation, interference, polarization and diffraction effects. This framework is essential for those who will work with imaging systems, lasers, optical communication systems, and optical measurements.

Course Grading

Your grade will come from a combination of homework (20%), midterms (20% each) and the final exam (40%)

- Homework (your lowest score will be dropped)
- Midterm 1
- Midterm 2
- Final Exam

You will receive a normalized score (based on a curve) for each component listed above.

It is your responsibility to convince me you have a conceptual understanding of the subject matter. When you solve problems, presenting your work in a clear, concise manner that shows the logical steps leading to the final answer will go along way towards this objective. Work will be graded on the quality of your solution, not just the correctness of your answer.

Class Resources

- Class web site:
  - Login required
    - Login with your student ID# but use an upper case 'W' in place of the leading two zeros. Example (W4821234)
    - Password default is "fall". You can change this once you log in.
  - View and download lectures and lecture notes
  - Browse and post in the discussion group
  - View your grades
  - Access any other class related information
- Class podcasts on iTunesU

Policies and Rules

- College and Departmental Policies
  - Students who wish to add or change lab sections must first obtain written permission from the lab instructor indicating that a lab space is available in the session for which the instructor is responsible.
  - You are responsible for understanding the policies and procedures about add/drops, academic renewal, withdrawals, incompletes, classroom behavior, and other policies described in the catalog. Please read your catalog thoroughly.
- Class Rules
  - Place your personal electronics in quiet mode, and refrain from using them in the classroom for non-class related work.
  - I encourage you to work on homework assignments in groups. You are able to learn much more from each other than you can from me, and you will find that if you take the time to help your classmates you will develop a better understanding of the material yourself. Of course I am also available, and am happy to meet with you during my scheduled office hours, and am available on-and-off outside of these hours - just stop by my office.
  - It's my job to help you learn. Help me help you - attend class, participate in discussions and problem solving sessions, discuss with me problems you are having and give me lots of feedback so I can teach more

What is Light?

- It is a wave
  - propagates as a disturbance in the electric and magnetic fields
  - can be described by a wavelength, frequency, and other wave-like properties
- It is made of particles
  - It interacts in a discreet manner
  - Its energy and momentum are quantized

What is light?

Maxwell's equations give rise to the "wave equation" that describes the propagation of electromagnetic waves. In one-dimension and in free-space this can be expressed as:

$$\frac{\partial^2 \vec{E}(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \vec{E}(x,t)}{\partial t^2}$$

Which has solutions that are traveling waves

$$\vec{E}(x,t) = \vec{E}_0 \cos\left(kx - \omega t + \phi_0\right)$$

What is light?

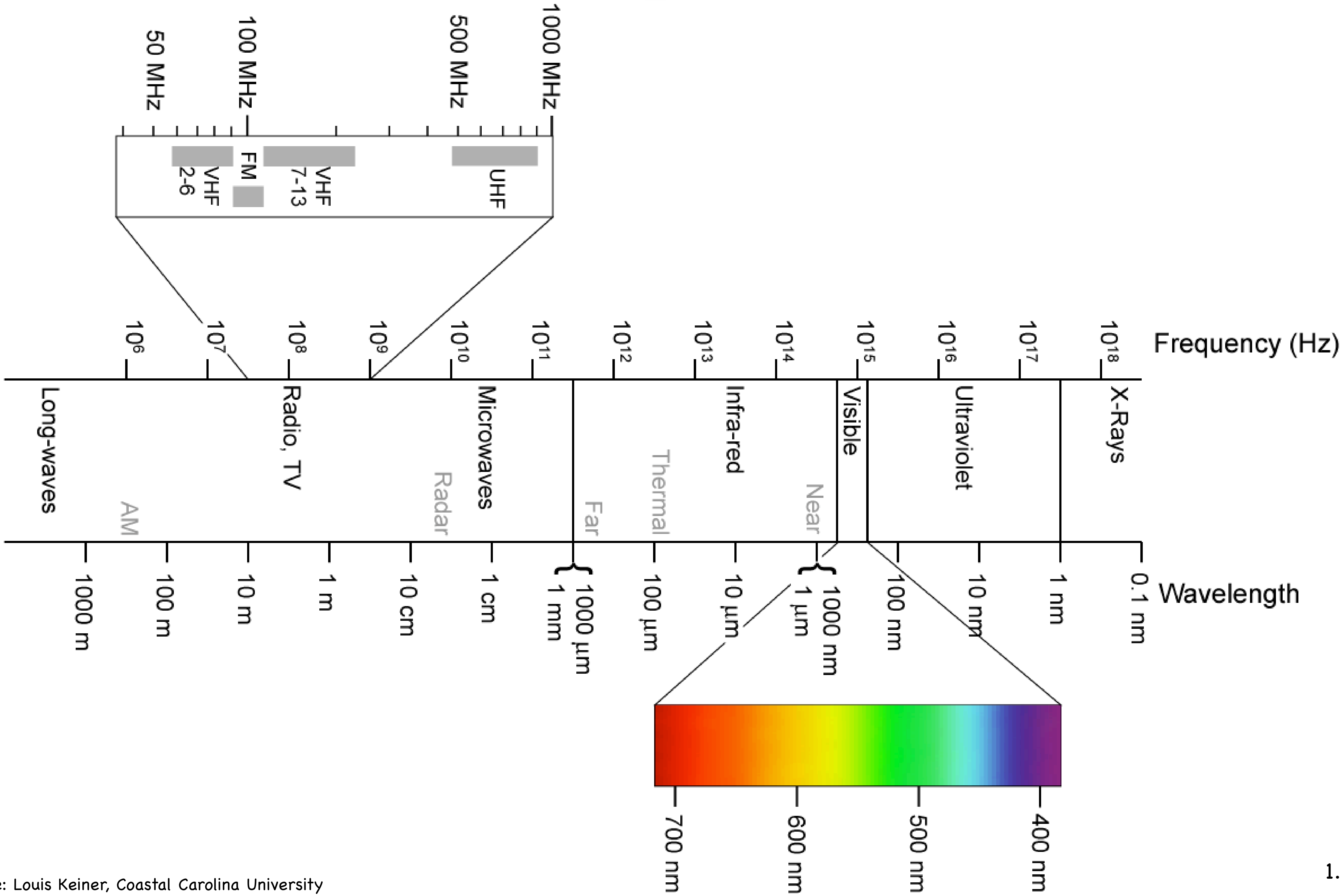
Like all electromagnetic waves, light

- is a transverse electromagnetic wave
- is a solution to the wave equation
- propagates at a velocity of c (in free space)
- is made up of quantum-mechanical particles called photons

What differentiates light from other forms of electromagnetic waves (microwaves, radio waves, etc), light

- The frequency of oscillation of light is too high for the changing electric & magnetic fields to be directly observed
- Only the average intensity of the electromagnetic wave can be observed

The Electromagnetic Spectrum



Source: Louis Keiner, Coastal Carolina University


FoxTrot
by Bill Amend
DO YOU SELL ULTRAVIOLET BULL?
EXCUSE ME?
WHAT ABOUT REGULAR VIOLET BULL?
NO.
BLUE BULL?
NO.
GREEN BULL?
NO.
YELLOW BULL?
NO.
ORANGE BULL?
NO.
www.foxtrot.com
RATS. OK, I'LL TAKE A RED BULL.
I WAS HOPING FOR SOMETHING A LITTLE HIGHER-ENERGY.
AT LEAST IT ISN'T INFRARED BULL.
AMEND 9-19

Properties of light

What are some distinguishing properties of a light wave?

- Wavelength (frequency)
- Direction of propagation (spatial mode)
- Irradiance (amplitude)
- Polarization
- Phase

Describing EM waves

- Amplitude E_0
- Polarization $\hat{E}_0$
- Direction of propagation $\hat{k}$
- Wavelength $\lambda = 2\pi/k$
- Frequency $f = \omega/2\pi$
- Initial phase ϕ_0

Expressions for a monochromatic wave

$$\vec{E}(x,t) = \vec{E}_0 \cos\left(kx - \omega t + \phi_0\right)$$

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cos\left(\vec{k}\cdot\vec{r} - \omega t + \phi_0\right)$$

useful relations

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

$$c = \frac{\omega}{k} = \lambda f$$

Complex representation of waves

Since manipulating trigonometric functions is difficult, it is common practice to express a wave in complex form instead

$$\vec{E}(\vec{r},t) = \Re\{\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_0)}\}$$

The time (and/or space) dependance is often omitted as it is understood the wave is oscillating at frequency ω, and so it can always be accounted for by adding it in explicitly after calculating how the wave evolves in space

$$\vec{E}(\vec{r}) = \Re\{\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}+\phi_0)}\}$$

Also, it is understood that when expressing a wave in complex form, the actual wave is the real part of the expression.

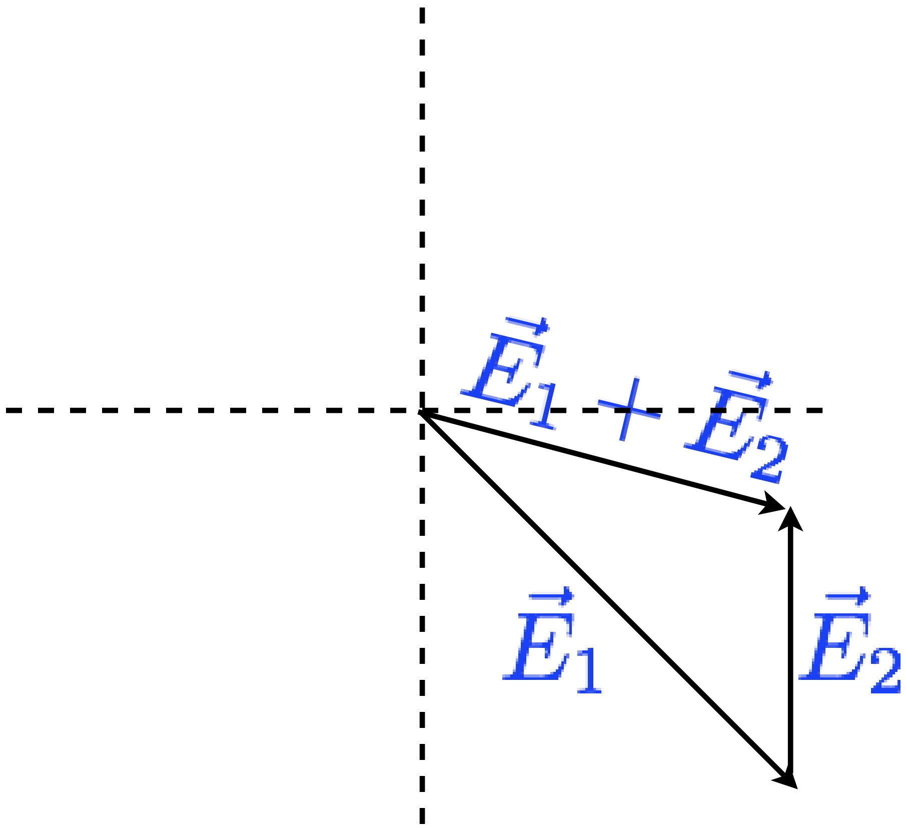
$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}+\phi_0)}$$

This form is referred to as the phasor representation of the wave

Phasor Example

- Two waves of equal frequency copropagate in the +x direction. One has an amplitude of 2 units and a (initial) phase of $-\pi/4$, while the other has an amplitude of 1 unit and a (initial) phase of $\pi/2$
  - Express their sum using phasors
  - Express their sum using the trigonometric expression

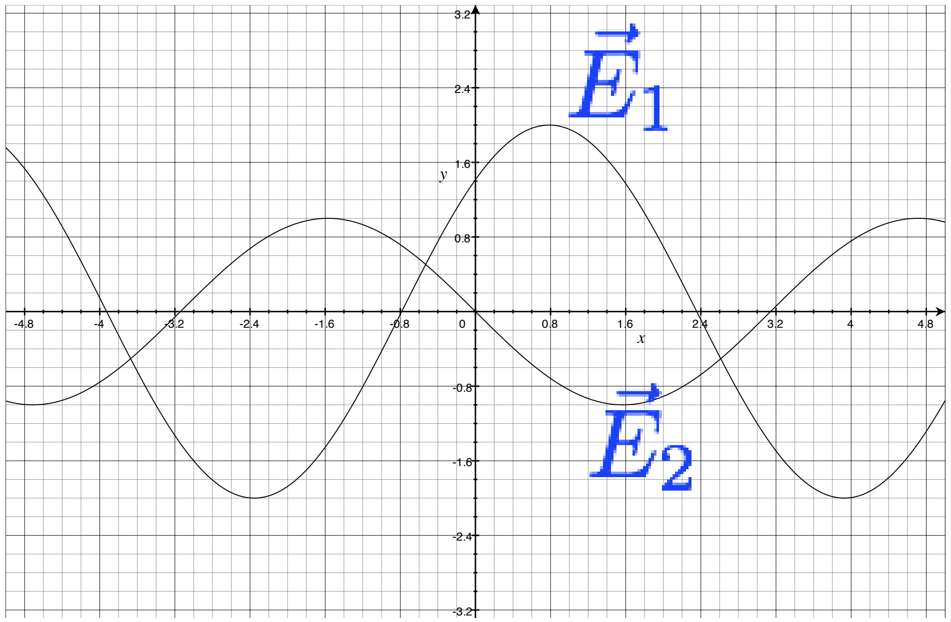
Phasor Example



$$
\begin{aligned}
\vec{E}_1 + \vec{E}_2 &= [2\cos(-\pi/4) + 1\cos\pi/2] \\
&+ i[2\sin(-\pi/4) + 1\sin\pi/2]
\end{aligned}
$$

$$\vec{E}_1 + \vec{E}_2 = 1.47e^{i(kx-0.28)}$$

Phasor Example



$$\begin{aligned}
\vec{E}_1 + \vec{E}_2 &= 2\cos(kx - \omega t - \pi/4) + \cos(kx - \omega t + \pi/2) \\
&= \cos(kx - \omega t)\left(\cos(-\pi/4) + \cos(\pi/2)\right) - \sin(kx - \omega t)\left(\sin(-\pi/4)\sin(\pi/2)\right) \\
&= \ldots
\end{aligned}$$

tedious use of trig relations makes this difficult

Irradiance of waves

- The oscillations of the electric and magnetic fields are too fast in an optical wave to be directly measured. We can only observe the power delivered by the wave
- Power per unit area is called irradiance and is proportional to the square of the(electric or magnetic) field
- The irradiance is given by

$$I(t) \propto < E(t)^2 > = \frac{1}{T}\int_{T-t}^{T} E^2(t')dt'$$

or in the phasor picture, by

$$I \propto |E|^2 = EE^* = E^*E$$

Phasor Example 2

A wave given by

$$\vec{E}_1(x,t) = \vec{E}_0 \cos\left(kx + \omega_1 t\right)$$

adds with a wave that is expressed by

$$\vec{E}_2(x,t) = \vec{E}_0 \cos\left(kx + \omega_2 t\right)$$

- Which direction are these waves going?
- Write an expression for the irradiance of the sum

Phasor Example 2

Using the phasor representation

$$\vec{E}_1(x,t) = \vec{E}_0 e^{i(kx+\omega_1 t)} \qquad \vec{E}_2(x,t) = \vec{E}_0 e^{i(kx+\omega_2 t)}$$

$$\begin{aligned}
\vec{E}_1(x,t) + \vec{E}_2(x,t) &= \vec{E}_0 e^{i(kx)} \left(e^{i\omega_1 t} + e^{i\omega_1 2t}\right) \\
&= \vec{E}_0 e^{ikx} e^{i(\omega_1+\omega_2)t/2} \Big(e^{i(\omega_1-\omega_2)t/2} \\
&\quad + e^{-i(\omega_1-\omega_2)t/2}\Big) \\
&= \vec{E}_0 \left(2\cos\left(\frac{\omega_1 - \omega_2}{2} t\right)\right) e^{ikx+i(\omega_1+\omega_2)t/2}
\end{aligned}$$

$$I \propto E^* E = 4|E_0|^2 \cos^2\left(\frac{\omega_1 - \omega_2}{2} t\right)$$

or

$$I \propto E^* E = 4 I_0 \cos^2\left(\frac{\omega_1 - \omega_2}{2} t\right)$$ with $$I_0 \equiv |E_0|^2$$

Example Problems

- A car radio antenna has a length that is $\lambda/4$ which gives the optimal reception efficiency. If the length of the antenna is 76cm what station is it optimized for?
- Show that $E(x,t) = E_0 e^{i2\pi(x/\lambda - ft)}$ is a solution to the 1-dimensional wave equation. What is the velocity of this wave?