Geometrical versus Physical optics

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Physics 158
Peter Beyersdorf
Class Outline

- Geometric optics versus physical optics
- Diffraction (Physical Optics)
  - Single slit calculation
  - Double slit calculation
- Snell’s law (Geometric optics)
Geometric Optics

- Light is treated as traveling as rays
- What are consequences of this?
  - Shadows are 1:1 mapping of the obstruction
  - Beams of light can propagate without diverging
- When is this a valid assumption?
Field at any point is considered the sum of the contributions from fields at all other points in space (i.e. light does not travel as rays)

What are the consequences of this?

**Diffraction**

- Shadows do not have sharp edges
- Beams of light diverge

When must we consider this formalism?
Huygen's Analysis of Diffraction

Consider a slit of width $d$ illuminated with a plane wave at normal incidence. What does its “shadow” look like at a distance $L$ where $L \gg d$?
Huygen’s Principle

Diffraction as considered by Huygen’s principle

Source: wikipedia
Huygen’s Analysis of Diffraction

\[ \vec{E}(x) = \int_{-d/2}^{d/2} \frac{E(x')}{\sqrt{L^2 + (x-x')^2}} K(x', x) dx' \]

\[ \vec{E}(x) = \frac{E_0}{L} e^{-ikL} e^{-ikx^2/2L} \int_{-d/2}^{d/2} e^{ikx'}/L dx' \]

\[ \vec{E}(x) = E_0 \frac{d}{L} e^{-ikL} e^{-ikx^2/2L} \text{sinc} \left( \frac{kxd}{2L} \right) \]

\[ \vec{I}(x) = I_0 \frac{d}{L} \text{sinc}^2 \left( \frac{kxd}{2L} \right) \]

\[ \sqrt{L^2 + (x-x')^2} \approx L \]

Since small changes to the value of the denominator hardly affect the value of the function

\[ K(x', x) = e^{-ik\left(\sqrt{L^2+(x-x')^2}\right)} \]

is called a Kernel and is a common mathematical tool for expressing how a quantity at one point in space and time affect other points

\[ \text{sinc}(u) \equiv \frac{\sin u}{u} \]

This is such a common function in optics it has been given its own name! (pronounced “sink”)

\[ \sqrt{L^2 + (x-x')^2} \approx L \left(1 + \frac{(x-x')^2}{2L}\right) \approx L + \frac{x^2}{2L} + \frac{xx'}{L} \]

Since \( L >> x' \) we can approximate this function by the first order expansion
Huygen’s Analysis of Diffraction

Consider a slit of width $d$ illuminated with a plane wave at normal incidence. What does its “shadow” look like at a distance $L$ where $L \gg d$?

This is not predicted by geometric optics!
Based on the diffraction from a slit found using Huygen’s principle, how would you set up a calculation of the diffraction pattern for a double slit as shown?
Young's Double Slit

\[ \vec{E}(x') = \frac{\vec{E}_A}{\sqrt{L^2 + x'^2}} e^{-ik\sqrt{L^2 + x'^2}} \]

\[ \vec{E}(x) = \int_{-D/2-d/2}^{-D/2+d/2} \frac{\vec{E}(x')}{\sqrt{L^2 + (x + D/2)^2}} e^{-ik\sqrt{L^2 + (x + D/2)^2}} dx' + \int_{D/2-d/2}^{D/2+d/2} \frac{\vec{E}(x')}{\sqrt{L^2 + (x - D/2)^2}} e^{-ik\sqrt{L^2 + (x - D/2)^2}} dx' \]

Later we’ll learn some tricks that will allow us to evaluate \( E(x) \),
For now consider the slits to be infinitely narrow...
Young's Double Slit

Considering
infinitesimal slits
with uniform illumination $E(x') = E_0$

\[
\vec{E}(x) = \frac{\vec{E}(-D/2)}{\sqrt{L^2 + (x + D/2)^2}} e^{-ik\sqrt{L^2 + (x + D/2)^2}} + \frac{\vec{E}(D/2)}{\sqrt{L^2 + (x - D/2)^2}} e^{-ik\sqrt{L^2 + (x - D/2)^2}}
\]

\[
\vec{E}(x) \approx \frac{\vec{E}_0}{\sqrt{L^2 + x^2}} \left( e^{-ik(L + (x + D/2)^2/2L)} + e^{-ik(L + (x - D/2)^2/2L)} \right)
\]

\[
\vec{E}(x) \approx \frac{2\vec{E}_0}{\sqrt{L^2 + x^2}} e^{-ik(L + (x^2 + D^2/4)/2L)} \cos \left( \frac{kxD}{2L} \right)
\]
Young's Double Slit

Considering infinitesimal slits with uniform illumination $E(x')=E_0$

\[
\vec{E}(x) \approx \frac{2\vec{E}_0}{\sqrt{L^2 + x^2}} e^{-ik(L+(x^2+D^2/4)/2L)} \cos \left( \frac{kxD}{2L} \right)
\]

\[
I(x) \approx \frac{I_0}{\sqrt{L^2 + x^2}} \cos^2 \left( \frac{kxD}{2L} \right)
\]
Snell’s law

Why does the spoon appear bent in this image?

Determine a relationship between the angle of the spoon and its apparent angle in a glass of water.
Snell’s law example

\[ n_{air} \sin \theta_t = n_{water} \sin \theta_i \]

\[ y = d \tan \theta_i = d' \tan \theta_t \]

\[ \frac{d'}{d} = \frac{\tan \theta_i}{\tan \theta_t} \approx \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_{air}}{n_{water}} \]

\[ \tan \theta_a = \frac{x}{h} \]

\[ \tan \theta_w = \frac{x}{h} \left( \frac{n_{air}}{n_{water}} \right) \]

\[ \tan \theta_w = \tan \theta_a \left( \frac{n_{air}}{n_{water}} \right) \]

Apparent distances get compressed in the direction normal to the interface.
Using the law of reflection, determine the angle for a ray of light exiting a corner cube reflector as a function of the incident angle.
Using the law of reflection, determine the angle for a ray of light exiting a corner cube reflector as a function of the incident angle.
Summary

Some phenomena can only be understood by considering physical optics

- diffraction

Many phenomena are well modeled by considering geometric optics

- refraction