Electrooptic Modulators for Cavity Locking

Case Study

Physics 208, Electro-optics

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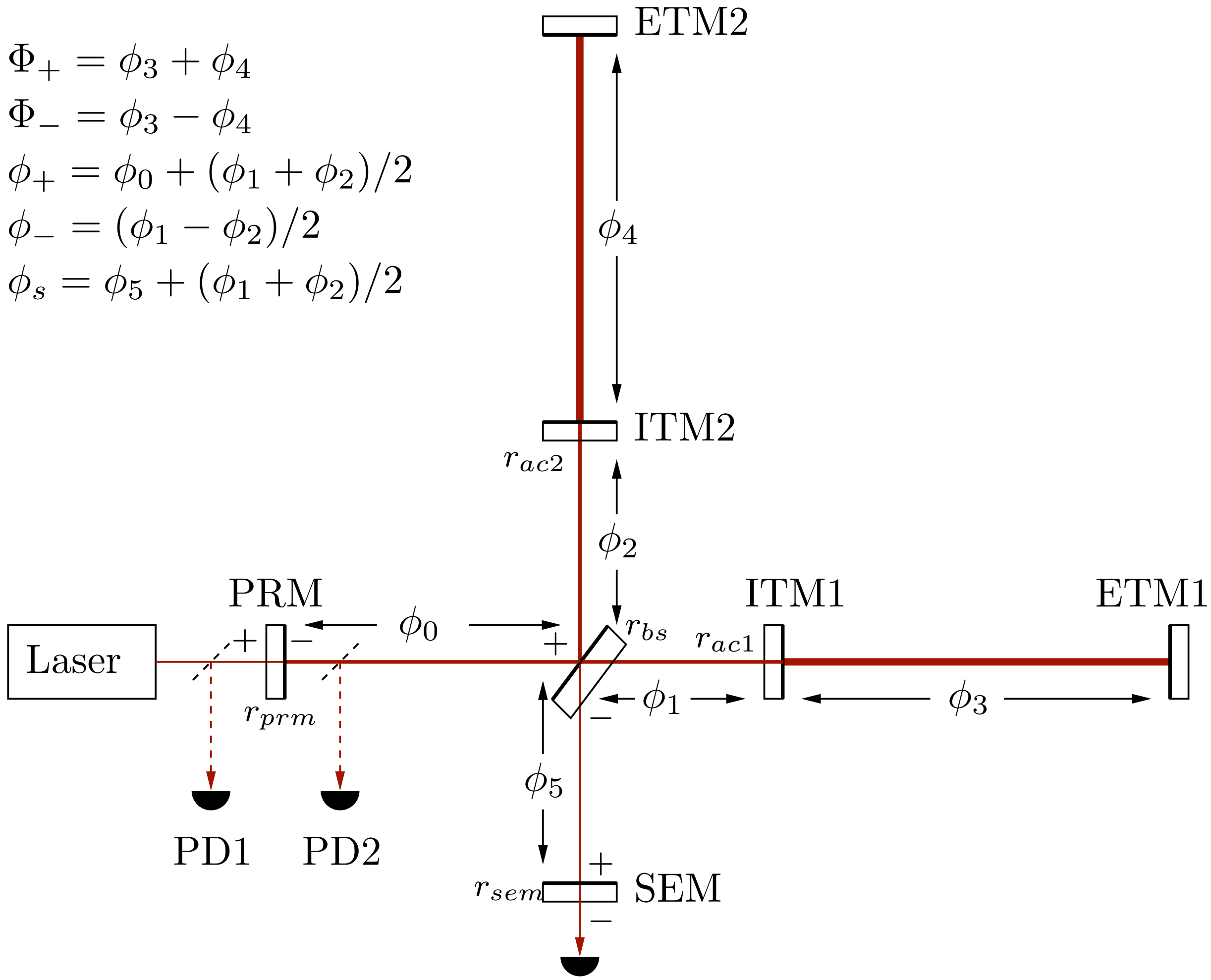
Class Outline

- Overview of cavity locking
- LIGO input spectrum
- The role of modulation sidebands
- Methods to generate various input spectra
- Advanced LIGO modulators

Advanced LIGO Configuration

$$\Phi_+ = \phi_3 + \phi_4$$
$$\Phi_- = \phi_3 - \phi_4$$
$$\phi_+ = \phi_0 + (\phi_1 + \phi_2)/2$$
$$\phi_- = (\phi_1 - \phi_2)/2$$
$$\phi_s = \phi_5 + (\phi_1 + \phi_2)/2$$

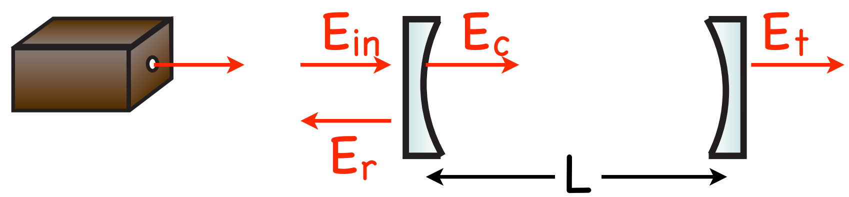


Cavity Locking

- Laser field must resonate in the cavities for interferometer to operate as intended
- External disturbances cause cavity length and laser frequency to fluctuate, thus active sensing and control is required to keep laser resonant in the cavities
  - Microwave technique for locking cavities developed by Pound was adapted to optical cavities by Drever and Hall, and used by Jan Hall for advances in laser stabilization that won he and Ted Hansch the 2005 Nobel prize in Physics
  - Pound-Drever-Hall technique for cavity locking (alternatively laser frequency stabilization) is widely used

Conceptual Model

Consider a Fabry-Perot cavity illuminated by a laser of frequency f, slightly detuned from the cavity resonance by $\delta f = f - f_{res}$



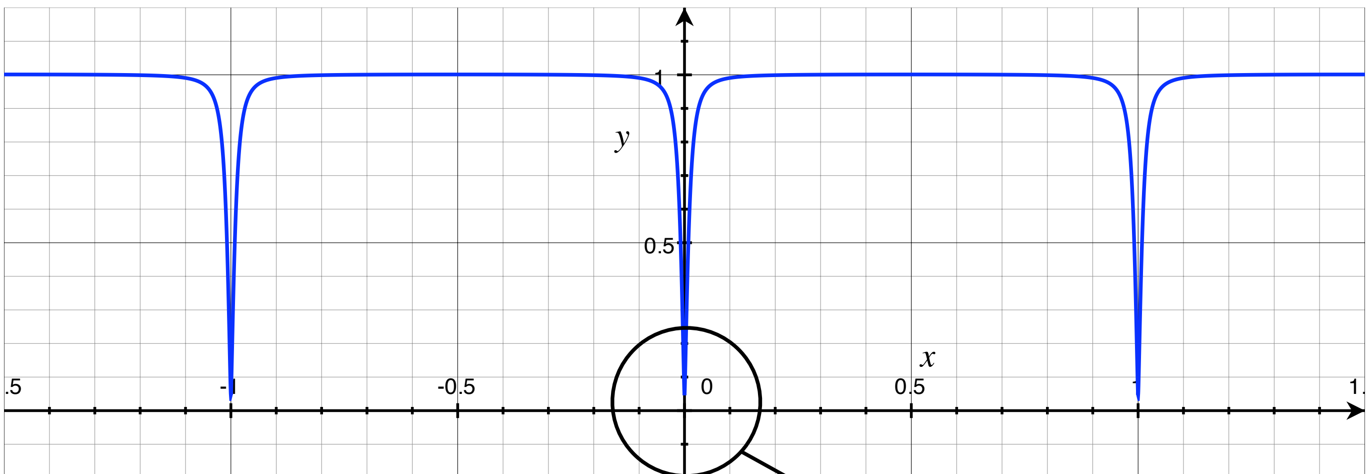
Treat the input mirror as having a reflectivity of $r_1>0$ when seen from inside the cavity, and the end mirror as having a reflectivity of $r_2>0$ when seen from inside the cavity.

The cavity reflectance is $r_{cav}(f) = -r_1 - \dfrac{t_1^2 r_2 e^{2ikL}}{1 - r_1 r_2 e^{2ikL}}$

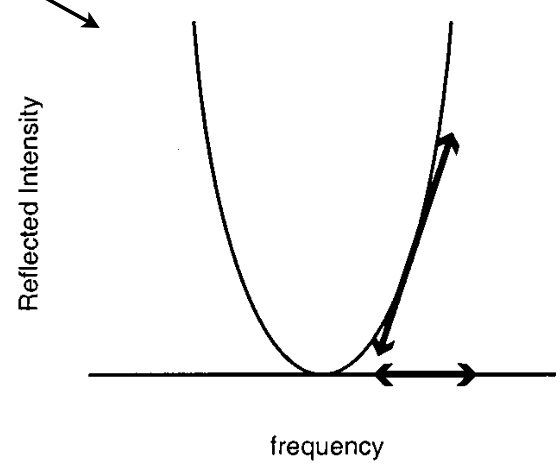
Conceptual Model

Cavity Reflectance

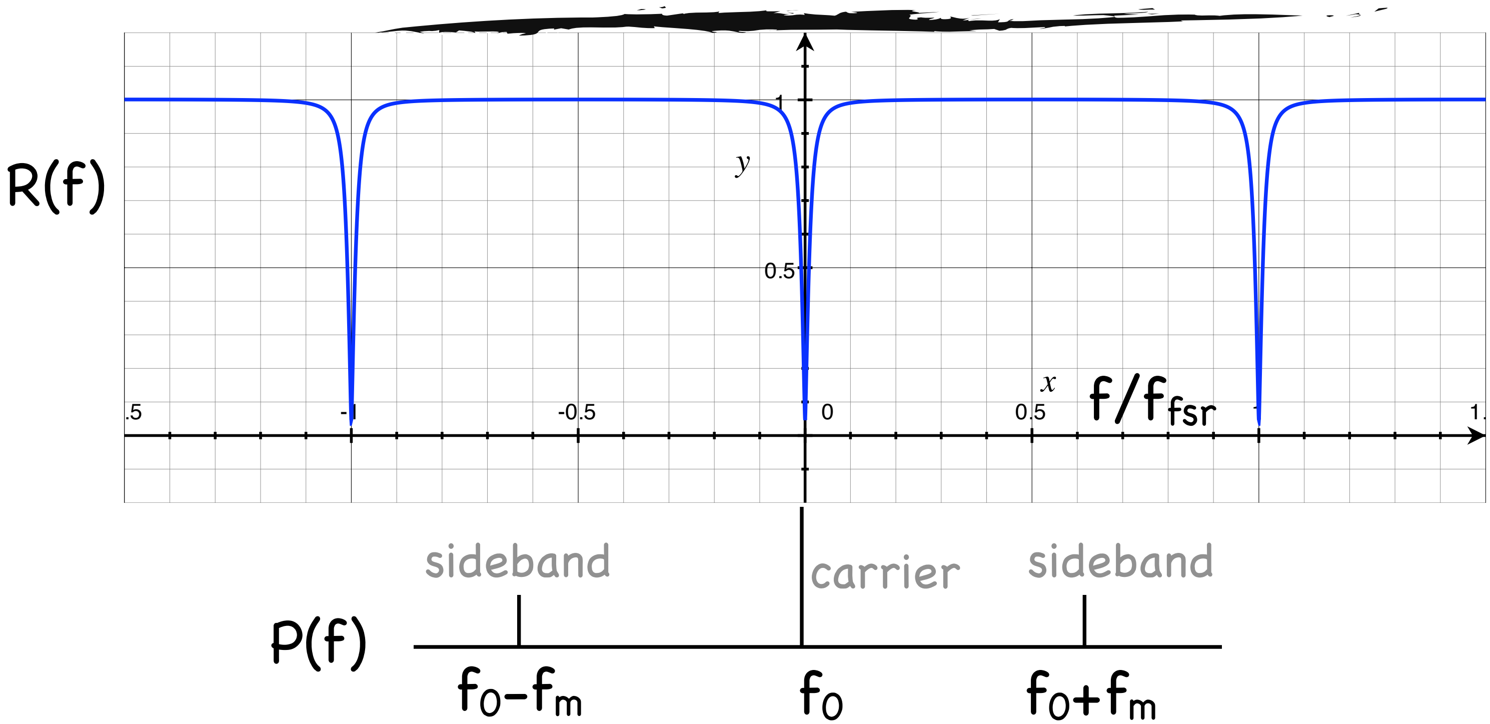
$$r_{cav}(f) = -r_1 - \frac{t_1^2 r_2 e^{2ikL}}{1 - r_1 r_2 e^{2ikL}}$$



Modulation of the round trip phase (by modulating laser frequency or cavity length) at a frequency f_m produces a modulation on the reflected power at f_m if cavity is off-resonance

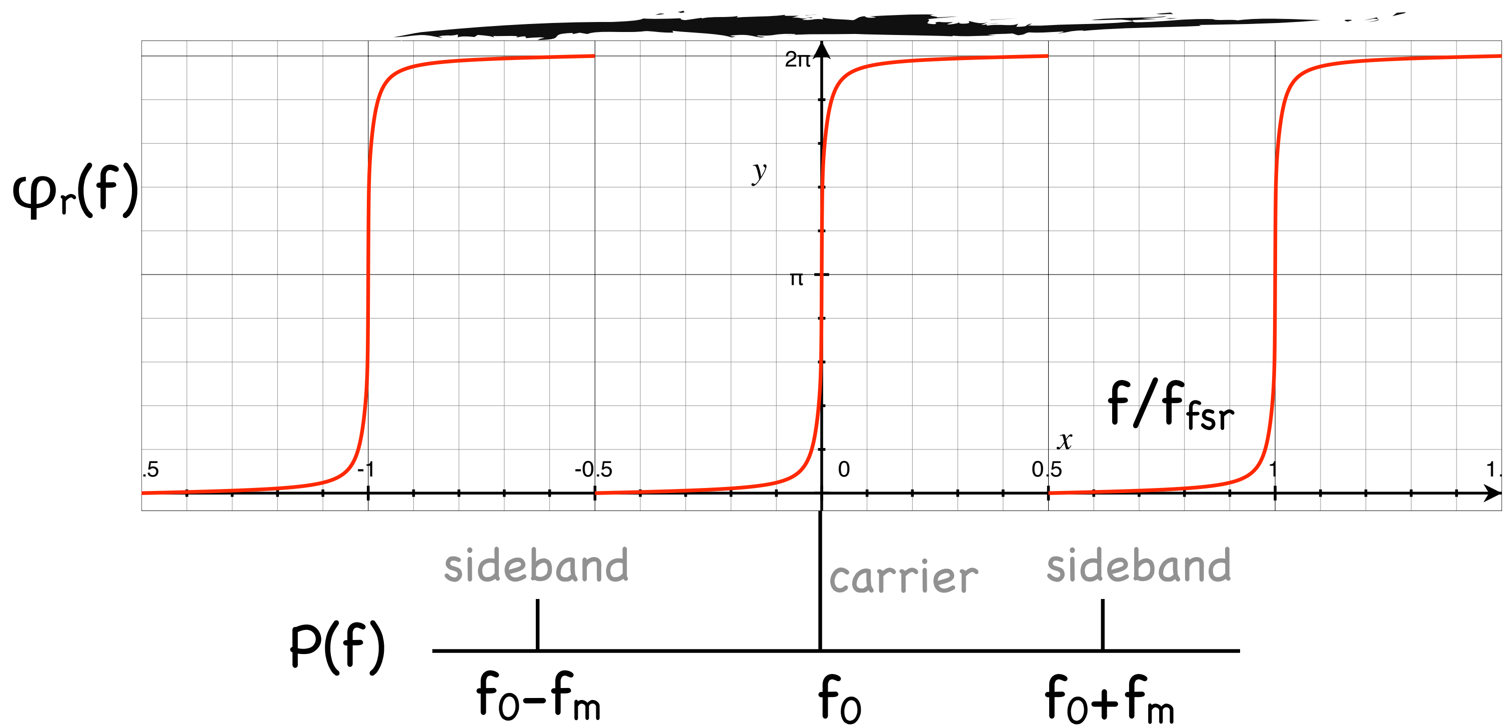


Carrier-Sideband Picture



A (phase) modulated beam has a carrier and sidebands spaced by frequency the modulation frequency f_m, as long as $f_m \neq n f_{fsr}$, the modulation frequency is not an integer multiple of the cavity's free spectral range ($f_{fsr}=c/2L$), the sidebands will reflect off the cavity with high efficiency, while the carrier will "see" a much lower reflection coefficient due ot the frequency dependence of the reflection coefficient

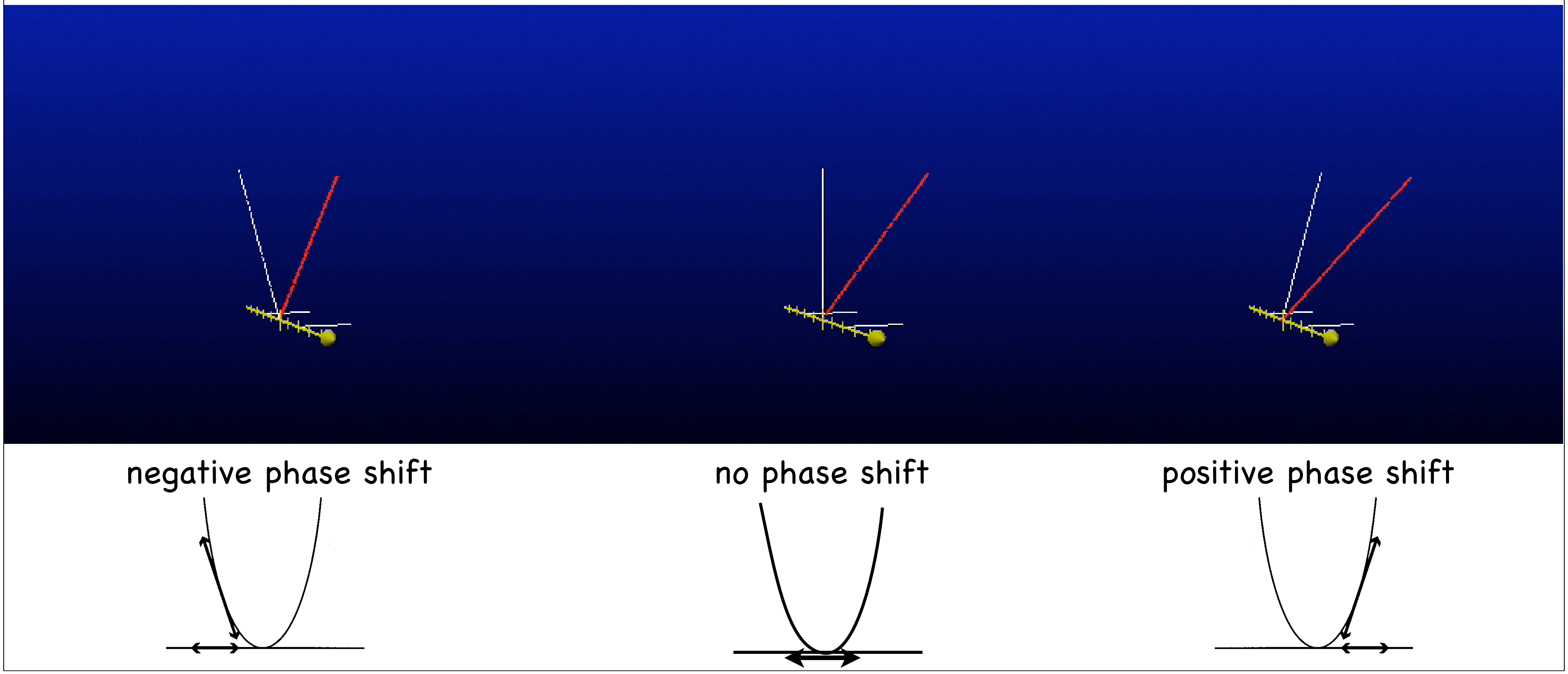
Carrier-Sideband Picture



The phase shift upon reflection from the cavity is essentially 0 for the sidebands, but the carrier has a phase shift that is a strong function of the detuning of the cavity (i.e. the difference between the cavities resonant frequency and the carrier frequency) when the carrier is near resonance (within a linewidth $\Delta f = f_{fsr}/\mathcal{F}$). Near resonance it has a slope of $2\pi\mathcal{F}/f_{fsr}$, when measured as a function of frequency

Carrier-Sideband Picture

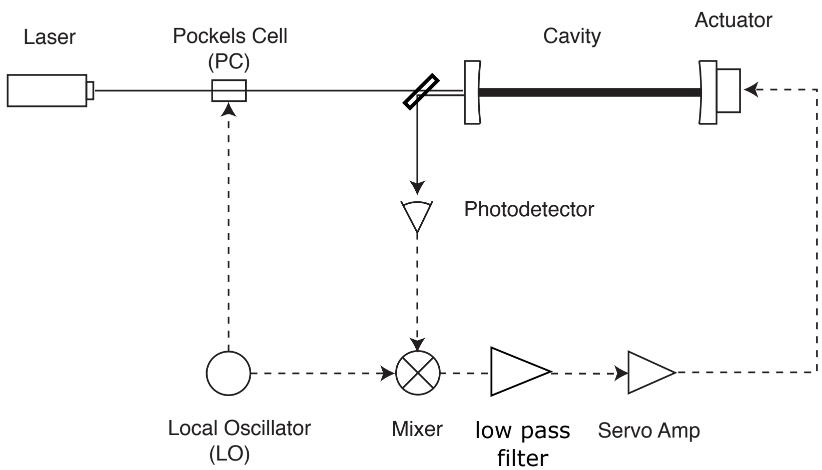
Phase shift of carrier transforms phase modulation into amplitude modulation



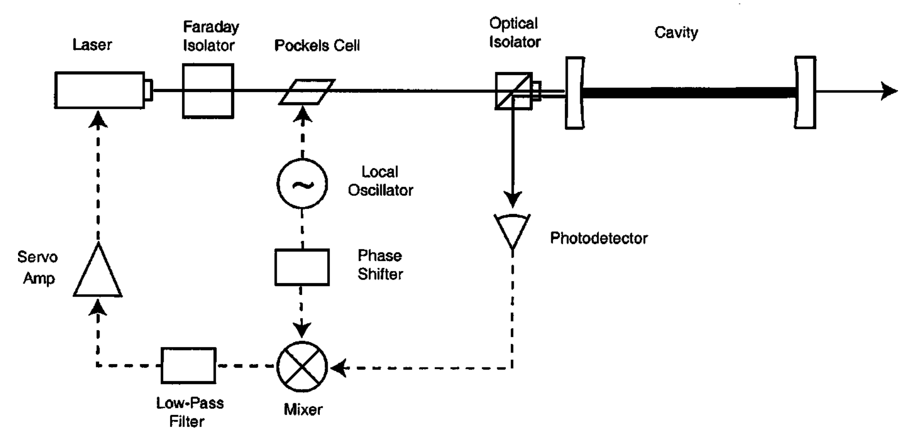
Experimental Schematic

Modulation of the laser frequency is accomplished by a pockels cell driven by a sinusoidal oscillator.

Any modulation at f_m on the light reflected from the cavity is detected (by mixing with the local oscillator) and used to provide feedback to the cavity (or laser).



PDH schematic for locking a cavity onto a laser



PDH schematic for locking a laser onto a cavity

Quantitative Model

Recall that the phasor amplitude for phase modulated light can be written as

$$E_{in} \approx E_0 \left(J_0(m) e^{i\omega t} + i J_1(m) \left(e^{i(\omega t - \Omega t)} + e^{i(\omega t + \Omega t)} \right) \right)$$

where m is the modulation depth, $\Omega = 2\pi f_m$ is the modulation frequency and $\omega = 2\pi f_0$ is the carrier frequency. The field reflected from the cavity is

$$E_r \approx E_0 \left(r(\omega) J_0(m) e^{i\omega t} + i J_1(m) \left(r(\omega - \Omega) e^{i(\omega t - \Omega t)} + r(\omega + \Omega) e^{i(\omega t + \Omega t)} \right) \right)$$

which can be approximated by

$$E \approx E_0 \left(-r_0 J_0(m) e^{i(\omega t + 2\pi \mathcal{F} \delta f / f_{fsr})} + i J_1(m) \left(e^{i(\omega t - \Omega t)} + e^{i(\omega t + \Omega t)} \right) \right)$$

when the carrier is near resonance, the sidebands are far off-resonance and $r_1, r_2 \approx 1$. Here r_0 is the magnitude of the cavity reflection on resonance and δf is the detuning of the laser from the cavity

Quantitative Model

The relative intensity detected by a photodetector is proportional to the magnitude of the field squared

$$E \approx E_0 \left(-r_0 J_0(m) e^{i(\omega t + 2\pi \mathcal{F} \delta f / f_{fsr})} + i J_1(m) \left(e^{i(\omega t - \Omega t)} + e^{i(\omega t + \Omega t)} \right) \right)$$

giving

$$\begin{aligned} I \propto E^* E &= E_0^2 \Bigg(r_0^2 J_0^2(m) + 2 J_1^2(m) \\ &- i r_0 J_0(m) J_1(m) \Bigg(\left(e^{i(2\pi \mathcal{F} \delta f / f_{fsr} + \Omega t)} - e^{-i(2\pi \mathcal{F} \delta f / f_{fsr} + \Omega t)} \right) \\ &+ \left(e^{i(2\pi \mathcal{F} \delta f / f_{fsr} - \Omega t)} - e^{-i(2\pi \mathcal{F} \delta f / f_{fsr} - \Omega t)} \right) \Bigg) \\ &+ J_1(m)^2 \left(e^{2i\Omega t} + e^{-2i\Omega t} \right) \Bigg) \end{aligned}$$

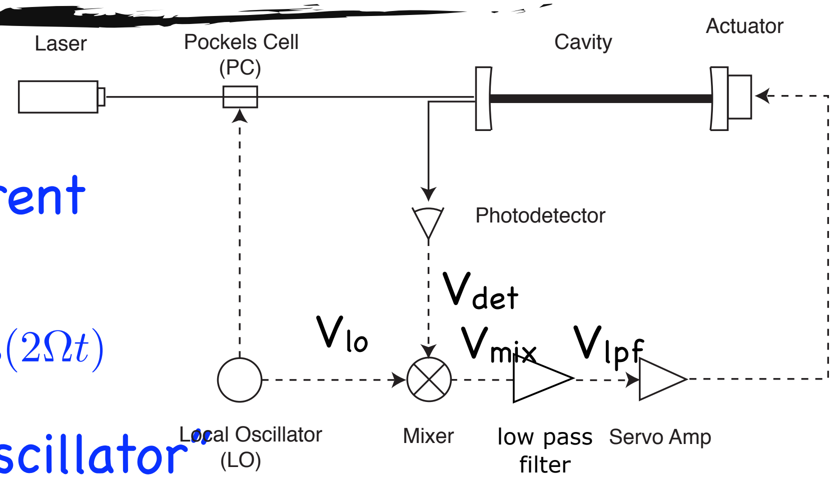
or

$$\begin{aligned} E^* E &= \text{DC terms} + 2\omega \text{ terms} \\ &+ 2E_0^2 r_0 J_0(m) J_1(m) \Big(\sin\left(2\pi \mathcal{F} \delta f / f_{fsr} + \Omega t\right) + \sin\left(2\pi \mathcal{F} \delta f / f_{fsr} - \Omega t\right) \Big) \end{aligned}$$

$$\begin{aligned} E^* E &= \text{DC terms} + 2\omega \text{ terms} \\ &+ \underline{4E_0^2 r_0 J_0(m) J_1(m) \sin(2\pi \mathcal{F} \delta f / f_{fsr})} \cos(\Omega t) \end{aligned}$$

This is the amplitude of the modulated power at the modulation frequency, near resonance ($\delta f \ll f_{fsr}/\mathcal{F}$) this is proportional to δf

Demodulation



Consider a detected photocurrent of the form

$$V_{det} = V_0 + V_\Omega \cos(\Omega t) + V_{2\Omega}\cos(2\Omega t)$$

it gets "mixed" with a "local oscillator" of the form $V_{lo}=\cos(\Omega t)$ which is equivalent to multiplication

$$V_{mix} = V_{lo}V_{det} = V_0\cos(\Omega t) + V_\Omega \cos^2(\Omega t) + V_{2\Omega}\cos(2\Omega t)\cos(\Omega t)$$

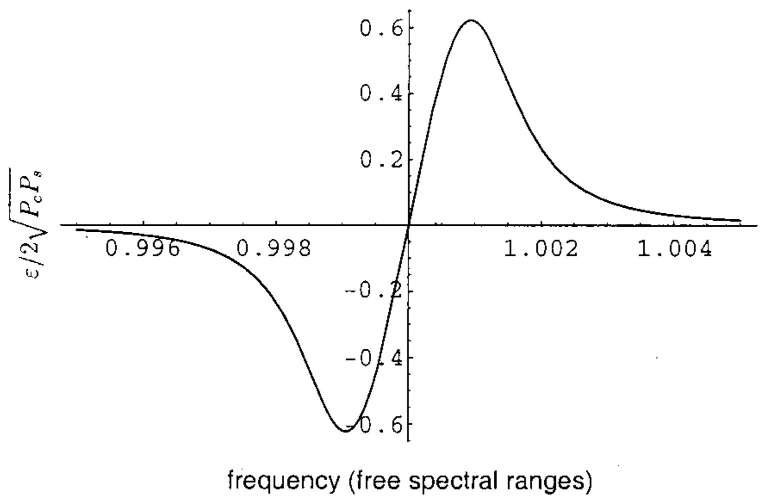
the mixed signal is then low pass filtered to get

$$V_{lpf} \approx \lim_{T\to\infty}\frac{1}{T}\int_{t-T}^{t} V_{mix}d\tau = \frac{1}{2}V_\Omega$$

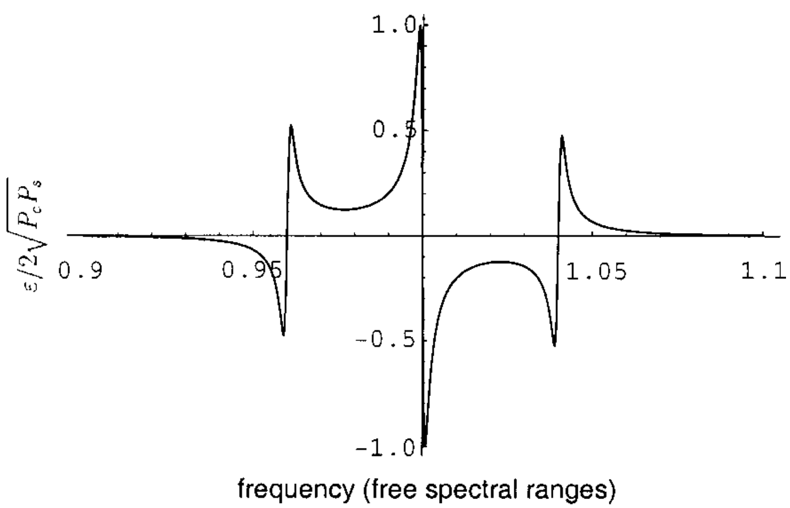
This is $4E_0^2 r_0 J_0(m)J_1(m)\delta f$ and provides a measure of the detuning of the laser from the cavity (or vice versa)

Error Signal

The demodulated (and low pass filtered signal) is a measure of the laser's detuning from the cavity resonance and is called an "error signal"

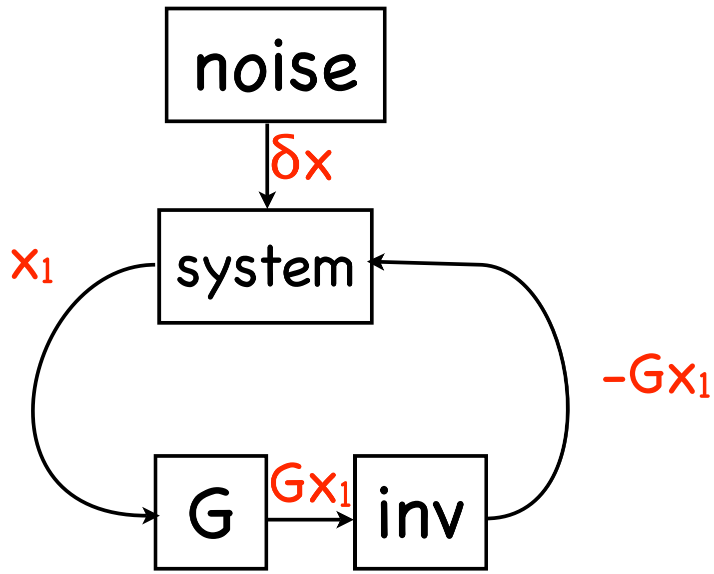


error signal as a function of detuning (for small tuning range)



error signal as a function of detuning (for large tuning range)

Negative Feedback



In the steady state we must have

$\delta x - G x_1 = x_1$

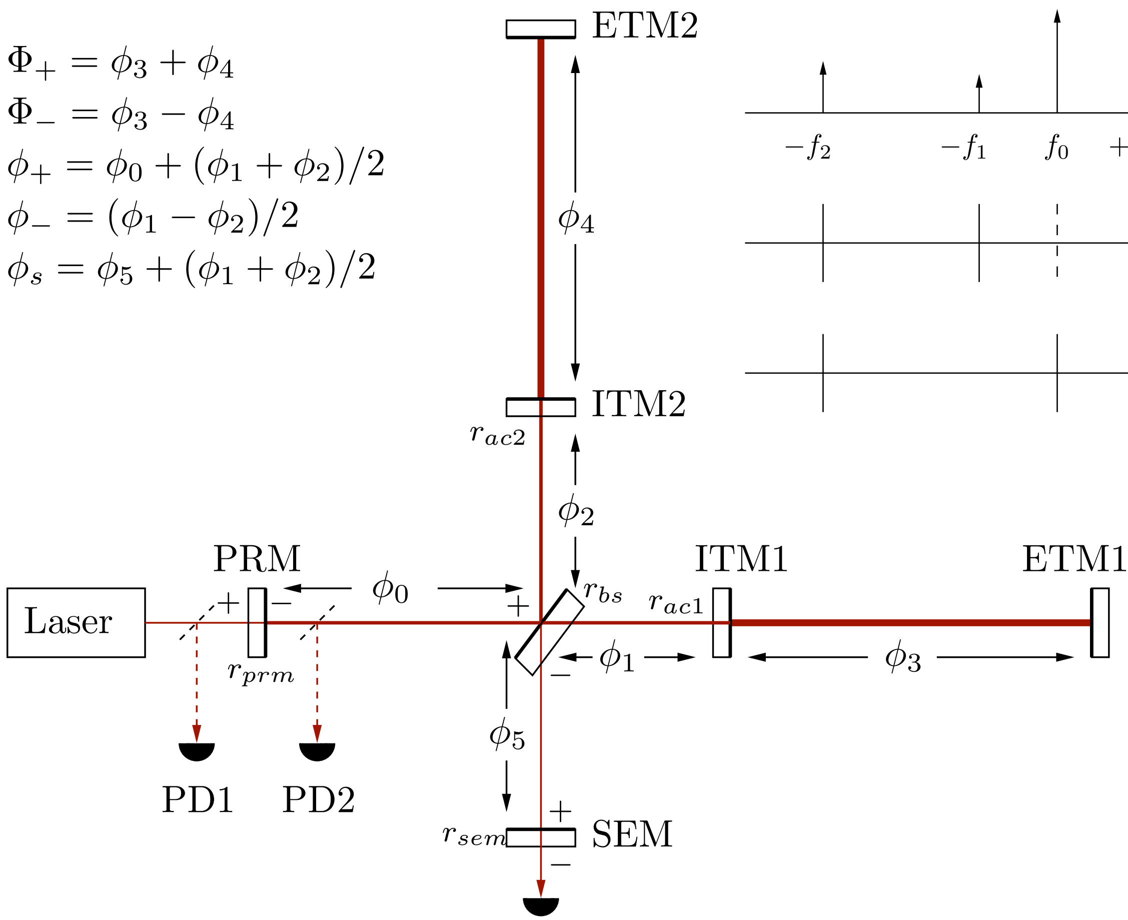
giving

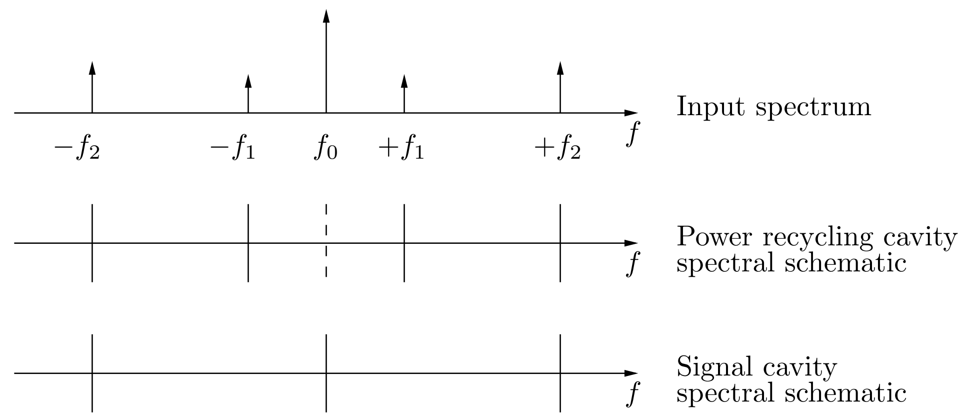
$$x_1 = \frac{\delta x}{1+G}$$

meaning the noise is suppressed by a factor of 1+G. The "noise" can be laser frequency or cavity length fluctuations

Cavities in LIGO

$$\Phi_+ = \phi_3 + \phi_4$$
$$\Phi_- = \phi_3 - \phi_4$$
$$\phi_+ = \phi_0 + (\phi_1 + \phi_2)/2$$
$$\phi_- = (\phi_1 - \phi_2)/2$$
$$\phi_s = \phi_5 + (\phi_1 + \phi_2)/2$$



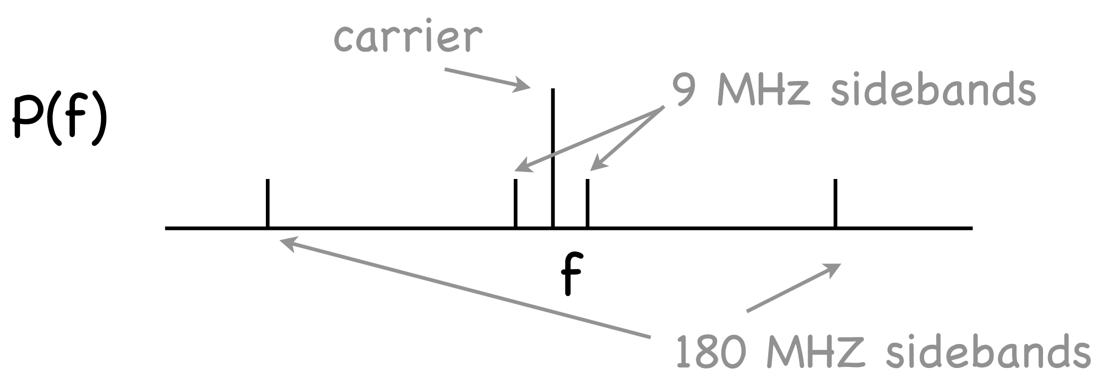


LIGO Input Spectrum

Various frequency components are necessary for length and alignment sensing of the many cavities and interferometers in the LIGO detector

phase modulation sidebands for locking "power recycling cavity" and Michelson interferometer (9 MHz)

phase modulation sidebands for locking the arms (180 MHz)



LIGO Modulators

Initial LIGO

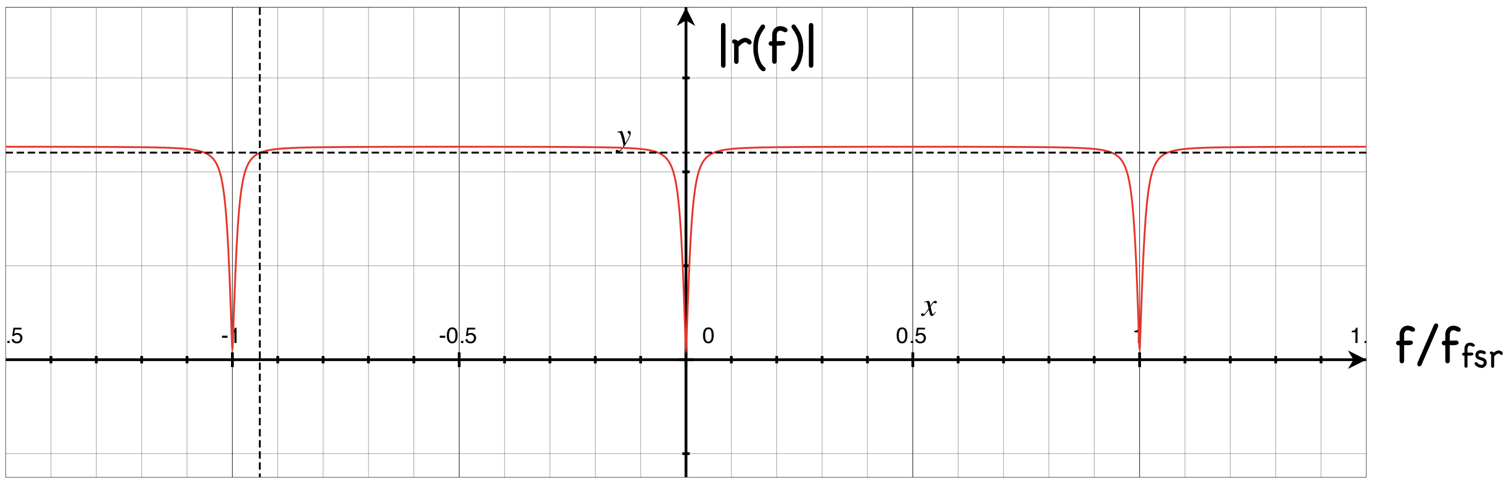
- $LiNbO_3$ slabs with 10mm x 10mm clear aperture for Initial LIGO (operates with up to 10W of power)
- Transverse modulation
- resonant circuit geometry
- 9MHz phase modulation sidebands for interferometer sensing

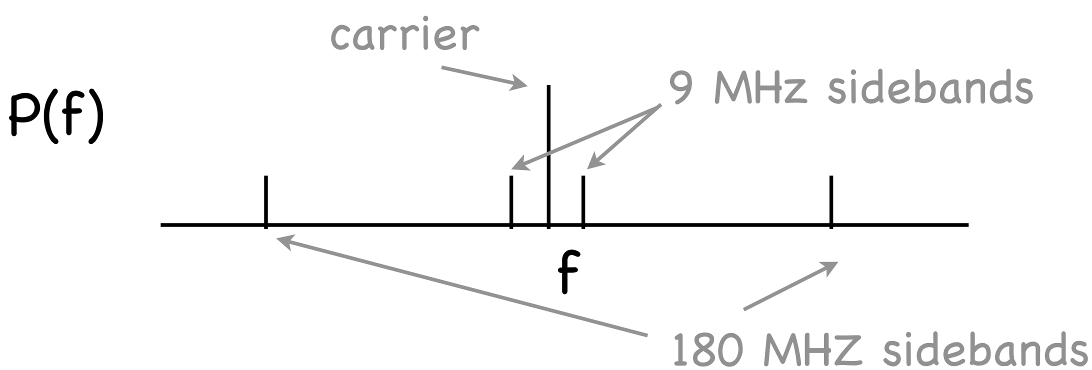
Advanced LIGO

- RTP ($RbTiOPO_4$) (operates with up to 300W of power)
- 9 MHz and 180 MHz PM sidebands for interferometer Sensing

Role of Modulation Sidebands

Cavity Reflectance $r_{cav}(f) = r - \dfrac{t^2}{1 - r^2 e^{i2\pi f/f_{fsr}}}$

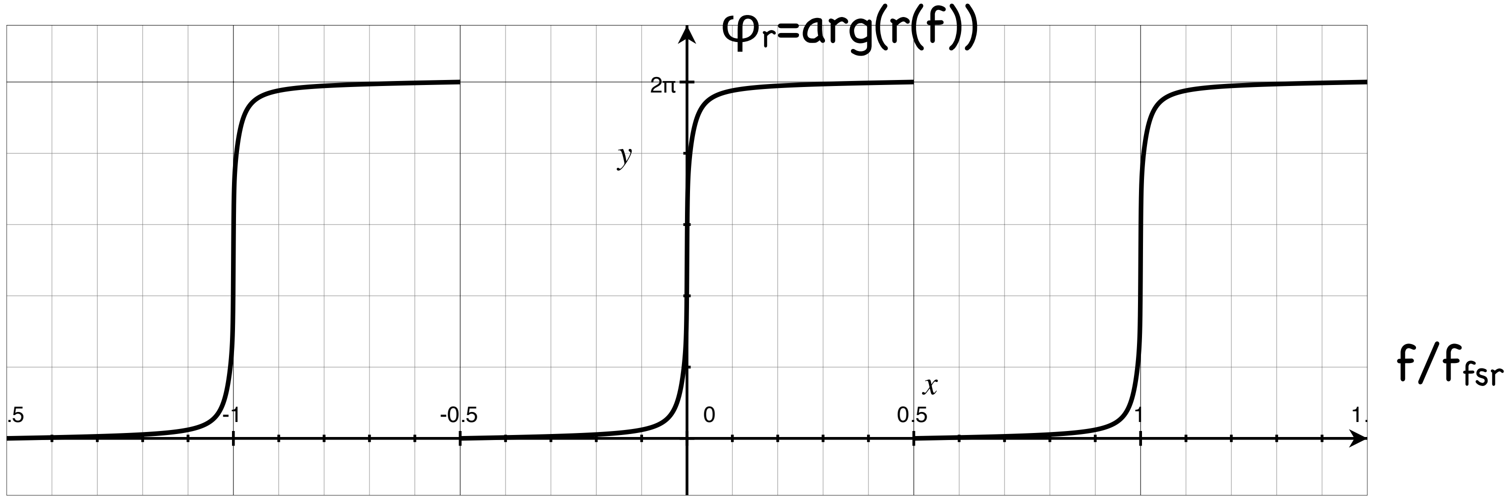


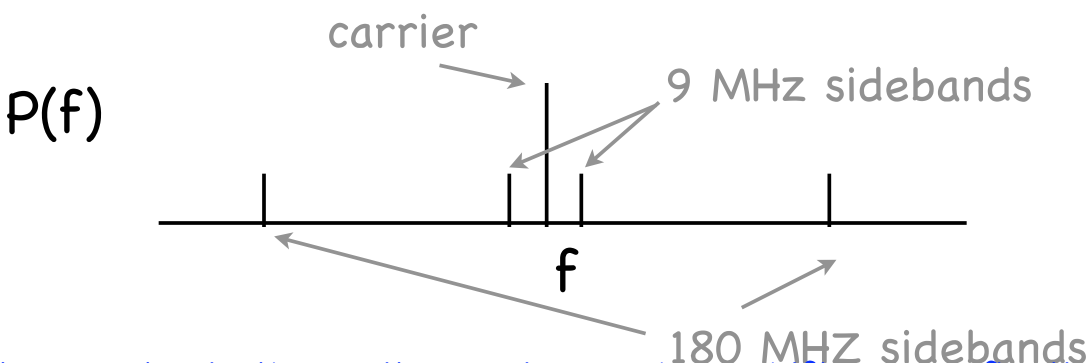


Sideband frequencies are such that only one component is resonant in the cavity of interest

Role of Modulation Sidebands

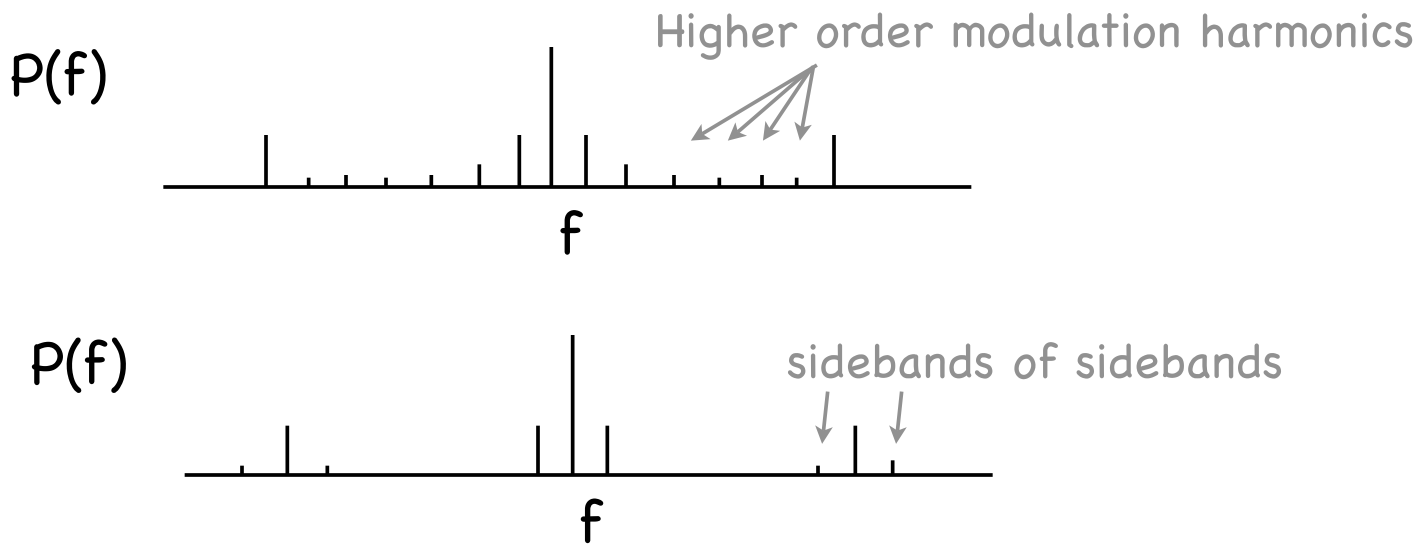
Cavity Reflectance $r_{cav}(f) = r - \frac{t^2}{1 - r^2 e^{i2\pi f/f_{fsr}}}$





Component that resonates in the cavity acquires a phase shift upon reflection that is a function of the cavities detuning.

Higher Order Sidebands



Demodulation gives the sum of all frequency components spaced by f_{lo}. Higher order modulation harmonics and sidebands-of-sidebands can produce unwanted contributions

Unintended Sidebands

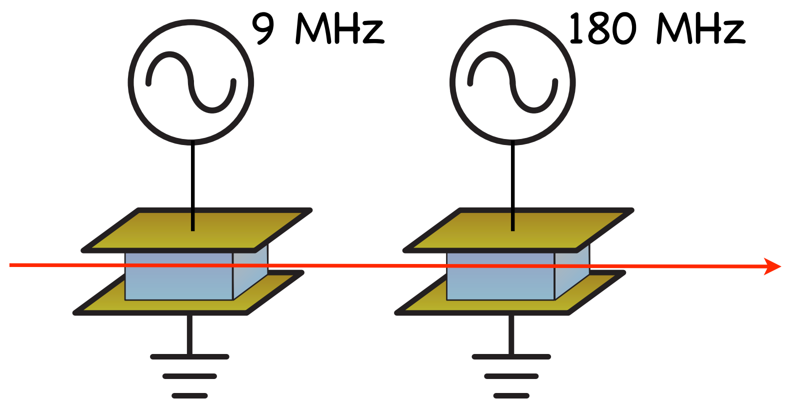
Higher order (for example 18 MHZ sidebands from the 9 MHz modulator)sidebands are problematic because the intermodulation they produce can obscure intended signals

Solutions:

- Low modulation depth
- Sideband cancellation
- Parallel modulation

Serial Modulation

For small modulation depth $J_1(m) \approx m/2$ and $J_2(m) \approx m/24$ so intermodulation depth is $m_1 m_2/48$

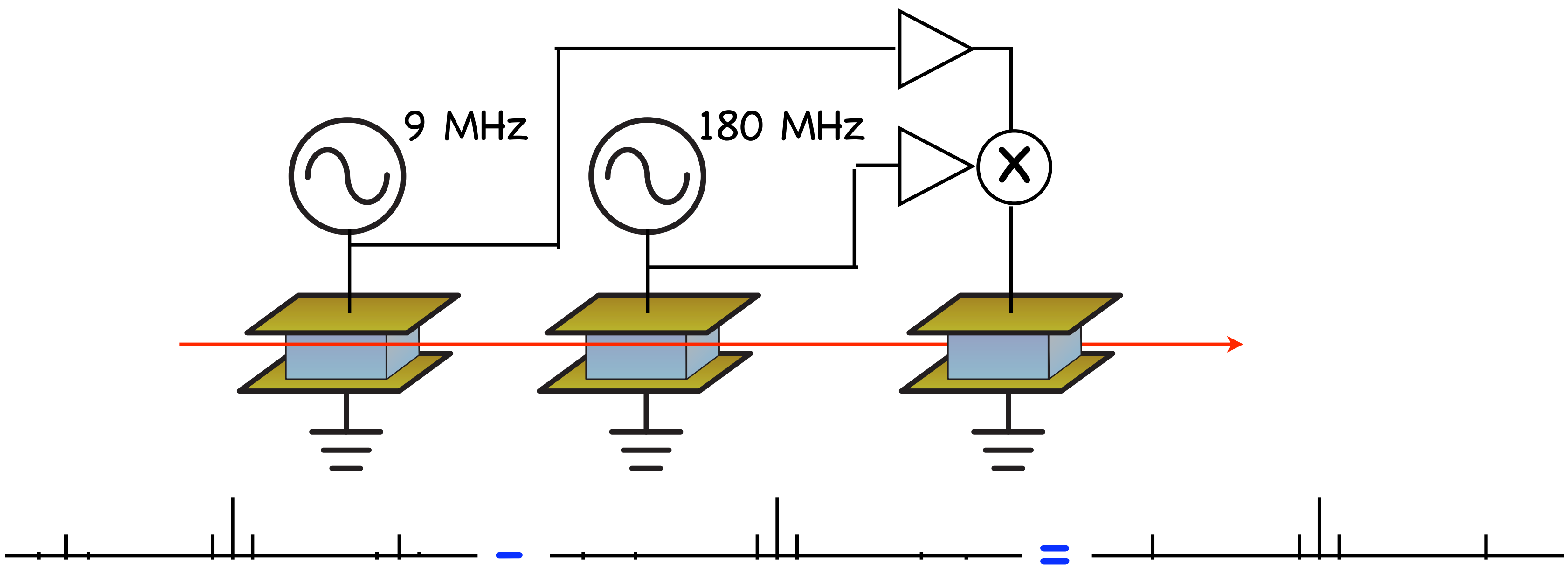


Power spectrum after 9 MHz modulator

Power spectrum after 9 MHz and 180 MHz modulators

Harmonic Compensation

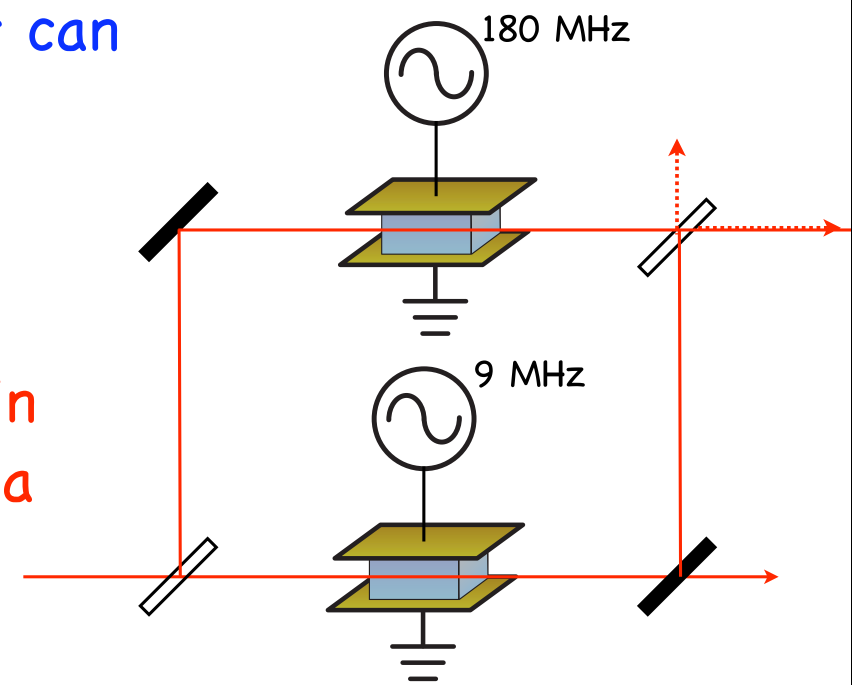
Through proper adjustment of amplitude and phase of 189 and 171 MHz signals, the intermodulation harmonics can be cancelled



Power spectrum after 9 MHz and 180 MHz modulators − Modulation spectrum from third modulator = Power spectrum after 9 MHz and 180 MHz modulators

Parallel Modulation

A Mach-Zehnder interferometer can be used to combine modulation sidebands from two independent modulators



Drawbacks include reduction in effective modulatin depth by a factor of 2 due to sidebands lost to the unused port and increased complexity of maintaining Mach-Zehnder interference condition

Single Sideband Generation

Some control and readout schemes require a "Single sideband" rather than a pair of phase modulated sidebands or amplitude modulated sidebands

Example: mapping out the frequency response of a detuned interferometer

Single Sideband Generation

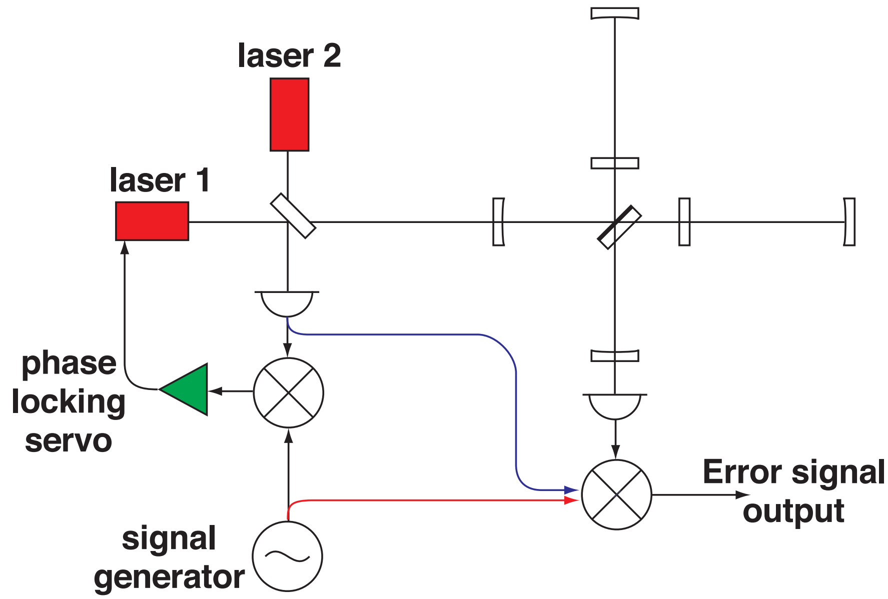
Phase modulation produces a pair of sidebands at the modulation frequency that are each in phase with the carrier (at some instant in time)

Amplitude modulation produces a pair of sidebands a the modulation frequency with one in phase with the carrier while the other is π out-of-phase with the carrier.

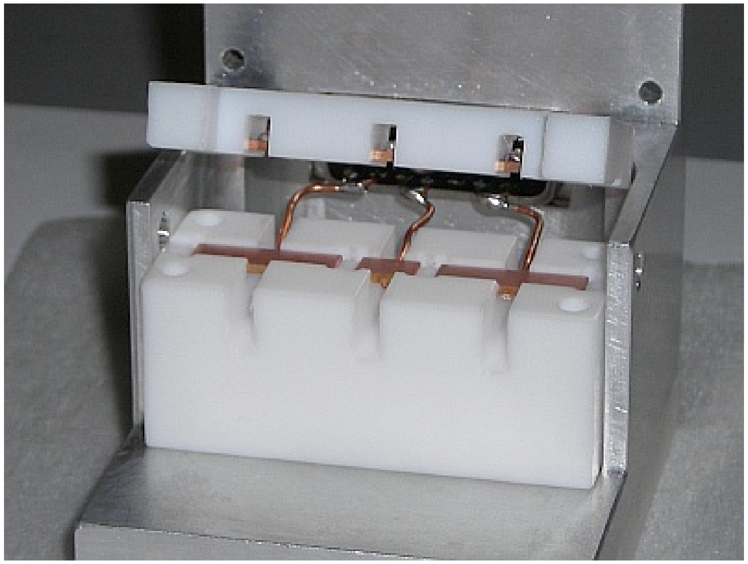
Combining amplitude and phase modulation at the same frequency, one sideband in a pair can be enhanced while the other is suppressed.

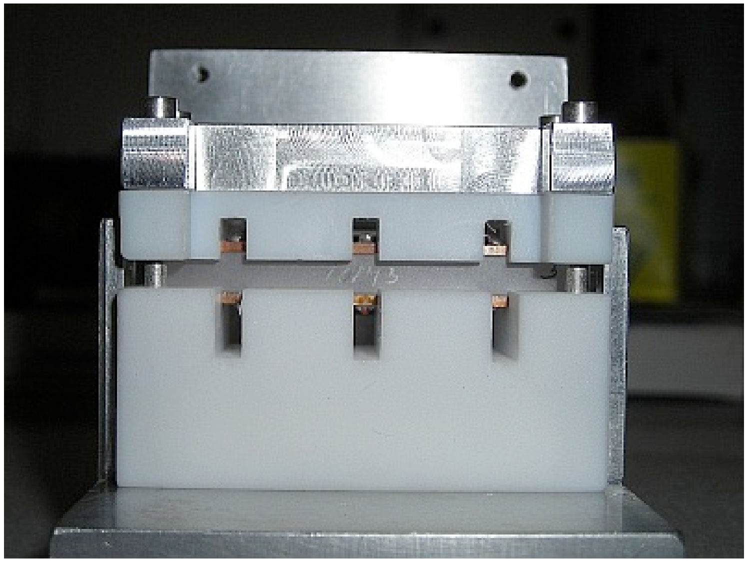
Sub-Carrier Generation

An alternative way to generate a single sideband is to phase lock two lasers with a given frequency offset. This had been proposed for Advanced LIGO



Advanced LIGO modulators





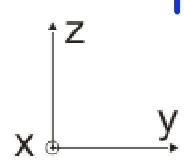
Three sets of modulation electrodes on one crystal

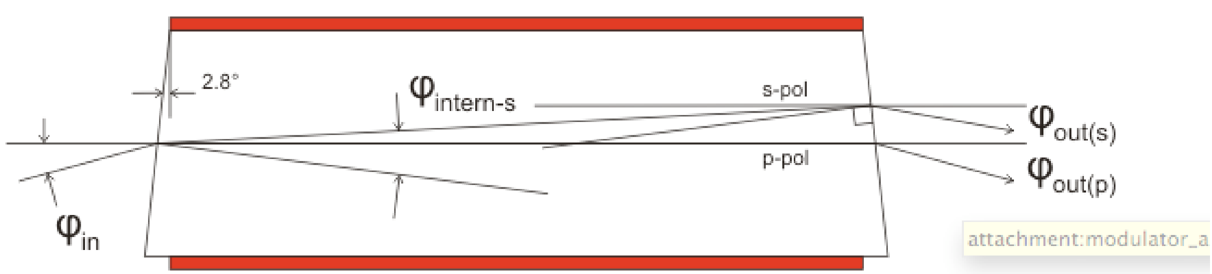
Modulator Crystals

To avoid interference effects from reflections off the modulator faces, the crystal has a 2x2.8° wedge

The wedge leads to polarization dependent transmission angle, and thus the modulator also acts like a polarizer

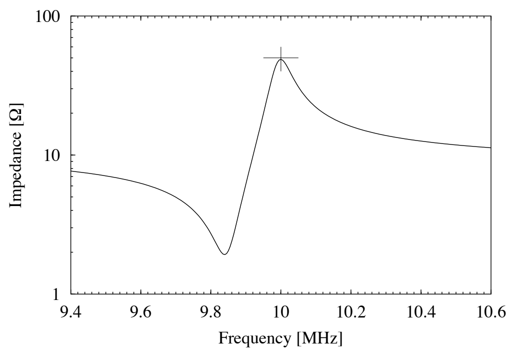
Steering by RTP crystal for s- and p- polarization	
Polarization	Angle [degrees]
p	4.81
s	4.31

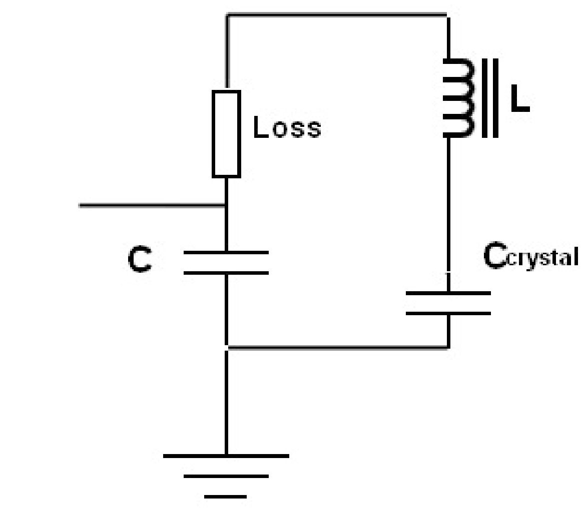




Resonant Circuit

The resonant circuit is designed to have 50Ω impedance at the resonant frequency, but high impedance at DC and low frequencies.



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References

- Guido Mueller, David Reitze, Haisheng Rong, David Tanner, Sany Yoshida, and Jordan Camp "Reference Design Document for the Advanced LIGO Input Optics", LIGO internal document T010002-00 (2000)
- James Mason, "Signal Extraction and Optical Design for an Advanced Gravitational Wave Interferometer" Ph.D. Thesis, CIT (2001)
- M Gray, D Shaddock, C Mow-Lowry, D. McClelland "Tunable Power Recycled RSE Michelson for LIGO II", Internal LIGO document G000227-00-D
- G. Heinzel "Advanced optical techniques for laser-interferometric gravitational-wave detectors" Ph.D. Thesis MPQ (1999)OSA
- E Black "An introduction to Pound - Drever - Hall laser frequency stabilization", Am. J. Phys. 69 (2001)