Electrooptic Modulators for Cavity Locking

Case Study
Physics 208, Electro-optics
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Class Outline

- Overview of cavity locking
- LIGO input spectrum
- The role of modulation sidebands
- Methods to generate various input spectra
- Advanced LIGO modulators
\[ \Phi_+ = \phi_3 + \phi_4 \]
\[ \Phi_- = \phi_3 - \phi_4 \]
\[ \phi_+ = \phi_0 + (\phi_1 + \phi_2)/2 \]
\[ \phi_- = (\phi_1 - \phi_2)/2 \]
\[ \phi_s = \phi_5 + (\phi_1 + \phi_2)/2 \]
Cavity Locking

- Laser field must resonate in the cavities for interferometer to operate as intended.

- External disturbances cause cavity length and laser frequency to fluctuate, thus active sensing and control is required to keep laser resonant in the cavities.

- Microwave technique for locking cavities developed by Pound was adapted to optical cavities by Drever and Hall, and used by Jan Hall for advances in laser stabilization that won he and Ted Hansch the 2005 Nobel prize in Physics.

- Pound-Drever-Hall technique for cavity locking (alternatively laser frequency stabilization) is widely used.
Consider a Fabry-Perot cavity illuminated by a laser of frequency $f$, slightly detuned from the cavity resonance by $\delta f = f - f_{\text{res}}$

The cavity reflectance is

$$r_{\text{cav}}(f) = -r_1 - \frac{t_1^2 r_2 e^{2ikL}}{1 - r_1 r_2 e^{2ikL}}$$

Treat the input mirror as having a reflectivity of $r_1 > 0$ when seen from inside the cavity, and the end mirror as having a reflectivity of $r_2 > 0$ when seen from inside the cavity.
Conceptual Model

Cavity Reflectance

\[ r_{cav}(f) = -r_1 - \frac{t_1^2 r_2 e^{2ikL}}{1 - r_1 r_2 e^{2ikL}} \]

Modulation of the round trip phase (by modulating laser frequency or cavity length) at a frequency \( f_m \) produces a modulation on the reflected power at \( f_m \) if cavity is off-resonance.
A (phase) modulated beam has a carrier and sidebands spaced by frequency the modulation frequency $f_m$, as long as $f_m \neq n f_{fsr}$, the modulation frequency is not an integer multiple of the cavity’s free spectral range ($f_{fsr} = c/2L$), the sidebands will reflect off the cavity with high efficiency, while the carrier will “see” a much lower reflection coefficient due to the frequency dependence of the reflection coefficient.
The phase shift upon reflection from the cavity is essentially 0 for the sidebands, but the carrier has a phase shift that is a strong function of the detuning of the cavity (i.e. the difference between the cavities resonant frequency and the carrier frequency) when the carrier is near resonance (within a linewidth $\Delta f = f_{\text{fsr}}/F$). Near resonance it has a slope of $2\pi F/f_{\text{fsr}}$, when measured as a function of frequency.
Carrier-Sideband Picture

Phase shift of carrier transforms phase modulation into amplitude modulation

negative phase shift

no phase shift

positive phase shift
Experimental Schematic

Modulation of the laser frequency is accomplished by a pockels cell driven by a sinusoidal oscillator.

Any modulation at $f_m$ on the light reflected from the cavity is detected (by mixing with the local oscillator) and used to provide feedback to the cavity (or laser).
Recall that the phasor amplitude for phase modulated light can be written as

\[ E_{in} \approx E_0 \left( J_0(m) e^{i \omega t} + i J_1(m) \left( e^{i(\omega t - \Omega t)} + e^{i(\omega t + \Omega t)} \right) \right) \]

where \( m \) is the modulation depth, \( \Omega = 2\pi f_m \) is the modulation frequency and \( \omega = 2\pi f_0 \) is the carrier frequency. The field reflected from the cavity is

\[ E_r \approx E_0 \left( r(\omega) J_0(m) e^{i \omega t} + i J_1(m) \left( r(\omega - \Omega) e^{i(\omega t - \Omega t)} + r(\omega + \Omega) e^{i(\omega t + \Omega t)} \right) \right) \]

which can be approximated by

\[ E \approx E_0 \left( -r_0 J_0(m) e^{i(\omega t + 2\pi F \delta f / f_{sr})} + i J_1(m) \left( e^{i(\omega t - \Omega t)} + e^{i(\omega t + \Omega t)} \right) \right) \]

when the carrier is near resonance, the sidebands are far off-resonance and \( r_1, r_2 \approx 1 \). Here \( r_0 \) is the magnitude of the cavity reflection on resonance and \( \delta f \) is the detuning of the laser from the cavity.
The relative intensity detected by a photodetector is proportional to the magnitude of the field squared giving

\[ E \approx E_0 \left( -r_0 J_0(m) e^{i(\omega t + 2\pi F \delta f / f_{fsr})} + iJ_1(m) \left( e^{i(\omega t - \Omega t)} + e^{i(\omega t + \Omega t)} \right) \right) \]

or

\[ E^* E = \text{DC terms} + 2\omega \text{ terms} \]

\[ + \quad 2E_0^2 r_0 J_0(m) J_1(m) \left( \sin(2\pi F \delta f / f_{fsr} + \Omega t) + \sin(2\pi F \delta f / f_{fsr} - \Omega t) \right) \]

\[ E^* E = \text{DC terms} + 2\omega \text{ terms} \]

\[ + \quad 4E_0^2 r_0 J_0(m) J_1(m) \sin(2\pi F \delta f / f_{fsr}) \cos(\Omega t) \]

This is the amplitude of the modulated power at the modulation frequency, near resonance (\( \delta f \ll f_{fsr} / f \)) this is proportional to \( \delta f \)
Consider a detected photocurrent of the form

\[ V_{det} = V_0 + V_\Omega \cos(\Omega t) + V_{2\Omega} \cos(2\Omega t) \]

it gets “mixed” with a “local oscillator” of the form \( V_{lo} = \cos(\Omega t) \) which is equivalent to multiplication

\[ V_{mix} = V_{lo} V_{det} = V_0 \cos(\Omega t) + V_\Omega \cos^2(\Omega t) + V_{2\Omega} \cos(2\Omega t) \cos(\Omega t) \]

the mixed signal is then low pass filtered to get

\[ V_{lpf} \approx \lim_{T \to \infty} \frac{1}{T} \int_{t-T}^{t} V_{mix} \, d\tau = \frac{1}{2} V_\Omega \]

This is \( 4E_0^2 r_0 J_0(m) J_1(m) \delta f \) and provides a measure of the detuning of the laser from the cavity (or vice versa)
Error Signal

The demodulated (and low pass filtered signal) is a measure of the laser’s detuning from the cavity resonance and is called an “error signal”
In the steady state we must have
\[ \delta x - G x_1 = x_1 \]
giving
\[ x_1 = \frac{\delta x}{1 + G} \]
meaning the noise is suppressed by a factor of 1+G. The “noise” can be laser frequency or cavity length fluctuations.
$\Phi_+ = \phi_3 + \phi_4$
$\Phi_- = \phi_3 - \phi_4$
$\phi_+ = \phi_0 + (\phi_1 + \phi_2)/2$
$\phi_- = (\phi_1 - \phi_2)/2$
$\phi_s = \phi_5 + (\phi_1 + \phi_2)/2$

Cavities in LIGO
Various frequency components are necessary for length and alignment sensing of the many cavities and interferometers in the LIGO detector.

- Phase modulation sidebands for locking “power recycling cavity” and Michelson interferometer (9 MHz)
- Phase modulation sidebands for locking the arms (180 MHz)
LIGO Modulators

Initial LIGO

LiNbO$_3$ slabs with 10mm x 10mm clear aperture for Initial LIGO (operates with up to 10W of power)

Transverse modulation

resonant circuit geometry

9MHz phase modulation sidebands for interferometer sensing

Advanced LIGO

RTP (RbTiOPO$_4$) (operates with up to 300W of power)

9 MHz and 180 MHz PM sidebands for interferometer Sensing
Role of Modulation Sidebands

Cavity Reflectance

\[ r_{cau}(f) = r - \frac{t^2}{1 - r^2 \cos(2\pi f/f_{fsr})} \]

Sideband frequencies are such that only one component is resonant in the cavity of interest.
Role of Modulation Sidebands

Cavity Reflectance

\[ r_{\text{cav}}(f) = r - \frac{t^2}{1 - r^2 e^{i2\pi f / f_{\text{fsr}}}} \]

\[ \phi_r = \arg(r(f)) \]

Component that resonates in the cavity acquires a phase shift upon reflection that is a function of the cavities detuning.
Higher Order Sidebands

Demodulation gives the sum of all frequency components spaced by $f_{lo}$. Higher order modulation harmonics and sidebands-of-sidebands can produce unwanted contributions.
Unintended Sidebands

Higher order (for example 18 MHz sidebands from the 9 MHz modulator) sidebands are problematic because the intermodulation they produce can obscure intended signals.

Solutions:

- Low modulation depth
- Sideband cancellation
- Parallel modulation
Serial Modulation

For small modulation depth $J_1(m) \approx m/2$ and $J_2(m) \approx m/24$
so intermodulation depth is $m_1 m_2/48$
Harmonic Compensation

Through proper adjustment of amplitude and phase of 189 and 171 MHz signals, the intermodulation harmonics can be cancelled.

Power spectrum after 9 MHz and 180 MHz modulators

- Modulation spectrum from third modulator

= Power spectrum after 9 MHz and 180 MHz modulators
Parallel Modulation

A Mach-Zehnder interferometer can be used to combine modulation sidebands from two independent modulators.

Drawbacks include reduction in effective modulation depth by a factor of 2 due to sidebands lost to the unused port and increased complexity of maintaining Mach-Zehnder interference condition.
Single Sideband Generation

Some control and readout schemes require a “Single sideband” rather than a pair of phase modulated sidebands or amplitude modulated sidebands

Example: mapping out the frequency response of a detuned interferometer
Single Sideband Generation

Phase modulation produces a pair of sidebands at the modulation frequency that are each in phase with the carrier (at some instant in time).

Amplitude modulation produces a pair of sidebands at the modulation frequency with one in phase with the carrier while the other is $\pi$ out-of-phase with the carrier.

Combining amplitude and phase modulation at the same frequency, one sideband in a pair can be enhanced while the other is suppressed.
Sub-Carrier Generation

An alternative way to generate a single sideband is to phase lock two lasers with a given frequency offset. This had been proposed for Advanced LIGO.
Advanced LIGO modulators

Three sets of modulation electrodes on one crystal
Modulator Crystals

To avoid interference effects from reflections off the modulator faces, the crystal has a 2x2.8° wedge

The wedge leads to polarization dependent transmission angle, and thus the modulator also acts like a polarizer

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Angle [degrees]</th>
</tr>
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<tbody>
<tr>
<td>p</td>
<td>4.81</td>
</tr>
<tr>
<td>s</td>
<td>4.31</td>
</tr>
</tbody>
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Steering by RTP crystal for s- and p- polarization
The resonant circuit is designed to have 50Ω impedance at the resonant frequency, but high impedance at DC and low frequencies.


M Gray, D Shaddock, C Mow-Lowry, D. McClelland “Tunable Power Recycled RSE Michelson for LIGO II”, Internal LIGO document G000227-00-D
