## Measurement \& Precision

When we use the measurements of two or more quantities to calculate some value the uncertainty in each measurement contributes to uncertainty in the calculated value.
For example, if we measure the circumference of a circle to be 31.0 cm $\pm 1.0 \mathrm{~cm}$, and its diameter to be $10.0 \mathrm{~cm} \pm 0.5 \mathrm{~cm}$ what value do we get for $\pi$, the ratio of the circumference and diameter?

The maximum value that is consistent with our measurements is $\Pi_{\max }=32.0 \mathrm{~cm} / 9.5 \mathrm{~cm}=3.37$, while the minimum value is $\Pi_{\text {min }}=30 \mathrm{~cm} / 10.5$ $\mathrm{cm}=2.86$, but our most likely value is $\Pi_{\text {meas }}=31 \mathrm{~cm} / 10 \mathrm{~cm}=3.10$

The formula for determining the uncertainty in a calculated quantity depends on how the quantity is calculated. If the calculation involves the product or quotient of (uncorrelated) measured values such that

$$
z=x y \text { or } z=x / y
$$

then the uncertainty is

$$
\left(\frac{\Delta z}{z}\right)^{2}=\left(\frac{\Delta x}{x}\right)^{2}+\left(\frac{\Delta y}{y}\right)^{2}
$$

If the calculation involves the sum or difference of measured quantities such that

$$
z=x+y \text { or } z=x-y
$$

then the uncertainty is

$$
\Delta z^{2}=\Delta x^{2}+\Delta y^{2}
$$

We express the calculated values as $z \pm \Delta z$. Note that the units of $\Delta z$ are the same as those of $z$ (they have to be!)
In the example above, what is $x$ and $y$ ? Which formula do we use to find the uncertainty in our measurement of $\pi$ ? What is $z$ in our uncertainty formula? What is $\Delta z$ ? How should we express our measured value of $\pi$ ?

Bonus:What happens to the uncertainty of our measurement if we average the results of several measurements?

