EXPERIMENT 1: MEASUREMENT AND PRECISION

Objective:

To determine the value of π from the ratio of the circumference to the diameter of a circle. (Actually, the real point of this lab is to introduce you to the concept of uncertainty in measurements.)

Theory:

It has been known since ancient times that the ratio of the circumference of a circle to its diameter is equal to the transcendental number represented by π. That is,

$$\pi = C/D$$

The instruments you use to measure this have a limit to their precision. You will have to estimate to the nearest mark on the scale you use (use centimeters!). As a general rule your uncertainty in a measurement can never be less than $\pm 1/2$ of the smallest ruled increment on your measurement tool, but can be higher if there are larger sources of uncertainty than those inherent in the measurement tool. Can you think of an example of a measurement where the uncertainty is greater than the precision of the measurement tool?

One method to estimate the precision of a measurement is to repeat the measurement several times. When you measure a value several times and get slightly different results, you express your measured value as $x_0 \pm \delta x$ where x_0 is the average value form your measurements and δx is called the uncertainty and is half the deviation from the largest to smallest measured values.

Note that the number of significant figures you include in your **results** is limited by the number of significant figures in the least precise measurement in your **data.** Suppose you measured the circumference to be $C = 5.08$ cm (which has 3 significant figures) and you measured the diameter to be $D=1.48$ cm (which also has 3 significant figures). According to your calculator, the ratio C/D is equal to 3.432432432. This has 10 significant figures! You must round off this answer to **three** significant figures and write $C/D = 3.43$.

Numbers containing too many significant figures are **wrong** because they imply a precision that is not justified by your measurements! For the same reason quantities calculated from measured values that do not include the uncertainty are **wrong**. To reiterate, your final result should never have more significant figures than your least-precise measurement, and should always be written with the associated uncertainty.

Procedure:

Work in groups of two. Each student will measure the diameter and the circumference of each disk several times. The group results will be recorded in a table similar to the one shown on the next page. Draw your own data table. Make sure it has a descriptive title and contains units in the headers of the appropriate columns.

Analysis:

1) For each measurement of circumference (C) and diameter (D), determine the value of π by taking the ratio C/D. Since both C and D have uncertainty in their measurements, be careful not to quote more than the appropriate number of significant figures in your value of π and to include the uncertainty in your result. To calculate the uncertainty in π given your measured uncertainties in C and D you can use the rules shown in the last section entitled "A note on error analysis"

2) Determine your measured value of π by averaging the results from each disk. You will have to use the formulas given in the last section title "A Note on Experimental Uncertainty" to determine the uncertainty in this value given the calculated uncertainty in π measured for each disk.

3) Determine the percent difference between each value of π you have calculated and the accepted value of $\pi=3.14159...$ Your accepted value of π should only contain the same number of significant figures as your calculated value.

$$\% \text{ Difference} = \left| \frac{\pi_{calculated} - \pi_{accepted}}{\pi_{accepted}} \right| \times 100\%$$

Example Data and Results Table:

Disk #	Diameter(D)	Circumference (C)	π=C/D
1	12.4±0.2 cm	39.3±0.8 cm	3.17±0.08

In the example in the data table the calculated value of π is 3.17±0.08 meaning, within the experimental uncertainty a value as low as 3.09 or as high as 3.25 would be consistent with our measurement, therefore this example measurement agrees with the accepted value of π. If the experimental uncertainty were not included however, the values would not agree since 3.17≠3.14.

Report:

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1) An appropriate title
2) The objective of the experiment
3) Your group's data in an appropriate format
4) A graph of your group's data
5) A summary of your results giving your value for π and a comparison to the accepted value (i.e. did your measured value agree with the accepted value for π?).

A Note on Experimental Uncertainty

The degree to which a measured value is regarded as representative of the actual value is called or divided to determine the value of a calculated quantity ($z=xy$ or $z=x/y$), the uncertainty of the calculated quantity, δz, is found from the relation

$$\frac{\delta z}{z}=\sqrt{\left(\frac{\delta x}{x}\right)^2+\left(\frac{\delta y}{y}\right)^2}$$

In contrast, when two measurements that each have uncertainty are added or subtracted to determine the value of a calculated quantity ($z=x+y$ or $z=x-y$), the uncertainty of the calculated quantity, δz, is found from the relation

$$\delta z=\sqrt{(\delta x)^2+(\delta y)^2}$$

The above rule can be applied to determine the uncertainty in an average of N measurements of the same quantity that each have an (uncorrelated) uncertainty of δz_i to give an uncertainty in the average of

$$\delta z_{avg}=\frac{1}{N}\sqrt{\sum_{i=1}^{N}(\delta z_i)^2}$$

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