

# Distributed Space-time Codes with Preprocessing over Multiple Stages of Relays

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**Abstract**—In a distributed wireless network that operates in low to moderate signal-to-noise (SNR) environment, it is crucial to design relay strategies so that the gains obtained from distributed space-time codes (DSTCs) are close to the gains of multi-antenna systems. For such networks, we propose a strategy that utilize multistage relays in which each stage maps their received signal through a non-linear function and utilizes DSTCs afterward. The proposed scheme outperforms the well-known amplify-and-forward and decode-and-forward strategies.

## I. INTRODUCTION

Wireless networks are required to provide higher and higher data rates due to increased consumer demand. In a point-to-point communication system, it was shown that multi-antennas can dramatically increase data throughput and enhance reliability of wireless channels [1]–[3]. However, in many wireless networks such as sensor networks, implementing multi-antennas may be impractical because the nodes require tiny size and simple hardware. In these cases, cooperative communication is introduced to achieve diversity and multiplexing gains without multi-antennas. The main idea is to allow multiple single-antenna nodes to jointly transmit and receive, forming a virtual multi-antenna system.

Numerous practical schemes have been shown to improve the performance of cooperative networks with a single relay [4], [5]. The performance of cooperative networks can be further improved by using multiple relays between source(s) and destination(s). Distributed space-time codes (DSTCs) are proposed for multi-relay networks [6]–[8]. In this case, relays either decode-and-forward (DF) the received message or amplify-and-forward (AF) it before using DSTCs. In our previous work [9], we studied a network formed by a single source-destination pair and multiple cooperative relays. We proposed a new relay strategy called *saturated amplify-and-forward* (SAF) to preprocess the received

signals at relays before forwarding it using DSTCs. This scheme outperforms commonly used strategies such as DF and AF. It is important to note that DF based schemes are reasonable when the relays are in high SNR environment but they are not applicable to relays in low SNR region because only a few relays (or maybe none of the relays) can successfully decode the received message. However, the nodes in wireless sensor network are limited in their battery power, so the system typically works in the low to moderate SNR range. The scheme we previous proposed [9] eliminates the need for error-free relay decoding and allows as many relays as possible to cooperate.

The main contribution of this work is that we extend single stage relaying strategies proposed in our earlier work [9] to multiple stages. This brings various questions to be answered. How many stages of relays are required? How do relay locations affect the performance? How should do processing be at each stage? In order to answer these questions, we explore the optimal preprocessing function in addition to optimal number of stages required.

The rest of the paper is organized as follows. In section II, we describe the system model, in which three subsections, matched filtering, nonlinear mapping, and Alamouti decoding, are explored in detail. The section III presents the simulation results. And finally the paper ends with conclusion in section IV.

## II. SYSTEM MODEL

We consider a network composed of a single source-destination pair and  $L$  relay stages with  $T$  relay nodes in each stage (see Fig. 1). Each relay node is assumed to have a single antenna and each relay stage can be treated as a virtual multi-antenna system. The transmission process consists of  $L + 1$  sessions (time-slots). In the first session, the source node broadcasts the message.

In the following  $L$  sessions, the relays at each stage cooperate to retransmit the source message to the next stage consecutively. Note that the retransmitted message depends only on the last received message at that relay stage.

We consider a network in which a source node ( $S$ ) transmits over multiple stages of relays to reach a destination node ( $D$ ). Each stage is composed of  $T$  relays ( $R_1^{(i)}, \dots, R_T^{(i)}$ ) where superscript ( $i$ ) denotes the stage index.

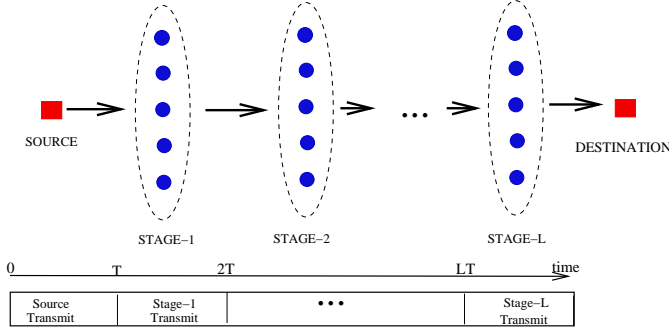


Fig. 1. Multistage cooperative relays

Let  $\mathbf{s} = [s[1] \dots s[n]]^t$  denote the message block transmitted by the source with average power  $P_s$ , i.e.,  $\mathbb{E}\{|s[k]|^2\} = P_s$ . We consider BSPK (or QPSK) modulated symbols. Let  $\mathbf{r}_i^{(1)} = [r_{i1}^{(1)} \dots r_{in}^{(1)}]^t \in \mathbb{C}^n$  denote the received signal at the  $i$ 'th relay in the first stage after source's transmission:

$$\mathbf{r}_i^{(1)} = h_i^{(1)} \mathbf{s} + \mathbf{w}_i^{(1)}, \quad (1)$$

where  $\mathbf{w}_i^{(1)} = [w_{i1}^{(1)} \dots w_{in}^{(1)}]^t \sim \mathcal{N}_c(0, \sigma^2 \mathbf{I})$  denotes the additive white Gaussian noise (AWGN) at the relay. The flat fading Rayleigh channel coefficient between source and the  $i$ 'th relay is denoted by  $h_i^{(1)}$ .

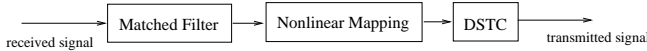


Fig. 2. Single relay processing: matched filtering, nonlinear mapping, and DSTC

The relays utilize a space-time code after preprocessing the received signal (see Fig. 2). The preprocessing is composed of matched filtering and symbol-by-symbol mapping of the matched filtered signal. Below we provide a detailed description of each block in Fig. 2.

### A. Distributed Space-time Codes

The received signal at the  $i$ th node in the  $\ell$ th stage due to transmission of  $T$  nodes in the previous stage is

given as follows:

$$\mathbf{r}_i^{(\ell)} := \begin{bmatrix} r_i^{(\ell)}[1] \\ \vdots \\ r_i^{(\ell)}[N] \end{bmatrix} = \mathbf{G}_{N \times T}^{(\ell-1)} \underbrace{\begin{bmatrix} h_{i1}^{(\ell)} \\ \vdots \\ h_{iT}^{(\ell)} \end{bmatrix}}_{\mathbf{h}_i^\ell} + \underbrace{\begin{bmatrix} n_i^{(\ell)}[1] \\ \vdots \\ n_i^{(\ell)}[N] \end{bmatrix}}_{\mathbf{n}_i^\ell}.$$

Here  $\mathbf{h}_i^\ell$  denotes the channel coefficient vector between the  $i$ th node in the  $\ell$ th stage and the nodes in the  $(\ell-1)$ th stage. The noise vector  $\mathbf{n}_i^{(\ell)} \sim \mathcal{N}_c(0, \sigma^2 \mathbf{I})$ . The matrix  $\mathbf{G}_{N \times T}^{(\ell-1)}$  denotes the space-time code and it is a function of preprocessed symbols in the previous stage relays, i.e.  $\tilde{\mathbf{s}}_1^{(\ell-1)}, \dots, \tilde{\mathbf{s}}_T^{(\ell-1)}$ . For example, if each relay stage have two relays  $T = 2$ , then the Alamouti space-time code can be utilized where

$$\mathbf{G}_{N \times T}^{(\ell)} = \begin{bmatrix} \tilde{s}_1^{(\ell)}[1] & \tilde{s}_2^{(\ell)}[2] \\ -\tilde{s}_1^{(\ell)*}[2] & \tilde{s}_2^{(\ell)*}[1] \end{bmatrix}.$$

Here  $\tilde{s}_i^{(\ell)}[k]$  denotes the  $k$ th symbol estimate at the  $i$ th node in the first-level. If the estimations are perfect, i.e.  $\tilde{s}_1^{(\ell)}[k] = \tilde{s}_2^{(\ell)}[k] = s[k]$ , then  $\mathbf{G}_{N \times T}^{(\ell)}$  is the same as standard Alamouti matrix in multi-antenna systems [12].

### B. Nonlinear Mapping

Let  $f_\ell: \mathbb{R} \rightarrow \mathbb{R}$  denote the mapping utilized at the  $\ell$ th stage relays. Let  $\tilde{\mathbf{s}}_i^{(\ell)} = [\tilde{s}_i^{(\ell)}[1] \dots \tilde{s}_i^{(\ell)}[n]]^t$  denote the preprocessed symbol block by the  $i$ 'th relay in the first stage such that

$$\begin{aligned} \text{Re}(\tilde{s}_i^{(\ell)}[k]) &= f_\ell(\text{Re}(\tilde{r}_i^{(\ell)}[k])), \\ \text{Im}(\tilde{s}_i^{(\ell)}[k]) &= f_\ell(\text{Im}(\tilde{r}_i^{(\ell)}[k])) \end{aligned} \quad (2)$$

for  $i = 1, \dots, T$ ,  $k = 1, \dots, n$ , where  $\text{Re}(x)$  and  $\text{Im}(x)$  denote the real and imaginary part of  $\mathbf{x}$ , respectively. We will define  $f_\ell(r_i^{(\ell)}[k]) := f_\ell(\text{Re}(r_i^{(\ell)}[k])) + j f_\ell(\text{Im}(r_i^{(\ell)}[k]))$ <sup>1</sup>. Note that the mapping should satisfy the transmission relay power constraint  $P_i$ , i.e.,

$$\mathbb{E}\{|\tilde{s}_i^{(\ell)}[k]|^2\} = \mathbb{E}\{|f_\ell(\tilde{r}_i^{(\ell)}[k])|^2\} \leq P_i^{(\ell)}.$$

Here  $\tilde{r}_i^{(\ell)}[k]$  denotes the output of the matched filter at the  $i$ th relay at  $\ell$ th level.

Our ultimate goal is to find the best function  $f_\ell(\cdot) = f(\cdot)$  such that the average error probability under the optimal maximum likelihood detector is minimized. Optimization over functionals is a challenging problem [10], [11]; hence, we narrow down the class of functions through parametrization.

<sup>1</sup>One can use two different functions for imaginary and real part mapping for different modulations.

In literature, two very popular relaying schemes are amplify-and-forward (AF) and decode-and-forward (DF). In AF, the relay node transmits the received signal after scaling it to its power level. In DF, the relay node decodes and re-encodes the message. We propose saturated amplify-and-forward scheme in which the nodes utilize a saturated version of the amplify-and-forward strategy [9]. Let

$$w_i[k] = \begin{cases} 1 & \text{if } \text{Re}(\tilde{r}_i[k]) > \tau \\ -1 & \text{if } \text{Re}(\tilde{r}_i[k]) < -\tau \\ \text{Re}(\tilde{r}_i[k])/\tau & \text{otherwise.} \end{cases}$$

The output of the mapping at the  $i$ 'th relay is  $\tilde{s}_i[k] = \sqrt{P_i}w_i[k]/\sqrt{\mathbb{E}|u_i[k]|^2}$ . Note that both DF (when  $\tau = 0$ ) and AF (when  $\tau = \infty$ ) strategies are subclasses of SAF.

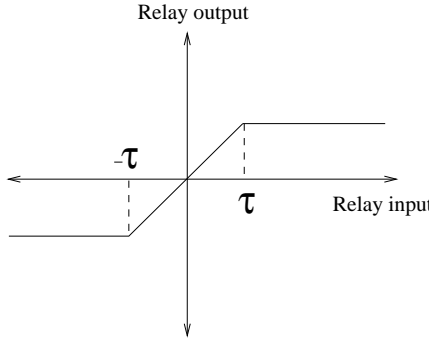


Fig. 3. Saturated amplify-and-forward

As shown in Fig. 3, the saturated amplify-and-forward (SAF) has a flexible parameter to control  $\tau$ . When it applies to a real system, we can use exclusive searching to figure out the best  $\tau$  so that the average symbol error probability under the optimal maximum likelihood detector is minimized.

### C. Matched Filtering

For the first stage relay, matched filtering is done simply as

$$\tilde{\mathbf{r}}_i^{(1)} := \frac{h_i^{(1)H}}{\|h_i^{(1)}\|} \mathbf{r}_i^{(1)}. \quad (3)$$

where  $\mathbf{r}_i^{(1)}$  is given in Eqn. 1 and  $h_i^{(1)}$  is the channel coefficient of the link from the source to the  $i$ th relay in the first stage.

The above equations for stage-1 can be easily generalized to stages  $\ell \geq 2$ . In that case, the matched filter is

defined as  $\mathbf{H}^h \bar{\mathbf{r}}_i^{(\ell)}$ . For example, in the case of Alamouti coding,

$$\mathbf{H}_i^{(\ell)} = \begin{bmatrix} h_{i1}^{(\ell)} & h_{i2}^{(\ell)} \\ h_{i2}^{*(\ell)} & -h_{i1}^{*(\ell)} \end{bmatrix},$$

and  $\bar{\mathbf{r}}_i^{(\ell)} = [r_i^{(\ell)} [1] r_i^{*(\ell)} [2]]^t$ .

We assume that the destination has perfect estimates of the channel coefficients.

### D. Simulations

We consider a network composed of one source-destination pair and  $L$  relay stages with two relays at each stage. It is assumed that the first relay decodes received message perfectly, the second relay has received signal at 5.3 dB and the transmission powers of source and relays are equal to 1. BPSK is chosen as modulation type. The nodes utilize standard Alamouti decoder for simplicity. The channel coefficients are independent and identically distributed as complex Gaussian random variables. The path loss model is  $\min\{1, 1/d^\alpha\}$ . The variance of AWGN noise is denoted by  $N_0$ . In Fig. 4, it shows average probability of error for the schemes, AD, DF, Minimum Mean Square Estimator (MMSE) and SAF, as a function of  $10 \log_{10}(1/N_0)$ .  $L$  stages of relays are regularly placed between source and destination that are 4m apart. The performance of SAF is optimized over the threshold  $\tau$  at each stage. The newly proposed scheme SAF outperforms uniformly for any SNR. In addition,  $L = 3$  is the optimal number of stages for the given source/destination distance.

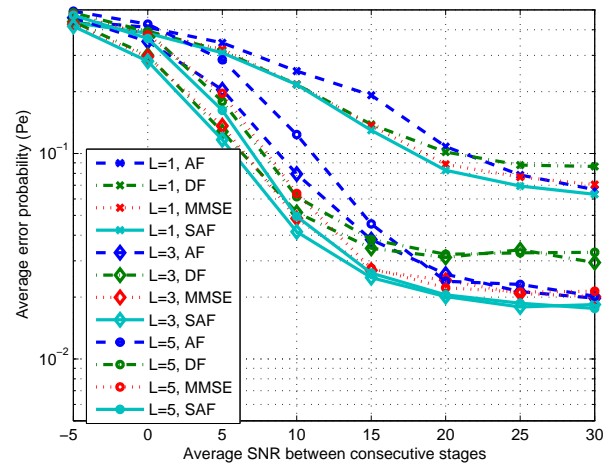


Fig. 4. The comparison of proposed schemes:  $\alpha = 2$ ,  $d_{SD} = 4m$

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