Basic Biostatistics

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Statistics for Public Health Practice

Chapter 17
Comparing Two Proportions

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Data conditions

- Binary response
- Binary explanatory factor via two independent groups:
  - Group 1 = "exposed"
  - Group 2 = "nonexposed"
- Notation:

Group	Number of successes	Sample size
1 (exposed)	a_1	n_1
2 (nonexposed)	a_2	n_2

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Sample Proportions

Sample proportion (average risk), group 1:

$$\hat{p}_1 = \frac{a_1}{n_1}$$

Sample proportion (average risk), group 2:

$$\hat{p}_2 = \frac{a_2}{n_2}$$

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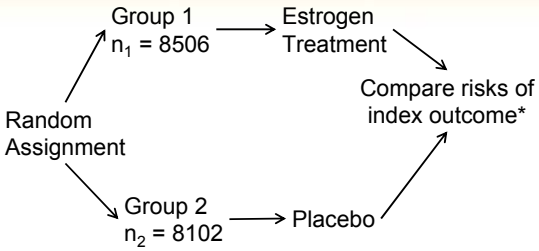
Illustrative Example: WHI Trial



*Death, MI, breast cancer, etc.

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2-by-2 Table

	Successes	Failures	Total
Group 1	a_1	b_1	n_1
Group 2	a_2	b_2	n_2
Total	m_1	m_2	N

$$\hat{p}_1 = \frac{a_1}{n_1} \qquad \hat{p}_2 = \frac{a_2}{n_2}$$

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WHI Data

	D+	D−	Total
E+	751	7755	8506
E-	623	7479	8102
Total	1374	15234	16608

$$\hat{p}_1 = \frac{751}{8506} = 0.08829$$

$$\hat{p}_2 = \frac{623}{8102} = 0.07689$$

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§17.3 Hypothesis Test

A. H_0: $p_1 = p_2$ (equivalently H_0: $RR = 1$)

B. Test statistic (three options)

- z (large samples)
- Chi-square (large samples, next chapter)
- Fisher's exact (any size sample)

C. P-value

D. Interpret $\Rightarrow$ evidence against the claim of H_0

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Fisher's Exact Test

A. Hypotheses.
H_0: $p_1 = p_2$ vs.
H_a: $p_1 \neq p_2$ [two-sided] OR H_a: $p_1 > p_2$ [left-sided] OR H_a: $p_1 < p_2$ [right-sided]

B. Test statistic. None, *per se;* reiterate data

***C.* *P*-value.** Use WinPepi > Compare2.exe > Program A

D. Interpretation: Level of evidence against H_0

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Example: Fisher's Test

Colonic necrosis. The incidence of colonic necrosis in an exposed group is 2 of 117. The incidence in a non-exposed group is 0 of 862.

Ask: Is this difference statistically significant?

A.Hypothesis statements. Under the null hypothesis, there is no difference in risks in the two populations. Thus:

H_0: $p_1 = p_2$

H_a: $p_1 > p_2$ (one-sided) or H_a: $p_1 \neq p_2$ (two-sided)

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Fisher's Test, Example

B. Test statistic $\Rightarrow$ none *per se,* other than data (right)

	D+	D−
E+	2	115
E−	0	862

C. *P*-value. Use WinPepi > Compare2.exe > A.

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Exact tests:
  Fisher's P:
    One-tailed:          P = 0.014 or 1.000
    Two-tailed:          P = 0.014
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D. Interpret. The P-value of .014 $\Rightarrow$ strong ("significant") evidence against H_0

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§17.4 Proportion Ratio **(Relative Risk)**

- Relative risk is used to refer to the RATIO of any two public health proportions
- Since incidence proportions represent average risks, then this ratio is also called a risk ratio:

$$\hat{RR} = \frac{\hat{p}_1}{\hat{p}_2}$$

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Example: RR (WHI Data)

	+	−	Total
Estrogen +	751	7755	8506
Estrogen −	623	7479	8102

$$\hat{p}_1 = \frac{751}{8506} = 0.08829; \hat{p}_2 = \frac{623}{8102} = 0.07689$$

$$\hat{RR} = \frac{\hat{p}_1}{\hat{p}_2} = \frac{0.08829}{0.07689} = 1.1483 \approx 1.15$$

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Example: RR (WHI Data)

- The *RR* is a risk multiplier, e.g., the RR estimate of 1.15 suggests the risk in the exposed group is "1.15 times" that of the nonexposed group
- When $p_1 = p_2$, $RR = 1$. Thus, the baseline RR, indicating no association, is 1 $\Rightarrow$ to get the percent change in risk, subtract 1 from the RR estimate: $RR - 1 = 1.15 - 1 = 0.15 = 15\%$ change in risk
- See pp. 389 – 390 for full discussion

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(1– α)100% CI for the *RR*

$$e^{\ln \hat{RR} \pm z_{1-\frac{\alpha}{2}} \cdot SE_{\ln \hat{RR}}}$$

$$\text{where } SE_{\ln \hat{RR}} = \sqrt{\frac{1}{a_1} - \frac{1}{n_1} + \frac{1}{a_2} - \frac{1}{n_2}}$$

ln ≡ natural log, base e

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90% CI for *RR*, WHI

	D+	D−	Total
E+	751	7755	8506
E−	623	7479	8102

$$\ln \hat{RR} = \ln(1.1483) = 0.1382$$

$$SE_{\ln \hat{RR}} = \sqrt{\frac{1}{751} - \frac{1}{8506} + \frac{1}{623} - \frac{1}{8102}} = 0.051920$$

For 90% confidence, $z = 1.645$

$$e^{0.1382 \pm (1.645)(0.051920)} = e^{0.1382 \pm 0.0854} = e^{0.0528, 0.2236}$$

$$= (1.05, 1.25)$$

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WinPepi > Compare2.exe > Program B

	D+	D−	Total
E+	751	7755	8506
E −	623	7479	8102

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RATIO [A:B] = 1.148     S.E. of log ratio = 0.052
 90% confidence interval = 1.05 to 1.25
 95% confidence interval = 1.04 to 1.27
 99% confidence interval = 1.00 to 1.31
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See prior slide for hand calculations

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§17.5 Systematic Error

- CIs and P-values address random error only
- In observational studies, systematic errors are more important than random error
- Consider three types of systematic errors:
  - Confounding
  - Information bias
  - Selection bias

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Confounding

- Confounding = mixing together of the effects of the explanatory variable with the extraneous factors.
- Example:
  - WHI trial found 15% increase in risk in estrogen exposed group.
  - Earlier observational studies found 40% lower in estrogen exposed groups.
  - Plausible explanation: Confounding by extraneous lifestyles factors in observational studies

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Information Bias

- **Information bias** - mismeasurement (misclassification) leading to overestimation or underestimation in risk
- **Nondifferential misclassification** (occurs to the same extent in the groups) ⇒ tends to bias results toward the null or have no effect
- **Differential misclassification** (one groups experiences a greater degree of misclassification than the other) ⇒ bias can be in either direction.

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Nondifferential & Differential Misclassification - Examples

Scenario A: Nondifferential misclassification: 10% of the cases in the exposed and nonexposed groups are misclassified as noncases.

	Accurate data +	Accurate data −	Total		Misclassified data +	Misclassified data −	Total
Exposed	100	900	1000	→	90	910	1000
Nonexposed	50	450	500		45	455	500

$\widehat{RR} = \frac{100/1000}{50/500} = 1.00$ $\widehat{RR} = \frac{90/1000}{45/500} = 1.00$

Scenario B: Differential misclassification: none (0%) of the cases are misclassified in the exposed group, while 10% are misclassified in the nonexposed group.

	Accurate data +	Accurate data −	Total		Misclassified data +	Misclassified data −	Total
Exposed	100	900	1000	→	100	900	1000
Nonexposed	50	450	500		45	455	500

$\widehat{RR} = \frac{100/1000}{50/500} = 1.00$ $\widehat{RR} = \frac{100/1000}{45/500} = 1.11$

Selection Bias

- Selection bias ≡ systematic error related to manner in which study participants are selected
- *Example.* If we shoot an arrow into the broad side of a barn and draw a bull's-eye where it had landed, have we identified anything that is nonrandom?

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Sample Size & Power for Comparing Proportions

Three approaches:

1. *n* needed to estimate given effect with margin of error *m* (not covered in Ch 17)
2. *n* needed to test H_0 at given α and power
3. Power of test of H_0 under given conditions

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Sample Size Requirements for Comparing Proportions

Depends on:

r ≡ sample size ratio = n_1 / n_2

$1-\beta$ ≡ power (acceptable type II error rate)

α ≡ significance level (type I error rate)

p_1 ≡ expected proportion, group 1

p_2 ≡ expected proportion in group 2, or expected effect size (e.g., *RR)*

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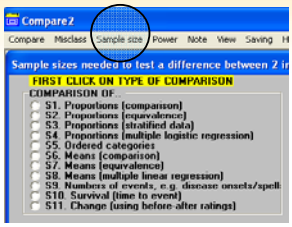
Calculation

- Formulas on pp. 396 – 402 (quite complex)
- In practice ⇒ use tables or (better yet) computer programs
- WinPEPI > Compare2.exe > Sample size



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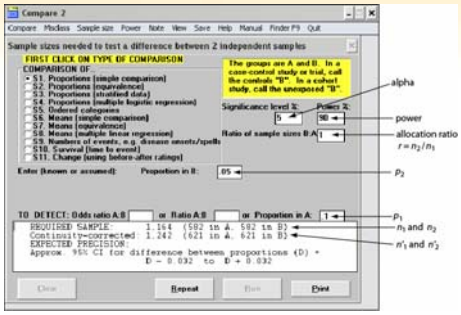
WinPepi > Compare2 > S1



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