5: Introduction to Estimation

Wisdom begins with the acknowledgement of uncertainty—of the *limits* of what we know.

Statistical Inference

p. 5.1

- Statistical inference = generalizing from a sample to a population with calculated degree of certainty
- Two traditional forms of statistical inference
  - Estimation (introduced this chapter)
  - Hypothesis testing (introduced next chapter)

Parameters and Estimates

p. 5.1

- Parameter ≡ numerical characteristic of a *population*, such as a population mean
- Estimate ≡ numerical characteristic of a *sample*, such as a sample mean
- Parameters and statistical estimates are related but are *not* the same

	Parameters	Estimates
Source:	From population	From sample
Notation:	Greek characters	Roman characters
Math:	Constants	Random variables
Calclated?	Not calculated	Calculated

The Sampling Distribution of the Mean:
A Model for Dealing With Random Error

p. 5.2

- A sample must be viewed as an example of other possible samples taken from the same population
- Each sample would have a different mean
- Each sample means would vary
- All possible means based on a given *n* would form a hypothetical distribution called:

  The Sampling Distribution of the Mean

Model for Dealing With Random Error (cont.)

p. 5.2

- SDM = *Sampling Distribution of Mean*
- Key facts:
  - Hypothetical (ya' won't see it)
  - Models random sampling error (not systematic error)
  - Basis for many inferential techniques
  - *Not* the same as sample distribution

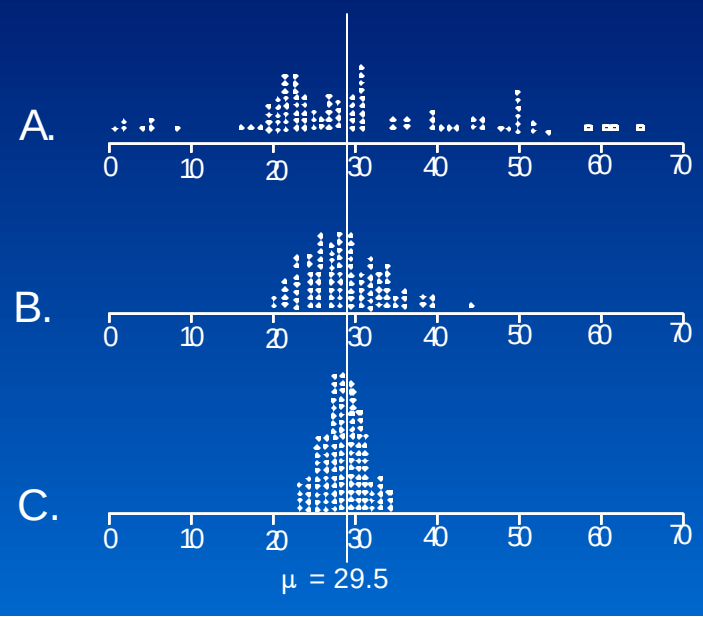
Sampling Experiment

p. 5.2

- Start with `populati.sav` , `AGE`
  - $N = 600$
  - $\mu = 29.5$
  - $\sigma = 13.6$
  - Data *not* normally distributed
- Sampling experiments:
  - Experiment A: 100 samples of $n = 1$ each, plot
  - Experiment B: 100 samples of $n = 10$ each, calculate 100 means, plot means
  - Experiment C: 100 samples of $n = 30$ each, calculate 100 means, plot means

Results of Sampling Experiments

p. 5.2



Three Properties of SDMs*

Recall SDM = Sampling Distribution of the Mean

- As *n* increases, becomes more normal
  Central Limit Theorem
- Centered on μ
  Law of Averages
- As *n* increases, clusters more tightly on μ
  Law of Large Numbers

Standard Error of The Mean

p. 5.4

- Related to Law of Large Numbers
- As *n* increases:
  - Standard deviation of SDM *decreases*
  - Precision of estimate *increases*
- Quantify imprecision with $\sigma_{xbar} \equiv$ SEM

$$SEM = \frac{s}{\sqrt{n}}$$

Formula 5.1

$$sem = \frac{s}{\sqrt{n}}$$

Formula 5.2

SEM: Illustrative Examples

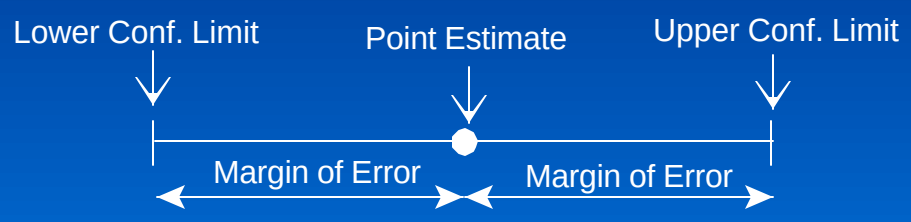
p. 5.4

- Illustrative example: `sample10.sav AGE`
  - $\sigma = 13.6$, $n = 10$
  - SEM = 13.6 / √10 = 4.30
- Illustrative example: `sample40.sav, AGE`
  - $\sigma = 13.6$, $n = 40$
  - SEM = 13.6 / √40 = 2.15
- Illustrative example: `sample10.sav, AGE`
  - If don't known σ, calculate *s*
  - $s = 15.4$, $n = 10$
  - sem = 15.4 / √10 = 4.87

Confidence Intervals: The General Idea

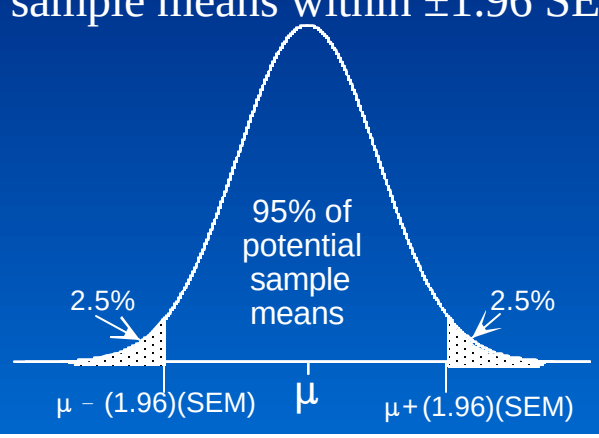
p. 5.5

- $\bar{x}$ is valid *point* of μ
  - but provides no info. about precision
- Surround estimate with margin of error (d)
  - lower confidence limit = sample mean $-$ d
  - upper confidence limit = sample mean $+$ d



Constructing a Confidence Interval
(p. 5.4 Notes)

- Recall that SDM is:
  - Normally distributed (Central Limit Theorem)
  - Centered on μ (Law of Averages)
  - Variability $\rightarrow \sigma_{xbar} \equiv SEM = \sigma / \sqrt{n}$ (Law of Large Numbers)
- $\therefore$ 95% of sample means within ± 1.96 SEM of μ:



95% Confidence Interval for μ, σ Given

p. 5.5

- Applying algebraic manipulation derives Form. 5.3:

$$\bar{x} \pm (1.96)(SEM)$$

- Illustrative example (`sample10.sav, AGE`)
  - n = 10, sample mean = 29.0, SEM = 4.30
  - 95% confidence interval for μ
    = 29.0 ± (1.96)(4.30)
    = 29.0 ± 8.4
    = (20.6, 37.4)
  - Interpretation of confidence interval: chance is in sampling process, not in parameter

Other Levels of Confidence

p. 5.6

- Confidence level
  - α ≡ lack of confidence; 1 − α ≡ confidence
  - e.g., when α = .05, 1 − α = .95
- Formula 5.4: $\bar{x} \pm (z_{1-a/2})(SEM)$

α	confidence level	$z_{1-\alpha/2}$
.10	90%	1.645
.05	95%	1.96
.01	99%	2.58

Other Levels of Confidence: Example

p. 5.6

$$\bar{x} \pm (z_{1-\alpha/2})(SEM)$$

- Recall `sample10.sav, AGE`
  - $n = 10$
  - sample mean = 29.0
  - SEM = 4.30
- For 90% confidence, $\alpha = .10 \rightarrow z_{1-(.10/2)} = z_{.95} = 1.645$
- Calculate using Formula 5.4:

  $= 29.0 \pm (1.645)(4.30)$
  $= 29.0 \pm 7.1$
  $= (20.6, 37.4)$

When σ is Not Known

p. 5.7

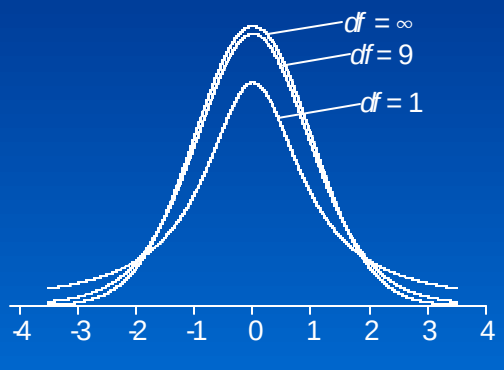
- Z error model assumes
  - normal distribution
  - σ was known without calculating s
- By calculating s, you add another source of uncertainty
- William Sealy Gosset developed method for addressing this additional uncertainty
  - Gosset published under the name "Student"

'Student' in 1908

Student's *t* Distribution

p. 5.7

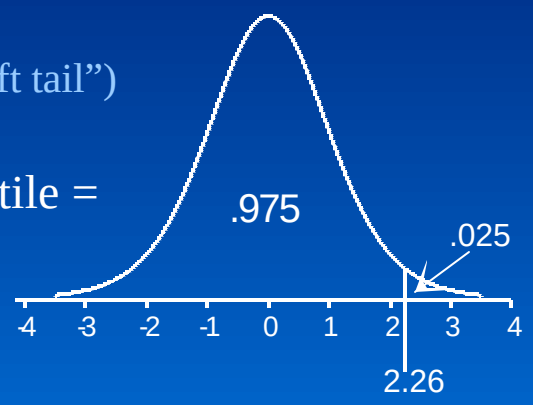
- Family of probability distributions w/ members identified by degrees of freedom (df)
- Looks like Z with broad tails
- Less df → broader tails



t Table

Appendix 2

- Probabilities for *t* w/df = 1–40, 50, 100, ∞
  - For other df, use closest listing
  - For df > 100, use ∞ df
- Table set-up
  - left column = df
  - top row = cumulative probability ("left tail")
  - second row = right tail
- Example: for df = 9, 97.5th percentile = 2.26



Notation

p. 5.7

- $t_{df,p}$ = *t* percentile with df and cumulative probability *p*
- Examples
  - $t_{9,.975}$ = 2.26 (see prior slide)
  - $t_{9,.025}$ = −2.26 (by symmetry)
  - $t_{anything,.5}$ = 0 (since *t* centers on 0)
  - $t_{13,.995}$ = 3.01 (from *t* table)

Confidence Interval for μ Using *s*

p. 5.8

- Formula 5.5 $\bar{x} \pm (t_{n-1,1-a/2})(sem)$
  - where
    - xbar = sample mean
    - $t_{n\text{-}1,1\text{-}\alpha/2}$ = *t* with *n* − 1 df and cumulative probability of 1 − α/2 (from *t* table)
    - sem = estimate standard error of mean = $s / \sqrt{n}$

Illustrative Example

p. 5.8

$\bar{x} \pm (t_{n-1,1-a/2})(sem)$

- `%ideal.sav` : Body weight expressed as percent of ideal (e.g., 100 = 100% of ideal body weight)
- Data
  - *n* = 18 diabetics
  - sample mean = 112.778 (calculated)
  - *s* = 14.424 (calculated)
- 95% confidence interval for μ

$$= 112.78 \pm (t_{18\text{-}1,1\text{-}.05/2})(14.424 / \sqrt{18})$$
$$= 112.78 \pm (t_{17,.975})(3.400)$$
$$= 112.78 \pm (2.11)(3.400)$$
$$= 112.78 \pm 7.17$$
$$= (105.61, 119.95)$$

Sample Size Requirements

p. 5.9

- Formula 5.6 $$n = \frac{4s^2}{d^2}$$
  - where
    - d = margin of error
    - σ = assumed standard deviation (from prior research or knowledge)
- Illustrative example
  - Suppose you want $d = 5$ and assume σ = 15

$$n = \frac{(4)(15^2)}{5^2} = 36$$