7: Paired Samples

p. 7.1

Prior chapters→ [mostly] one-sample

This chapter & next→ two-samples

Two types of two-sample problems
- Paired (“dependent”) samples
- Independent

Paired samples = each data point of first sample *matched* to unique point in second sample
- e.g.,

Independent samples = data points of one sample ***unrelated*** to the points in second
- e.g., group 1 cf. to [unrelated] group 2

Illustrative OATBRAN.SAV

p. 7.1

Outcome variable = LDL (“bad”) cholesterol (mg/dl)

Sample 1 = LDL on `CORNFLK` diet

Sample 2 = LDL on `OATBRAN` diet

Cross-over trial with washout period

ID	CORNFLK	OATBRAN
1	4.61	3.84
2	6.42	5.57
3	5.40	5.85
4	4.54	4.80
5	3.98	3.68
6	3.82	2.96
7	5.01	4.41
8	4.34	3.72
9	3.80	3.49
10	4.56	3.84
11	5.35	5.26
12	3.89	3.73
13	2.25	1.84
14	4.24	4.14

Exploration

p. 7.3

- $n \equiv$ number of *pairs*
  - e.g., $n = 14$ (based on 28 measurements)
- Means of individual samples
  - Mean `CORNFLK` = 4.444
  - Mean `OATBRAN` = 4.081
  - ∴ `OATBRAN` lower on average
- **Create** **`DELTA`**
  - Let `DELTA = CORNFLK - OATBRAN`
  - Order of subtraction does not materially effect results (but do pay attention!)

```
ID   CORNFLK OATBRAN DELTA
---- ------- ------- -----
1       4.61    3.84   0.77
2       6.42    5.57   0.85
3       5.40    5.85  -0.45
.
.
.
```


Focus on `DELTA`

`DELTA` : 0.77,

- $\sum x = 0.77 + 0.85 + \ldots + 0.10 = 5.08$
- $n = 14$
- Subscript $_d$ denotes statistics for `DELTA`
- Mean difference (Formula 3.2): $\bar{x}_d = \frac{5.08}{14} = 0.363$
- Sum of square (Formula 3.7):

  $SS_d = (0.77 - 0.363)^2 + (0.85 -.363)^2 + \ldots + (0.10 = .363)^2 = 2.143$
- Standard deviation (Formula 3.11): $s_d = \sqrt{\frac{2.143}{14 - 1}} = 0.406$

Did I say all analyses focus on **DELTA**?

P Yes!

P This makes it like a one-sample analysis

P Apply what you've already learned

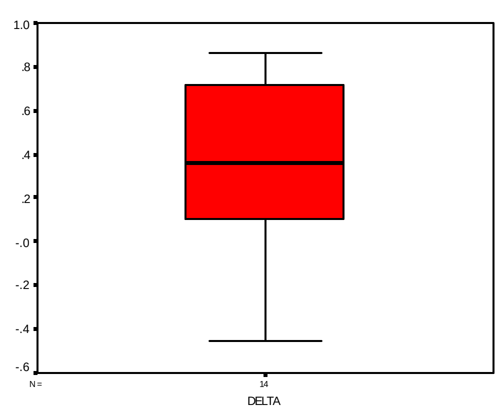
Exploring **DELTA**

DELTA: 0.77, 0.85, -0.45, -0.26, 0.30, 0.86, 0.60, 0.62, 0.31, 0.72, 0.09, 0.16, 0.41, 0.10

Stem-and-leaf

```
|-0|42
|+0|011334
|+0|667788
DELTA (mg/dl x 0.1)
```

Box-and-whiskers



P Center ≈ +0.3

P Spreads from -0.4 to +0.8

P No obvious outliers

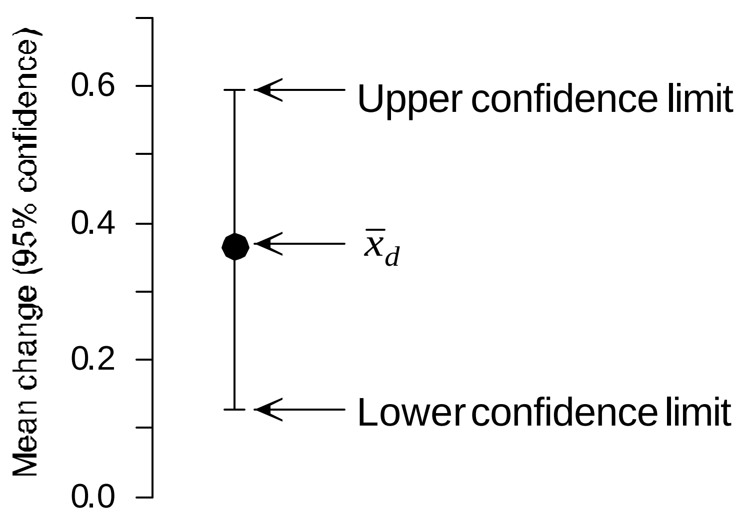
P Shape difficult to assess when *n* small

Confidence Interval for μ_d

p. 7.6

- Formula 7.1: $\bar{x}_d \pm (t^*)(se_{\bar{x}_d})$
  - $\bar{x}_d$ ≡ mean of DELTA
  - df = n − 1
  - t^* ≡ t score with given confidence
  - se ≡ standard error of mean DELTA = $s_d / \sqrt{n}$
- Illustrative example (95% confidence)
  - sample mean of DELTA = 0.3629 (prior slide)
  - df = n − 1 = 14 − 1 = 13
  - t^* for 95% confidence = 2.16 (from new table)
  - se = .4060 / √14 = .1085
- 95% confidence interval for μ_d
  - = 0.3629 ± (2.16)(0.1085)
  - = 0.3629 ± 0.2344
  - =
- ∴ 95% confident *population* mean difference lies within

95% Confidence Interval Graph



Hypothesis

p. 7.7

(A) Hypotheses

- H_0: $\mu_d = 0$ (no mean difference in *pop'n*)
- Alternative can be
  - H_1: $\mu_d < 0$ (one-sided to left)
  - H_1: $\mu_d > 0$ (one-sided to right)
  - H_1: $\mu_d \neq 0$ (two-sided)

(B) Set α

(C) Test statistics (formula 7.2) $t_{\text{stat}} = \dfrac{\bar{x}_d}{se_{\bar{x}_d}}$ with df = $n - 1$

(D) Conclusion

- Convert t_{stat} to p value
- Retain or reject H_0

Illustrative **OATBRAN.SAV**

p. 7.7

$$t_{\text{stat}} = \frac{\bar{x}_d}{se_{\bar{x}_d}} \text{ with df} = n - 1$$

P Does oatbran diet alter LDL levels (on average)?

- H_0: $\mu_d = 0$
- H_1: $\mu_d \neq 0$ (two-sided alternative)

P Let $\alpha = .05$ (or whatever)

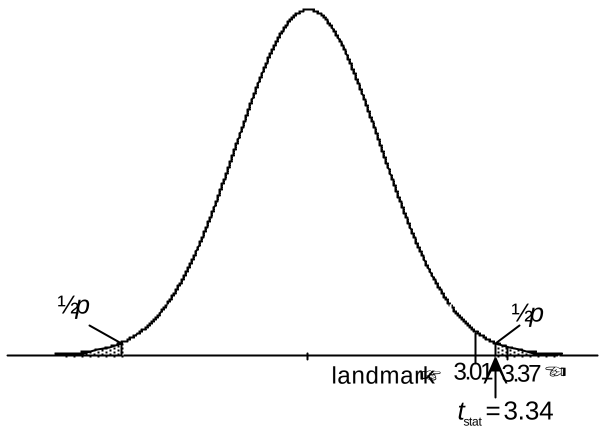
P Teststatistic

- $t_{\text{stat}} = \dfrac{0.3629}{0.1085} = 3.34$
- df = 14 − 1 = 13

13 df row in *t* table

	Upper Tail Probability										
	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
13	0.69	0.87	1.08	1.35	1.77	2.16	2.28	2.65	3.01	3.37	3.85	4.22

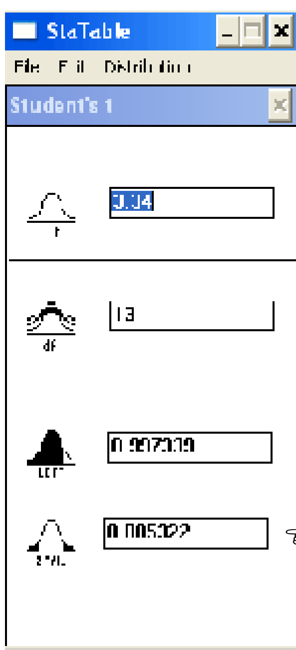
$t_{stat} = 3.34$



One tail between .0025 and .005
Two-tails between .005 and .01 (.005 < p < .01)
Since $p < \alpha$, reject H_0

Conversion of *t* stat to *p* value with Computer

p. 7.7



☜ Two-tailed *p* value

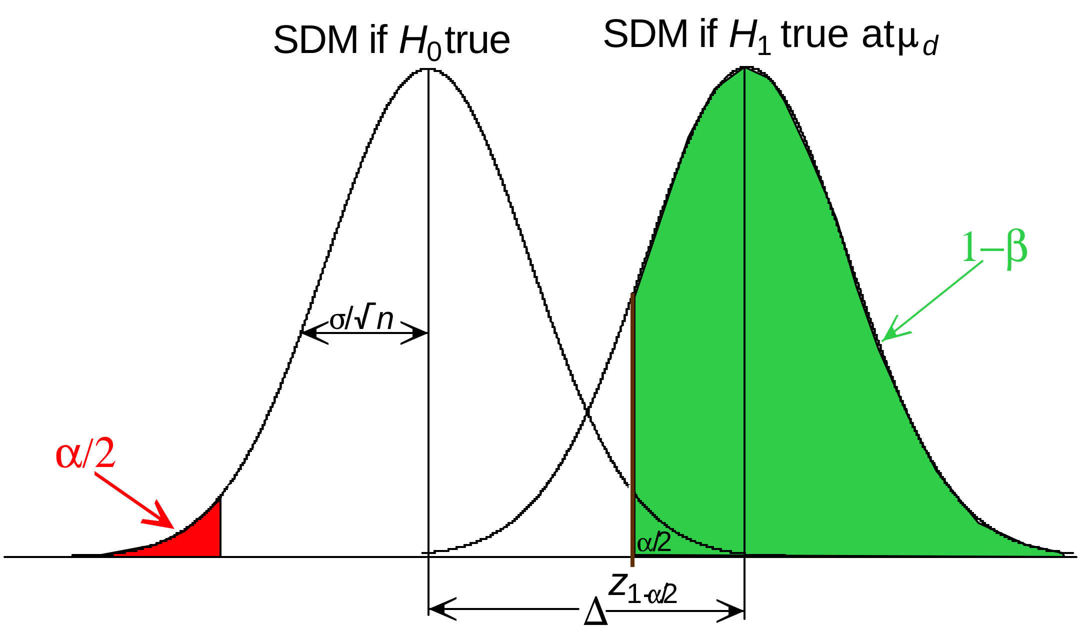
Power (1 − β), Basics

p. 7.8 - 7.9

- If H_0 is false and we retain it, we commit a type II error.
- Pr(type II error) = β
- Pr (no type II error) = 1 − β = power
- Power function:
  - Δ ≡ difference worth detecting (actual μ_d for paired samples)
  - σ ≡ standard deviation of variable
  - α ≡ type I error rate (and whether one-sided or two-sided)
  - n ≡ sample size

Power (1 − β), Basis in Probability

Competing Hypotheses: *not* in Reader



Calculation of 1 – β

p. 7.8

P Conditions

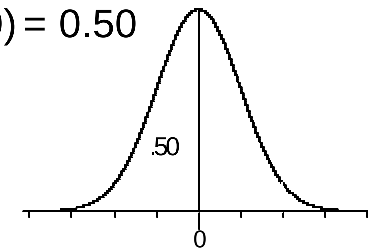
- *n* given
- σ assumed (e.g., based on prior knowledge or study)
- α = .05 two-sided
- Δ ≡

$$Power = f\left(-1.96 + \frac{\Delta\sqrt{n}}{s}\right)$$

Formula 7.3

where ϕ(*z*) represents area to left of *z* on a standard normal curve

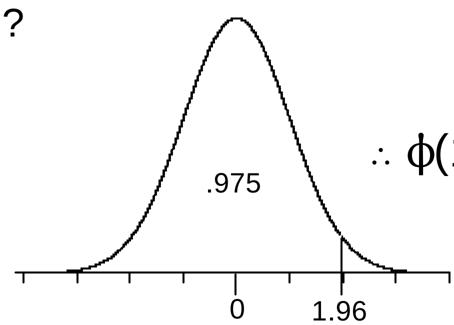
e.g., ϕ(0) = 0.50



Phi (ϕ) Function

p. 7.8

- **P** ϕ(z) → cumulative probability on standard normal curve
- **P** Say it words: Area under curve to left of *z* is . . .
- **P** Draw it
- **P** Use landmarks
- **P** Use z table (or StaTable) to lookup value
- **P** e.g., ϕ(1.96) = ?



∴ ϕ(1.96) = .975

Illustrative

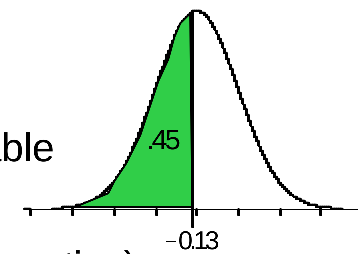
p. 7.9

P ***Conditions***

- n=30 (given)
- σ=6 (based on prior knowledge)
- α=.05 two-sided
- Δ=2 (specified by researcher)

P Calculate:

$$Power = f\left(-1.96 + \frac{|2|\sqrt{30}}{6}\right) = f(-0.13)$$



Convert z score to cumulative probility w/ z table

$\phi(-0.13) = .4483$

Power < .80 considered "inadequate" (by convention)

Power for α = .01 (two-sided)

p. 7.8

P **Use Formula 7.4**

Schematic of Power for Lab 7, Problem 7

Lab Workbook p. 37