DEPARTMENT OF MATHEMATICS
2009 PROBLEM SOLVING COMPETITION

Information For Participants

The problems for Part A of the Competition appear below. Even if you do only one or two problems, we encourage you to submit your solutions. It is important that you justify the key steps in your solutions to obtain full credit.

You can submit hard copies of your solutions at the Department office (MH 308), or you can e-mail your solutions to jackson@math.sjsu.edu. Include your name and ID number, your major and year in school, and your contact information (e-mail and/or phone) with your submission. Solutions for Part A are due Thursday, October 15th at 4 pm, and late submissions will NOT be considered.

The problems for Part B will be available on Monday, October 19th at 9 am. Final results of the Competition will be posted at the Department office, and also online at www.math.sjsu.edu, by November 24, 2009. Winners will be contacted directly regarding their prizes.

If you have any questions about the Competition, please call the Department at (408) 924-5100. We hope you enjoy working on the problems!

PART A

Problem A-1

Find the volume of the solid in R^3 bounded by the six planes $x = 0$, $y = 0$, $z = 0$, $x + y = 1$, $x + z = 1$, and $y + z = 1$.

Problem A-2

Let $n \geq 2$ be an integer. Select arbitrarily n red and n blue points in the plane, all distinct and no three collinear. Show the n red points can be paired up with the n blue points using n noncrossing line segments.

Problem A-3

Consider a simple quadrilateral Q in the plane. For each edge e in Q, let $\mathbf{n_e}$ denote the unit vector normal to the edge e and directed towards the exterior of Q. Show that $\sum_e \text{length}(e)\cdot\mathbf{n_e}$ is the zero vector.

Problem A-4

A row (column) operation on an n×n (0,1)-matrix consists of selecting a single row (column) in the matrix, and then complementing the bits in just that row (column). Determine which n×n (0,1)-matrices can be converted to the n×n zero matrix by a sequence of such row or column operations.

Ex. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is such a 2×2 matrix, since

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \xrightarrow{Complement\_Row\_1} \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \xrightarrow{Complement\_Column\_1} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

SOLUTIONS ARE DUE THURSDAY, OCTOBER 15, 2009 AT 4 PM