DEPARTMENT OF MATHEMATICS
2009 PROBLEM SOLVING COMPETITION

The problems for Part B of the Competition appear below. You do not need to have submitted any solutions for Part A to participate in Part B. Even if you do only one or two of the problems, we encourage you to submit your solutions. It is important that you justify the key steps in your solutions to receive full credit.

You can submit hard copies of your solutions at the Department office (MH 308), or can e-mail your solutions to jackson@math.sjsu.edu. Include your name and ID number, your major and year in school, and your contact information (e-mail and/or phone) with your submission. Solutions for Part B are due Thursday, October 29th at 4 pm, and late submissions will NOT be considered.

Final results of the Competition will be posted at the department office, and also online at www.math.sjsu.edu, by November 24, 2009. Winners will be contacted directly regarding their prizes.

If you have any questions about the Competition, please call the Department at (408) 924-5100. We again hope you enjoy working on the problems!

PART B

Problem B-1

Consider $n+1$ people sitting in seats numbered $0,1,2,\cdots,n$. The person in seat 0 randomly selects one of the other seats, and exchanges places with the person in the selected seat. This exchange procedure is repeated five more times, for a total of six such exchanges altogether. At the end, what is the probability that everyone is in their original seat?

Problem B-2

Given three arbitrary n-point-sets $\{A_1,A_2,\cdots,A_n\}$, $\{B_1,B_2,\cdots,B_n\}$, and $\{C_1,C_2,\cdots,C_n\}$ on the standard unit sphere S^2 in $\Re^3$, show there exists a point P on S^2 such that $\sum_{i=1}^{n}|P-A_i|^2=\sum_{i=1}^{n}|P-B_i|^2=\sum_{i=1}^{n}|P-C_i|^2$.

Problem B-3

Does the infinite series $\sum_{n=1}^{\infty} \frac{|\sin n|}{n}$ converge or diverge?

Problem B-4

Given four points in $\Re^3$ such that the $\binom{4}{2} = 6$ interpoint distances are all odd integers, show the four points cannot all lie in a single plane.

SOLUTIONS ARE DUE THURSDAY, OCTOBER 29, 2009 AT 4 PM