#6

(a) Ch. Poly. = \( D(s) = (2s + 1)(s + 8) \)

\( D'(s) = 2s^2 + 16s + 8 \) \( = 2s^2 + 17s + 8 \)

\( T(s) = \frac{N(s)}{D(s)} \)

(b) Ch. Eq. = \( D(s) = 0 \)

\( D(s) = 2s^2 + 17s + 8 = 0 \)

(c) Finite poles & zeros (not infinity).

Poles: \( D(s) = 0 \) \( \Rightarrow (2s + 1)(s + 8) = 0 \)

\( s_1 = \frac{-1}{2}, \ s_2 = -8 \)

Zeros: \( N(s) = 0 \) \( \Rightarrow 4(s + 1) = 0 \)

\( s_1 = -1 \)

(d) All poles & zeros (including infinity).

Poles: \( D(s) = 0 \) \( \Rightarrow (2s + 1)(s + 8) = 0 \)

\( s_1 = \frac{-1}{2}, \ s_2 = -8 \)

Zeros: \( N(s) = 0 \) \( \Rightarrow 4(s + 1) = 0 \)

\( s_1 = -1, \ s_2 = \infty \)

The \# of poles = \# of zeros, therefore, \( 3 \) poles & \( 3 \) zeros

(e) Dominating or Dominant pole is the one with a real part closer to the origin (zero) on the left half-plane as the system is required to be a stable system.

Poles are \( s_1 = \frac{-1}{2} \) & \( s_2 = -8 \)
# 6 (continued)

Dominating \( s = -1 \) has a smaller negative real part, closer to the origin.

f) Time constant is defined as \( \tau = \left| \frac{1}{\text{pole value}} \right| \)

\( s \) the dominating time constant corresponds to the dominating pole ..., \( \tau = \left| \frac{1}{-1/2} \right| = 2 \) see

g) Output is defined in \( s \) domain, therefore,

\[
C(s) = T(s) R(s) = \frac{4(s+1)}{(2s+1)(s+8)} \left( \frac{1}{s} \right) = \frac{4(s+1)}{s(2s+1)(s+8)}
\]

\[\text{unit step } = \frac{1}{s}\]

\[\text{Note: if it is asked for system response, then we need to determine } C(t) = \mathcal{L}^{-1}[C(s)]\]

h) System output due to a unit ramp input, \( R(s) = \frac{1}{s^2} \)

\[
C(s) = \frac{4(s+1)}{(2s+1)(s+8)} \left( \frac{1}{s^2} \right) = \frac{4(s+1)}{s^2(2s+1)(s+8)}
\]

Steady-state value of \( C(t) \) is obtained as \( t \to \infty \)

to determine \( C(t) = \mathcal{L}^{-1}[C(s)] \) and to set \( t \to \infty \) in quite time consuming, therefore, we could use the Laplace Transform theorem, Final-Value theorem, as follows:

\[\lim_{s \to 0} \left( s C(s) \right) \]

\[
\lim_{t \to \infty} C(t) = C = \lim_{s \to 0} \frac{\frac{4(s+1)}{s^2(2s+1)(s+8)}}{s^2} = \frac{\lim_{s \to 0} \frac{4(s+1)}{s^2(2s+1)(s+8)}}{s^2} = \frac{4}{s^2(2s+1)(s+8)} \]

\[\therefore C(t \to \infty) = \frac{4}{s^2(2s+1)(s+8)} \to 0 \text{ as } s \to 0\]
# 7

\[ Q(s) = \frac{k}{s(s+7)(s+11)} \]

\[ T(s) = \frac{Q(s)}{1+Q(s)} = \frac{k}{s^3 + 13s^2 + 77s + k} = \frac{N(s)}{D(s)} \]

Unity feedback

Let's use Routh-Hurwitz Stability Criterion

\[
\begin{array}{c|cccc}
 & s^3 & 1 & 77 \\
 & s^2 & 18 & k \\
 & s^1 & 77 \cdot \frac{k}{18} & 0 \\
 & s^0 & k & 0 \\
\end{array}
\]

Check the column for sign change.

For stability, no sign change, that is 

\[ 1 > 0, \quad 18 > 0, \quad 77 - \frac{k}{18} > 0, \quad k > 0 \]

But for the case of an unstable system, then 

\[ 1 > 0, \quad 18 > 0, \quad 77 - \frac{k}{18} < 0, \quad k > 0 \]

\[ k \text{ must be } > 0 \]

\[ 77 - \frac{k}{18} < 0 \]

\[ 77 < \frac{k}{18} \quad \Rightarrow \quad 1386 < k \]

and \( 0 < k \)

\[ \therefore \quad k > 1386 \] will result in instability
#7 (Continued)

\[ T(s) = \frac{k}{s(s+2)(s+3)} \]

\[ T(s) = \frac{k}{s^3 + 5s^2 + 6s + k} \]

unity feedback

Let's use Routh-Hurwitz stability criterion

\[
\begin{array}{cccc}
 s^3 & 1 & 6 \\
 s^2 & 5 & k \\
 s^1 & 6 - \frac{k}{5} & 0 \\
 s^0 & k & 0 \\
\end{array}
\]

check the column for sign change

For stability, no sign changes, that is

\( 1 > 0, \ 5 > 0, \ 6 - \frac{k}{5} > 0, \ k > 0 \)

But for the case of instability, sign change is allowed, therefore

\( 1 > 0, \ 5 > 0, \ 6 - \frac{k}{5} < 0, \ k > 0 \)

\[ k \text{ must be } > 0 \]

\[
6 - \frac{k}{5} < 0 \\
6 < \frac{k}{5} \Rightarrow 30 < k \\
\therefore \ k > 30 \text{ will result in instability.} \]
\( G_2 = \frac{10}{s(s+2)}\frac{1}{1 + \frac{10}{s(s+2)}(k_5)} = \frac{10}{s(s+2)}\frac{1}{\frac{s(s+2) + 10k_5}{s(s+2)}} = \frac{10}{s^2 + (2 + 10k) s} \)

\[ T(s) = \frac{C}{R} = \frac{\frac{10k_1}{s^2 + (2 + 10k_2) s}}{1 + \frac{10k_1}{s^2 + (2 + 10k_2) s}} = \frac{\frac{10k_1}{s^2 + (2 + 10k_2) s}}{1 + \frac{10k_1}{s^2 + (2 + 10k_2) s}} = \frac{N(s)}{D(s)} \]

\[ D(s) = s^2 + (2 + 10k_2) s + 10k_1 = 0 \quad \text{ch. Eq} \]

Compare with \( s^2 + 2\omega_n s + \omega_n^2 = 0 \)

Equate the coefficients:

\[ 2 + 10k_2 = 2\omega_n \quad \text{and} \quad \omega_n^2 = \frac{2 + 10k_2}{2} \quad (1) \]

\[ 10k_1 = \omega_n^2 \quad \Rightarrow \quad \omega_n = \sqrt{10k_1} \quad (2) \]

Performance characteristic due to a unit step input.
#8 (continued)

\[ t = 1 \text{ sec} \quad \frac{t}{5} = 2 \text{ sec} \]

From page 2-33, we have:

\[ t = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}} \]

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\[ \frac{t}{5} = 4 \xi \quad \Rightarrow \quad \xi = \frac{2}{\omega_n} \]

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\[ \omega_n \sqrt{1 - \xi^2} = \frac{\pi}{\omega_n} \]

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\[ \omega_n^2 \left(1 - \xi^2\right) = \pi^2 \]

\[ \omega_n^2 - \omega_n \xi \pi = \pi^2 \quad \Rightarrow \quad \omega_n^2 - \omega_n \left(\frac{2}{\omega_n}\right) = \pi^2 \]

\[ \omega_n^2 - \omega_n \frac{2}{\omega_n} = \pi^2 \quad \Rightarrow \quad \omega_n^2 - \omega_n \frac{2}{\omega_n} = \pi^2 \]

\[ \omega_n^2 = \pi^2 + \frac{2}{\omega_n} \]

\[ \omega_n^2 = \pi^2 + \frac{2}{\omega_n} \]

\[ \omega_n = \sqrt{\pi^2 + \frac{2}{\omega_n}} \]

\[ \omega_n = \sqrt{\pi^2 + \frac{2}{\omega_n}} \]

\[ 10 \xi = \pi^2 + 4 = 13.9 \]

\[ k_1 = \frac{13.9}{10} = 1.39 \]

\[ k_1 = 1.39 \quad , \quad k_2 = 0.2 \]
$T(s) = \frac{G}{1 + G}$

\[ T(s) = \frac{k}{s(s + \alpha)} = \frac{N(s)}{D(s)} \]

\[ D(s) = s^2 + \alpha s + k = 0 \quad \text{ch. 8} \]

Compare with $s^2 + 2\xi \omega_n s + \omega_n^2 = 0$

\[ \alpha = 2\xi \omega_n, \quad k = \omega_n^2 \]

Performance characteristics due to unit step input,

\[ \text{PO} = 30\% \quad , \quad t_s = 0.2 \text{ sec} \]

From Page 2-33

\[ \text{PO} = \frac{100}{\text{max}} \left( \frac{-\pi \delta}{\sqrt{1 - \delta^2}} \right) \]

\[ t_s = 4 \xi, \quad \tau = \frac{1}{2\%} \]

\[ 30 = 100 \delta \]

\[ \ln \left( \frac{30}{100} \right) = \frac{-\pi \delta}{\sqrt{1 - \delta^2}} \]

\[ -1.2 = \frac{-\pi \delta}{\sqrt{1 - \delta^2}} \quad \Rightarrow \quad \delta = 0.36 \quad \text{underdamped} \]

\[ t_s = 0.2 = \frac{4}{\delta \omega_n} = \frac{4}{0.36 \omega_n} \quad \Rightarrow \quad \omega_n = \frac{4}{0.2(0.36)} = 55.6 \]

\[ \alpha = 2\xi \omega_n = 2(0.36)(55.6)^2 \approx 40, \quad k = \omega_n^2 = (55.6)^2 = 3109 \]
# 11

a) \( f_n = ? \)

b) \( \xi = ? \)

c) \( \% OS = ? \)

d) System rise time?

From the graph:

\[
\xi_d = (0.7 - 0.3) = 0.4 \text{ sec}
\]

\[
x_1 = 0.4, \quad x_2 = 0.08
\]

From vibrations, log decrement:

\[
\delta = \ln \left( \frac{0.4}{0.08} \right) = 1.6
\]

b) \[
\xi = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = 0.26 < 1 \quad \text{under-damped oscillations shown in graph}
\]

c) \[
\% OS = \frac{100(1.4 - 1.0)}{1.0} = 0.4 \times 100 = 40
\]

From page 2-33...

\[
\% OS = 100 \left[ e^{-\frac{n\delta}{\sqrt{1-\xi^2}}} \right]
\]

\[
40 = 100 \left[ e^{-\frac{2\pi}{\sqrt{1-\xi^2}}} \right]
\]

\[
\xi = 0.27 \quad \text{close to above, but not}
\]

\[
\xi = 0.26 < 1
\]

\[
\omega_d = \omega \sqrt{1-\xi^2} = 2\pi f_d = \frac{2\pi}{\xi_d} = \frac{2\pi}{0.4}
\]

\[
\omega_d \sqrt{1-(0.26)^2} = \frac{2\pi}{0.4} \Rightarrow \omega = 16.27 \text{ rad/sec}
\]

\[
f_n = \frac{\omega_d}{2\pi} = \frac{16.27}{2\pi} = 2.6 \text{ Hz}
\]

From page 2-33...

\[
t_r = \frac{2\pi - \phi}{\omega \sqrt{1-\xi^2}}, \quad \phi = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}
\]

\[
\phi = \tan^{-1} \frac{\sqrt{1-(0.26)^2}}{0.26} = 1.3 \text{ rad}
\]
\# 11 (Continued)

\[ t_1 = \frac{2\pi - 1.3}{16.27 \sqrt{1 - (0.26)^2}} = \frac{4.98}{15.7} = 0.3 \text{ sec} \]

From graph \(0\) to \(100\%\) \(= 0.2\) sec