\[ \dot{c} + 2 \dot{c} + 3 c = 0 \quad \Rightarrow \quad c(0) = 3, \quad \dot{c}(0) = -2 \]

To obtain the system transfer function, we need to put the differential equation into a non-homogeneous form, i.e., \[ \ddot{c} + 2 \dot{c} + 3 c = f(t) \quad \text{Eq. (1)} \]

Let's take the Laplace transform of Eq. (1) and let all IC's = 0.

\[
\mathcal{L} \left[ \ddot{c} + 2 \dot{c} + 3 c = f(t) \right]
\]

\[
\begin{align*}
\left\{ s^2 C(s) - s \dot{c}(0) - c(0) \right\} & + 2 \left\{ s \dot{c}(s) - \dot{c}(0) \right\} + 3 \{ C(s) \} = F(s) \\
\underbrace{y_0}_{y_0} & \quad \underbrace{\dot{y}_0}_{\dot{y}_0} \\
\text{IC}'s = 0 & \quad \text{IC}'s = 0
\end{align*}
\]

\[ s^2 C + 2 s \dot{c} + 3 c = F(s) \]

\[ (s^2 + 2s + 3) C(s) = F(s) \]

\[ T(s) = \frac{C(s)}{F(s)} = \frac{1}{s^2 + 2s + 3} \]

Compare coefficients of the denominator with those of the original differential equation.

To obtain transient response, we need to solve the original differential equation incorporating the IC's.

\[
\mathcal{L} \left[ \ddot{c} + 2 \dot{c} + 3 c = 0 \right]
\]

\[
\begin{align*}
\left\{ s^2 C - s \dot{c}(0) - c(0) \right\} & + 2 \left\{ s \dot{c}(s) - \dot{c}(0) \right\} + 3 \{ C \} = 0 \\
\underbrace{\dot{y}_0}_{\dot{y}_0} & \quad \underbrace{-2}_{-2} \quad \underbrace{1}_{1} \\
\left\{ s^2 C - 3 s + 2 \right\} & + \left\{ 2 s C - 6 \right\} + 3 \{ C \} = 0
\end{align*}
\]
Page 2-109, 1a (continued)

\[(3^2 + 2s + 3) \frac{C}{s} = 3s - 2 + 6 = 3s + 4\]

\[C(s) = \frac{3s + 4}{s^2 + 2s + 3}\]

Response is given in time domain, therefore,

\[c(t) = \mathcal{L}^{-1} \left[ C(s) \right] = \mathcal{L}^{-1} \left[ \frac{3s + 4}{s^2 + 2s + 3} \right]\]

Let's use partial fraction method to solve for \(c(t)\)

Page 2-109, 1c

\[\ddot{c} + 3\dot{c} + 7c = 0\]

\[c(0) = 1\]

\[\dot{c}(0) = -3\]

Same as 1a.

\[\ddot{c} + 3\dot{c} + 7c = f(t)\]

\[\mathcal{L} \left[ \ddot{c} + 3\dot{c} + 7c = f(t) \right] = \mathcal{L} \left[ \ddot{c} + 3\dot{c} + 7c \right] = \mathcal{L} \left[ f(t) \right]
\]

\[\left\{ s^2C - sC(0) - \dot{C}(0) \right\} + 3\left\{ sC - c(0) \right\} + 7\left\{ C \right\} = F(1)\]

Transfer Function with \(IC(s) = 0\)

\[(s^2 + 3s + 7) C(s) = F(s)\]

\[T(s) = \frac{C(s)}{F(s)} = \frac{1}{s^2 + 3s + 7}\]

To obtain Transient Response, \(c(t)\)

\[\ddot{c} + 3\dot{c} + 7c = 0\]

\[\mathcal{L} \left[ \ddot{c} + 3\dot{c} + 7c = 0 \right] = \mathcal{L} \left[ \ddot{c} + 3\dot{c} + 7c = 0 \right] = 0\]

\[\left\{ s^2C - sC(0) - \dot{C}(0) \right\} + 3\left\{ sC - c(0) \right\} + 7\left\{ C \right\} = 0\]

\[\left( s^2 + 3s + 7 \right) C = s - 3 + 3 = s\]
\[
(5^2 + 5s + 3) \quad C(s) = S
\]

\[
c(t) = \mathcal{L}^{-1} \left\{ \frac{S}{5^2 + 3s + 7} \right\}
\]

Page 2-109, # 2 b

\[
\ddot{c} + 5\dot{c} + 3c = u(t)
\]

This is a non-homogeneous differential equation where

\[ f(t) = u(t) \] unit step input

Transfer function is obtained from

\[
\mathcal{L} \left\{ \ddot{c} + 5\dot{c} + 3c = f(t) \right\}
\]

\[
\left\{ 5^2 c - 5c(0) - \dot{c}(0) \right\} + 5 \left\{ 5c - c(0) \right\} + 3 \left\{ C \right\} = F(s)
\]

To find transfer function, let all IC's = 0

\[
\therefore (5^2 + 5s + 3) \quad C(s) = F(s)
\]

\[
T(s) = \frac{C(s)}{F(s)} = \frac{1}{5^2 + 5s + 3}
\]

to determine the steady-state response, \( c(t) \), due to the input \( f(t) = u(t) \), let IC's = 0, then

\[
C(s) = T(s) F(s) = \frac{1}{5^2 + 5s + 3} \left( U(s) \right) = \frac{1}{5^2 + 5s + 3} \left( \frac{1}{S} \right)
\]

when \( \mathcal{L} \left\{ u(t) \right\} = U(s) = \frac{1}{S} \)
\[ c(t) = L^{-1} \left[ C(s) \right] = L^{-1} \left[ \frac{1}{s(s^2 + 5s + 3)} \right] \]

Page 2-109, # 2b (Continued)

\[ c'' + 2c' + 3c = e^t \]

TF:
\[ L\left[ c'' + 2c' + 3c = e^t \right] = f(t) \]
\[ s^2 C - sc(0) - c'(0) + 2(sC - c(0)) + 3C = F(s) \]

To determine TF, let \( IC' = 0 \)
\[ (s^2 + 2s + 3)C = F \]
\[ \frac{C(s)}{F(s)} = \frac{1}{s^2 + 2s + 3} \]

To determine the steady-state response, \( c(t) \), then
\[ c(t) = \frac{1}{s+1} \left[ \frac{1}{(s+1)(s^2 + 2s + 3)} \right] \]

where \[ L\left[ f(t) = e^t \right] = \frac{1}{s+1} \]

\[ c(t) = L^{-1} \left[ C(s) \right] = L^{-1} \left[ \frac{1}{(s+1)(s^2 + 2s + 3)} \right] \]
where \( G_x \) is replaced by

\[
\begin{align*}
R & \overset{+}{\longrightarrow} \quad G_1 \quad \overset{+}{\longrightarrow} \quad G_2 \quad \overset{-}{\longrightarrow} \quad G_3 \\
R & \overset{+}{\longrightarrow} \quad G_4 \\
\end{align*}
\]

\[
\begin{align*}
M & \overset{+}{\longrightarrow} \quad G_2 \quad \overset{-}{\longrightarrow} \quad G_3 \\
\end{align*}
\]

\[
G_3 = \frac{\text{Feed Forward Gain}}{1 - \text{Loop Gain}}
\]

\[
G_3 = \frac{G_2 \cdot G_3}{1 - [G_4 \cdot G_3 \cdot H_2 (-1)]} = \frac{G_2 \cdot G_2}{1 + G_2 \cdot G_3 \cdot H_2}
\]

Now let's consider

\[
\begin{align*}
R & \overset{+}{\longrightarrow} \quad G_1 \quad \overset{-}{\longrightarrow} \quad G_2 \\
\end{align*}
\]

use Mason's Gain Formula

\[
\frac{C}{R} = T(1) = \frac{G_1 \cdot G_3}{1 - [(G_1)(G_2)(H_1)(-1)]} = \frac{G_1 \cdot G_2}{1 + G_1 \cdot G_3 \cdot H_1}
\]

Substitute for \( G_3 \) in (1), then

\[
\frac{C}{R} = \frac{G_1 \cdot G_2 \cdot G_3}{1 + G_2 \cdot G_3 \cdot H_2 + G_1 \cdot G_2 \cdot G_3 \cdot H_1}
\]
a) Open-loop Transfer function

\[
\frac{C}{R} = (4)(\frac{1}{5})(5)(\frac{2}{s^2 + 1}) = \frac{40}{s^2(s^2 + 1)}
\]

b) Closed-loop Transfer function

Similar to problem #7, we need to simplify the loop.

\[
\frac{C}{R} = \frac{\text{inner loop}}{1 - \text{inner loop}} = \frac{\left(\frac{1}{5}\right)(5)(\frac{2}{s^2 + 1})}{1 - \left[\frac{1}{5}(5)(\frac{2}{s^2 + 1})(-1)\right]}
\]

where \(G(1) = \frac{\text{inner loop}}{1 - \text{inner loop}}\)

\[
G(1) = \frac{\frac{10}{s(s^2 + 1)}}{1 + \frac{10}{s(s^2 + 1)}} = \frac{10}{s(s^2 + 1) + 10}
\]

\[
T_c = \frac{C}{R} = \frac{(4) G(\frac{1}{5})}{1 - [(4)(G)(\frac{1}{5})(-1)]} = \frac{\frac{4G}{s}}{1 + \frac{4G}{s}} = \frac{4G}{s + 4G}
\]

Substituting \(G\) from (1), the

\[
T_c = \frac{C}{R} = \frac{4(\frac{10}{s^2 + 10})}{s + 4(\frac{10}{s^2 + 10})} = \frac{40}{s(s^2 + 10) + 10} = \frac{40}{s^3 + 10s^2 + 10s + 10}
\]
c) Closed loop system characteristic Eq.

\[ \text{ch. Eq } \Rightarrow D(s) = 0 \Rightarrow \frac{C}{R} = \frac{N(s)}{D(s)} \]

\[ D(s) = 0.1s^3 + s^2 + 10s^4 + 40 \] - Characteristic Polynomial

\[ D(s) = 0.1s^3 + s^2 + 10s^4 + 40 = 0 \] - ch. Eq.

e) Loop gain function

\[ (4)(G)(\frac{1}{s})(-1) = -\frac{4}{s} G = -\frac{4}{s} \left( \frac{10}{1s^2 + s^2 + 10} \right) \]

LG function: 

\[ \frac{40}{s^3 + s^2 + 10s^4 + 40} \]

f) All closed loop system zeros:

\[ T_{\text{closed loop}} = \frac{C}{R} = \frac{N(s)}{D(s)} = \frac{40}{s^3 + s^2 + 10s^4 + 40} \]

All zeros:

\[ N(s) = 0 \] No finite zero but

Since system is a 3rd order \( (s^3) \)

Then there are 3 zeros at \( \infty \)

\[ \text{zeros } s_1 = \infty \]

\[ s_2 = \infty \]

\[ s_3 = \infty \]

g) All poles...are the roots of \( D(s) = 0 \)

\[ 0.1s^3 + s^2 + 10s^4 + 40 = 0 \]

3rd order polynomial will result in three poles:

\[ s_1 = -a \]

\[ s_2 = -b \]

\[ s_3 = -c \]

I use your equation solver

1. To determine the roots.