Let us choose PI controller

$$T(s) = \frac{(k_c + k_i)}{(1 + \frac{1}{s})} = \frac{(k_c s + k_i)}{s(s+1)} = \frac{k_c s + k_i}{s^2 + (1+k_c)s + k_i}$$

Given \( \delta = 0.5 \), then compare ch. Eq.

$$s^2 + (1+k_c)s + k_i = 0$$

with \( s^2 + 2\zeta \omega_n s + \omega_n^2 = 0 \)

Furthermore, if \( \frac{t_s}{T_s} = 0.25 \) due to a unit ramp \( R = \frac{1}{T_s} \)

K, \( t_s = 4 \) sec (assume 2% accuracy)

Then \( T_s = 4 \), \( C = 4 \), \( T = 4 \) sec

$$\Rightarrow T = C = 1 \text{ sec}$$

Note that

$$T = C = \left| \frac{1}{\zeta \omega_n} \right|$$

Both conditions (a) & (b) are met.

Real part of the roots of the ch. Eq.

$$T = C = 1 = \left| \frac{1}{\zeta \omega_n} \right| = \frac{1}{\zeta \omega_n} \Rightarrow \zeta \omega_n = \frac{1}{1} = 1$$

From

$$1 + k_c = 2 \zeta \omega_n$$

$$\zeta = \frac{1}{\xi} = \frac{1}{0.5} = 2$$

$$1 + k_c = 2(0.5)(2) = 2$$

$$k_c = 1$$

$$k_i = \omega_n^2 = (2)^2 = 4$$

Now let's see if condition (a) - \( \xi = 0.25 \) due to \( R = \frac{1}{T_s} \).

$$E = R - TR = (1 - T) R = \left( \frac{1 - \frac{5+4}{s^2+2s+4}}{\frac{1}{5^2}} \right) \text{ not!}$$
\[ E = \frac{s^2 + 2s + 4}{s^2(s^2 + 2s + 4)} = \frac{s^2 + s}{s^2(s^2 + 2s + 4)} = \frac{s(s + 1)}{s^2(s^2 + 2s + 4)} = \frac{s}{s^2 + 2s + 4} = 1 \rightarrow 0.25 \]

Due to ramp:

\[ E(s) = \lim_{s \to 0} s \frac{s(s + 1)}{s^2(s^2 + 2s + 4)} = \lim_{s \to 0} \frac{s + 1}{s^2 + 2s + 4} = 1 \]

\[ T(s) = \frac{C}{R} = \frac{G_e G}{1 - \left[ G_e G \right] (-1)} = \frac{G_e G}{1 + G_e G} \]

\[ T(1) = \frac{(k_p + k_i)(\frac{1}{s + 5})}{1 + (k_p + k_i)(\frac{1}{s + 5})} = \frac{k_p s + k_i}{s(s + 5) + k_p s + k_i} \]

\[ T(1) = \frac{k_p s + k_i}{s^2 + (s + k_p) s + k_i} \]

a) Let \( k_p = k_i = 10 \) \( \Rightarrow \)

\[ T(1) = \frac{10(s + 1)}{s^2 + 10s + 10} \]

Unit step response:

\[ C(s) = T(1) \left( \frac{1}{s} \right) \]

\[ C(s) = \frac{10(s + 1)}{s(s^2 + 10s + 10)} \rightarrow \quad \mathcal{L}^{-1} \left[ C(s) \right] \]

b) Let \( k_p = k_i = 45 \) \( \Rightarrow \)

\[ T(1) = \frac{45(s + 1)}{s^2 + 50s + 45} \]

Unit step response:

\[ C(s) = T(1) \left( \frac{1}{s} \right) \]

\[ C(s) = \frac{45(s + 1)}{s(s^2 + 50s + 45)} \rightarrow \quad \mathcal{L}^{-1} \left[ C(s) \right] \]
c) Speed of response is defined by response rise time, \( t_r \)

From page 2-34:
\[
\tau = \frac{\pi - \phi}{\omega_n \sqrt{1 - \xi^2}}
\]
(third quadrant) \( \xi = \frac{\omega_n}{\omega} \sqrt{1 - \xi^2} \)

when \( \xi = 1 \Rightarrow \phi = \tan^{-1} \frac{\sqrt{1 - \xi^2}}{\xi} \)

\[
\phi = \tan^{-1} \sqrt{1 - \xi^2} \\
\phi = \tan^{-1} \sqrt{1 - (1)^2} \\
\phi = \tan^{-1} 0 \\
\phi = 0
\]

Compare with \( 5^2 + 25 \xi + \xi^2 = 0 \)

\[
25 = 10 \quad , \quad 25 \xi = 10 \Rightarrow \xi = 3.16 \ \text{rad/sec}
\]

\[
\xi = \frac{10}{2 \omega_n} = \frac{10}{2(3.16)} = 1.58 > 1
\]

when \( \xi = 2 \Rightarrow \phi = \tan^{-1} \sqrt{1 - \xi^2} \)

\[
\phi = \tan^{-1} \sqrt{1 - (2)^2} \\
\phi = \tan^{-1} \sqrt{1 - 4} \\
\phi = \tan^{-1} (-1) \\
\phi = \frac{3\pi}{2} < 0
\]

\[
\phi = \tan^{-1} \sqrt{1 - \xi^2} \\
\phi = \tan^{-1} \sqrt{1 - (2)^2} \\
\phi = \tan^{-1} (-1) \\
\phi = \frac{3\pi}{2} < 0
\]

Both systems with \( k_p = k_i = 10 \) & \( k_p = k_i = 45 \) are overdamped and the rise time could not be used as a measure of their speed. Since \( \phi = \tan^{-1} \frac{\sqrt{1 - \xi^2}}{\xi} \) requires \( \xi < 1 \).

Therefore, we could use the time constant \( \tau = \frac{1}{\xi \omega_n} \) as a measure of the system speed.

when \( \xi = 1 \Rightarrow \tau = \frac{1}{(1.58)(3.16)} = 0.2 \ \text{Sec} \)

when \( \xi = 2 \Rightarrow \tau = \frac{1}{(3.73)(6.7)} = 0.04 \ \text{Sec} \)
Therefore, the system with \( k_p = k_i = 45 \) is a more robust system. As \( k_p \) increases, the system becomes more responsive.

Steady State Error due to unit step:

\[
E = R - C = R - TR = (1 - T)R
\]

For \( k_p = k_i = 10 \),

\[
T(s) = \frac{10(s+1)}{s^2 + 10s + 10}
\]

\[
E = \left[ 1 - \frac{10(s+1)}{s^2 + 10s + 10} \right](\frac{1}{5}) = \frac{1}{5} \left[ \frac{s^2 + 70s + 10 - 70s - 10}{s^2 + 10s + 10} \right]
\]

\[
E = \frac{1}{5} \left[ \frac{s^2}{s^2 + 10s + 10} \right]
\]

\[
e_{ss} = \lim_{s \to 0} sE = \lim_{s \to 0} s \cdot \frac{1}{5} \left[ \frac{s^2}{s^2 + 10s + 10} \right] = 0 \quad e_{ss} = 0
\]

For \( k_p = k_i = 45 \),

\[
T(s) = \frac{45(s+1)}{s^2 + 50s + 45}
\]

\[
E = \left[ 1 - \frac{45(s+1)}{s^2 + 50s + 45} \right](\frac{1}{5}) = \left( \frac{s^2 + 50s + 45 - 45s - 45}{s^2 + 50s + 45} \right)(\frac{1}{5})
\]

\[
E = \frac{1}{5} \left[ \frac{s^2 + 5s}{s^2 + 50s + 45} \right] = \frac{s + 5}{s^2 + 50s + 45}
\]

\[
e_{ss} = \lim_{s \to 0} sE = \lim_{s \to 0} s \cdot s \left[ \frac{s + 5}{s^2 + 50s + 45} \right] = 0
\]
d) Increase in \( k_i \) from 10 to 45 helps to increase the system response.

Increase in \( k_i \) from 10 to 45 helps the \( \zeta \) to reach a settling time faster.

@ \( k_i = 10 \) .... \( \zeta = 4 \cdot 2 = 4 \cdot 0.2 = 0.8 \) sec

@ \( k_i = 45 \) .... \( \zeta = 4 \cdot 0.2 = 0.16 \) sec

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\[
T_{(s)} = \frac{k G_{(s)}}{1 + k G_{(s)}} = \frac{k}{(s+2) (s+7)} = \frac{k}{s^2 + 9s + (14+k)}
\]

a) Dominating time constant corresponds to the negative real part of the closest pole to the imaginary axis.

For the plant \( G_{(s)} = \frac{1}{(s+2) (s+7)} \)

\( \text{ch. eq: } (s+2) (s+7) = 0 \)

\( s = -2 \quad \& \quad s = -7 \)

\( \eta \)

\( \zeta = \left| \frac{-\frac{1}{5}}{-\frac{1}{2}} \right| = \left| -\frac{1}{5} \right| = \frac{1}{2} = 0.5 \) sec
b) for the closed-loop system:

Ch Eq: \[ s^2 + 9s + (14+k) = 0 \]

\[ s = \frac{-9 \pm \sqrt{81 - 4(1)(14+k)}}{2} \]

At \( k=1 \)

\[ s_{1,2} = \frac{-9 \pm \sqrt{81-64}}{2} = \frac{-9 \pm 4.6}{2} \]

\[ s_{1,2} = -6.8 \text{ and } -2.2 \]

To have a time constant of \( \tau = 0.25 \)

Then \( s = \left| -\frac{1}{\tau} \right| = -\frac{1}{0.25} = -4 \)

We need to have a pole with a negative real value of \( s = -4 \)

\[ -4 = \frac{-9 \pm \sqrt{81 - 56 - 4k}}{2} = \frac{-9 \pm \sqrt{25 - 4k}}{2} \]

\[ -8 = -9 \pm \sqrt{25 - 4k} \]

\[ i = \frac{+\sqrt{25 - 4k}}{2} \Rightarrow 1 = 25 - 4k \]

\[ +4k = 24 \]

\[ k = 6 \]

c) \[ \lim_{s \to 0} \ s E_{10} \]

d) Determine \( k \) for \( s = 0.5 \) in \( s_{ss} \) above
From figure:

Peak magnification = 2

\[ = 20 \log 2 = 6 \text{ dB} \]

Peak (circular) freq = \(25 \frac{\text{rad}}{\text{sec}}\)

Bandwidth: is a range from \(\omega = 0\) to \(\omega = 70 \frac{\text{rad}}{\text{sec}}\)

Cutoff frequency: --- the upper limit of bandwidth

(also break freq \(\omega_{cutoff}\) = \(70 \frac{\text{rad}}{\text{sec}}\))

Also defined at which the signal transmission has gone to 0 dB --- here above 100 rad/ sec

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\[ T(s) = \frac{10}{s^2 + 3s + 8} \]

Compare

Ch: \[ Y(s) = D(s) = s^2 + 3s + 8 = 0 \quad \text{with} \]

\[ s^2 + 2\omega_n s + \omega_n^2 = 0 \]

\[ 3 = 2\omega_n \quad , \quad 8 = \omega_n^2 \]

\[ \Rightarrow \omega_n = \sqrt{8} = 2.83 \frac{\text{rad}}{\text{sec}} \]

\[ \Rightarrow \gamma = \frac{3}{2\omega_n} = 0.53 < 1 \]

undamped oscillation will occur!
a) Resonant peak amplitude is the Max % overshoot

\[ M_p = \% OS = 100 \cdot e^{-(\pi/\sqrt{1-\xi^2})} \]
\[ = 100 \cdot e^{-\left(\frac{\pi}{\sqrt{1-(.53)^2}}\right)} = 100 \cdot (1.14) = 14\% \]

b) Resonant frequency at peak amplitude

\[ \omega_p = \omega_0 \sqrt{1-2\xi^2} = 2.83 \sqrt{1-2(.53)^2} = 1.87 \text{ rad/sec} \]

See page 1-39

c) System bandwidth

\[ \omega_b = \omega_0 \sqrt{1-2\xi^2 + \sqrt{2 - 4\xi^2 (1-\xi^2)}} \]
\[ \omega_b = \left(2.83\right) \sqrt{1-2(.53)^2 + \sqrt{2 - 4(.53)^2 (1-(.53)^2)}} \]
\[ \omega_b = 3.49 \text{ rad/sec} \]

\[ \frac{\omega_b}{\omega_0} = \frac{3.49}{2.83} > 1 \text{ checks } \checkmark \]