7-1) \( u(x) = Ke^{10x} \quad K = \text{constant} \)

7-2) \( u(x) = -\frac{3}{2} + Ke^{2x} \quad K = \text{constant} \)

7-3) \( u(x) = -\frac{2}{\ln \frac{c \cdot x}{x^2}} \quad C = \text{constant} \)

7-4) \( u(x) = -2x \ln x \quad K = \text{constant} \)

7-5) \( u(x) = \frac{3}{2} e^{2x} - \frac{1}{2} \)

7-6) \( u(x) = x + 9/x \)

7-7) \( t_e = 46.33 \text{ s} \)

7-8) (a) \( \frac{dH(t)}{dt} = -\sqrt{2g} \left( \frac{\pi d^2}{4 \rho L W} N^2 \right) \)

(b) \( H(t) = \left[ -\sqrt{\frac{g}{2}} \frac{\pi d^2}{4 \rho L W} t + \sqrt{D} \right]^2 \)

(c) \( t_e = 12.55 \text{ hrs} \)

7-9) \( \frac{y^2}{\sqrt{y}} \frac{dy(t)}{dt} + \frac{d^2}{dt^2} \sqrt{2g} = 0 \)

\( t_e = 4.372 \text{ min} \)

7-10) \( \left[ -57740y^{-\frac{3}{2}} + 115.48y^{\frac{1}{2}} \right] \frac{dy(t)}{dt} = 323564.96 \)

\( t_e = 5.31 \text{ s} \)

7-11) (a) \( V = 2186 \text{ cm}^3 \)

(b) \( 0.533344 \left[ y(t) \right]^{2.5} + 11.591 \left[ y(t) \right]^{1.5} + 113.36 \left[ y(t) \right]^{0.5} = 250.2t + 988.36 \)

(c) \( t_e = 0.4 \text{ s} \)
7.12) (b) $H = 13.416$ cm
(c) $\frac{[y(t)]^{15}}{3} \frac{dy(t)}{dt} = -\frac{(1.5)^2}{4} \sqrt{2 \times 981} = -24.9156$
$[y(t)]^{15} = -186.8673 t + 659.7529$
t_e = 3.535 s

7.13) (a) Top volume $V_1 = 703.72$ cm$^3$
Bottom volume $V_2 = 134$ cm$^3$
Total volume $V = 837.72$ cm$^3$
(b) Time for draining the IV bottle:
Top portion: $t_{e1} = 67.6 s$
Bottom portion: $t_{e2} = 12.04 s$
(c) $L = 21.22$ cm
$t_e = 83.2 + 12.04 = 95.24 s$

7.14) (a) 23.56 kW
(b) $T(x) = 140 \div 120 x$

7.15) (a) $T(t) = 92 e^{-2 \times 10^{-6} \times t} + 8$
$\alpha = 265.88 / m^2 \cdot s$ for $T(2$ hrs$) = 10^\circ C$
(b) $\alpha = 53.176 / m^2 \cdot s$ to shorten cooling time

7.16) (a) $T(t) = 80 \exp(-0.62 \times 10^{-4} t) - 60$
(b) $-59.73^\circ C$
(c) Required time = 1.2 hour
(d) Increase $\alpha$ to shorten the cooling time to $-40^\circ C$

7.17) Similar to Problem 7.16)
7-18) (a) \[ \frac{dT(t)}{dt} = - \alpha A [T(t) - T_f] \]

\[ T(t) = 100 e^{-0.02t} - 80 \]

(b) \[ T(1200s) = -76.23^\circ F \]

(c) \[ T(te) = -40 \rightarrow te = 455 \]

(d) \[ \alpha = 0.06363 \text{ m}^2 \text{s}^{-1} \]

7-19) (a) \[ \frac{dV(t)}{dt} + \frac{c}{m} V(t) = -g \]

(b) \[ \frac{dV(t)}{dt} + \frac{c}{m} V(t) = +g \]

(c) \[ V(t) = -\frac{mg}{c} + (V_0 + \frac{mg}{c}) e^{-\frac{c}{m}t} \]

(d) \[ V(t) = \frac{mg}{c} (1 - e^{-\frac{c}{m}t}) \]

With \[ V(0) = 0 \]

(e) Max. height \((H_{\text{max}})\) and the time to reach this height \((t_m)\):

\[ H_{\text{max}} = \int_0^{t_m} V(t) \, dt \]

\[ = 47.89 \text{ m} \]

\[ t_m = 1.8715 \text{ s} \]

7-20) (a) \[ m \frac{dV(t)}{dt} + 20 [V(t)]^2 = mg \]

\[ \rightarrow 5.097 \frac{dV(t)}{dt} = 5.0 - [V(t)]^2 \]

(b) \[ V(t) = \frac{7.071(e^{6.3817t} - 1)}{1 + e^{6.3817t}} \]

(c) Time to reach ground \(t_e = 2.72 \text{ min} \)

(d) Impact velocity \(V_f \approx 7.071 \text{ m/s} \)

(e) Momentum at landing \(= mV_f = 720.8 \text{ kg-m/s} \)