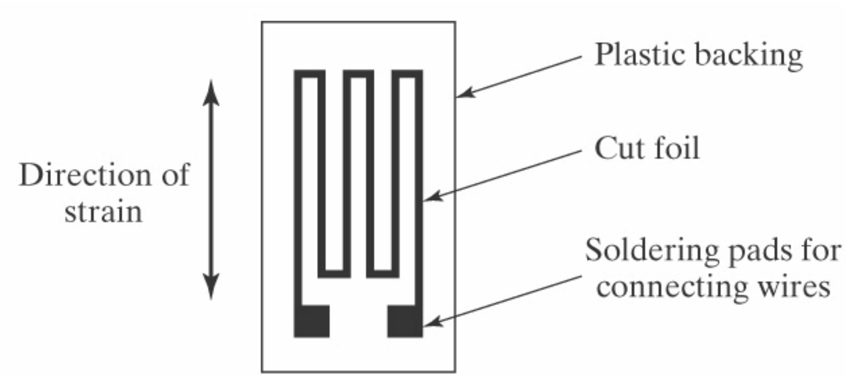
Strain and Force

Strain Gage

- Measures strain as a change in length L, observed by change in resistance R, for a given resistivity ρ and cross-sectional area A.
- For elastic materials that follow Hooke's law, strain ε is related to stress σ by the modulus of elasticity E.
- Strain has units of length per length. For many materials the strain is so small that it is convenient to express in units of "microstrain" ($1/10^6$).
- Common strain gages are fabricated as etched metal foil on plastic backing.

$$R = \rho \frac{L}{A}$$

$$\sigma = E\varepsilon$$

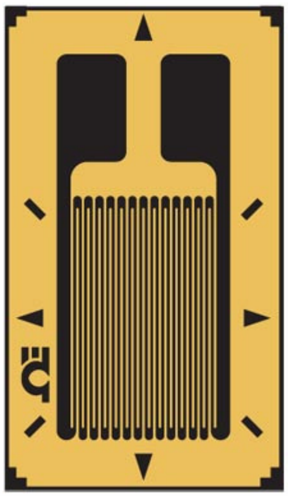


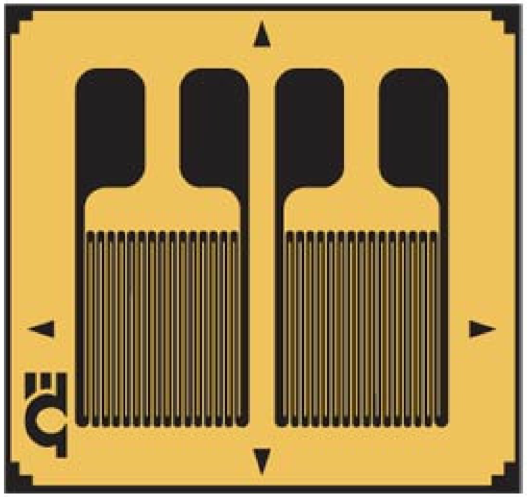
Why meander in the serpentine pattern?

Measuring more than just strain…

- Strain gages are one of the most versatile types of sensors because in addition to strain itself, they can also measure:
  - Acceleration
  - Force
  - Torque
  - Pressure
- How?

Commercial Strain Gage Example



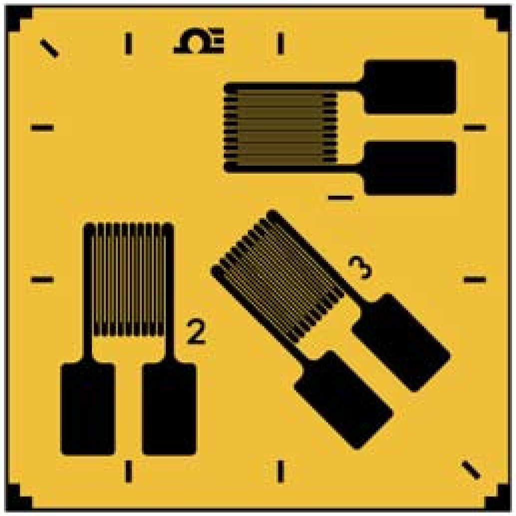


	SGD SERIES
Foil Measuring Grid	Constantan foil 5 microns thick
Carrier	Polyimide
Substrate Thickness	20 microns
Cover Thickness	25 microns
Connection Dimensions: mm (in)	Solder pads or ribbon leads, tinned copper flat wire 30 L x 0.1 D x 0.3 mm W (1.2 x 0.004 x 0.012"); other wire types available upon request
Nominal Resistance	Stated in "To Order" box
Resistance Tolerance Per Package	±0.15% to ±0.5% depending on gage spec
Gage Factor (Actual Value Printed on Each Package)	2.0 ±5%
Gage Factor Tolerance Per Package	1.00%
THERMAL PROPERTIES	
Reference Temperature	23°C (73°F)
SERVICE TEMPERATURE	
Static Measurements	-75 to 200°C (-100 to 392°F)
Dynamic Measurements	-75 to 200°C (-100 to 392°F)
TEMPERATURE CHARACTERISTICS	
Steel (and Certain Stainless Steels)	11 ppm/°C (6.1 ppm/°F)
Aluminum	23 ppm/°C (12.8 ppm/°F)
Uncompensated	±20 ppm/°C (11.1 ppm/°F)
Temperature Compensated Range	-5 to 120°C (5 to 248°F)
Tolerance of Temp Compensation	2 ppm/°C (1.0 ppm/°F)
MECHANICAL PROPERTIES	
Maximum Strain	3% or 30,000 microstrain
Hysteresis	Negligible
Fatigue (at ±1500 microstrain)	>10,000,000 cycles
Smallest Bending Radius	3 mm (⅛")
Transverse Sensitivity	——

Image(s) from http://www.omega.com/
Retrieved September 2008

Multiaxial Strain

- Strain gages are generally useful only in one direction, so multiple strain gage are arranged in a "rosette" pattern to calculate the complete set of planar strains ε_x, ε_y, γ_{xy} .



SGD-1/120-RYT21

Image(s) from http://www.omega.com/
Retrieved September 2008

Gage Factor for Strain Gages

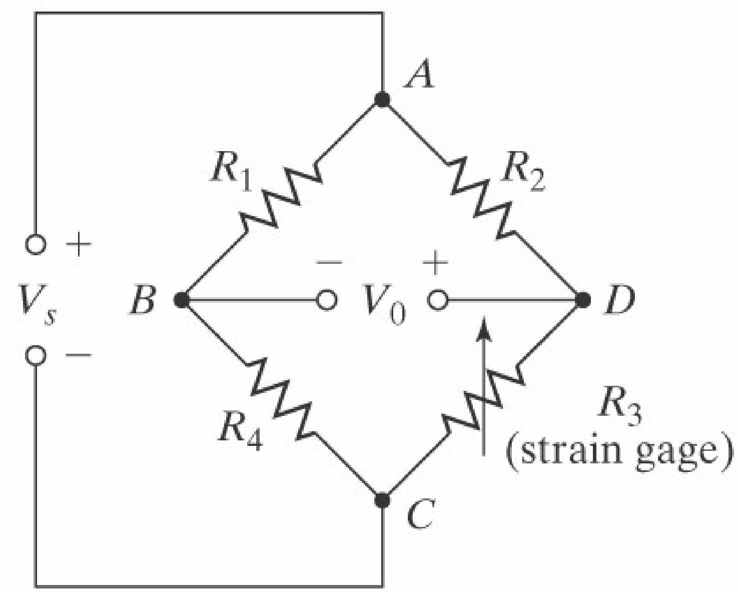
- The gage factor S for a strain gage expresses how the strain ε is related to the observed change in resistance R:

$$S = \frac{\Delta R / R}{\varepsilon}$$

- Typical value for conventional strain gages made of constantan metal is ~2.
- Gage factor is approximately constant for a given strain gage (although subject to temperature effects).

Bridge Circuits for Strain Gages

- Strains and their associated changes in resistance are generally very small in magnitude.
- A **Wheatstone bridge** circuit is necessary to measure small changes in resistance.
- If the bridge is initially balanced such that V_o is zero at zero strain, V_o can be used as a proportional measurement of the change in resistance.



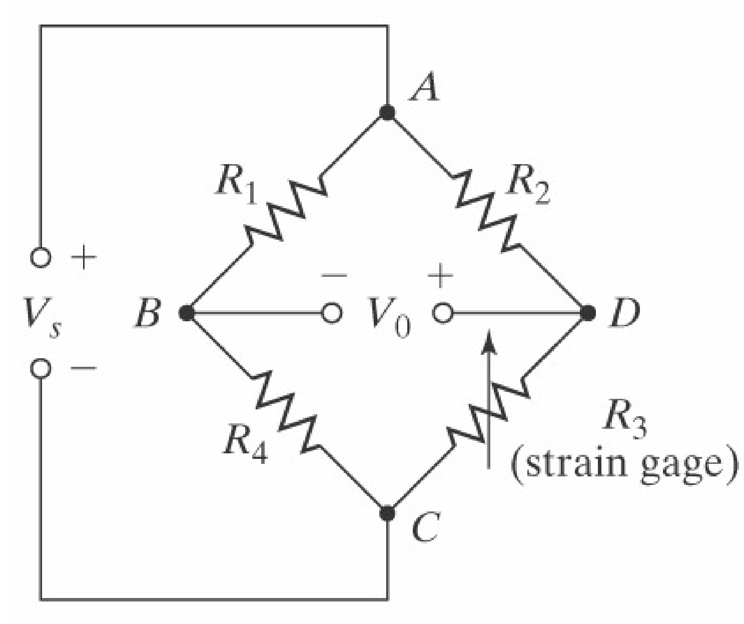
$$V_o = \frac{R_3 R_1 - R_4 R_2}{(R_2 + R_3)(R_1 + R_4)} V_s$$

Measures change in the resistance than resistance itself

Bridge Circuits for Strain Gages (continued)

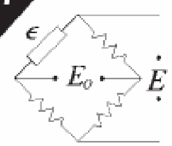
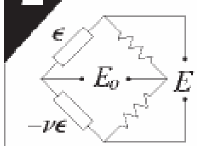
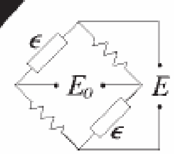
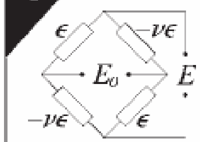
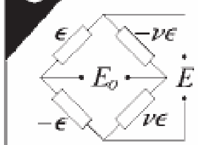
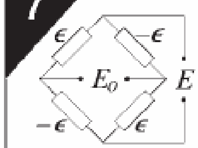
- A "quarter bridge" uses a single strain gage with three fixed resistors, and gives strain as (where R_{3i} is the initial value of R_3):

$$\varepsilon = \frac{(R_2 + R_{3i})^2}{S(V_s R_2 R_{3i})} V_o$$



"Half bridge" and "full bridge" circuits provide 2X or 4X the output, respectively. These are sometimes possible if the arrangement symmetry is such that paired strains will be the same with equal and opposite magnitude.

Bridge Circuit Application Examples

Bridge/Strain Arrangement (Note 1)	Description	Bridge Output, E_o/E mV/V (Notes 2, 3)
1 	Single active gage in uniaxial tension or compression.	$\frac{E_o}{E} = \frac{F\epsilon \times 10^{-3}}{4 + 2F\epsilon \times 10^{-6}}$
2 	Two active gages in uniaxial stress field — one aligned with maximum principal strain, one "Poisson" gage.	$\frac{E_o}{E} = \frac{F\epsilon(1+\nu) \times 10^{-3}}{4 + 2F\epsilon(1-\nu) \times 10^{-6}}$
3 	Two active gages with equal and opposite strains — typical of bending-beam arrangement.	$\frac{E_o}{E} = \frac{F\epsilon}{2} \times 10^{-3}$
4 	Two active gages with equal strains of same sign — used on opposite sides of column with low temperature gradient (bending cancellation, for instance).	$\frac{E_o}{E} = \frac{F\epsilon \times 10^{-3}}{2 + F\epsilon \times 10^{-6}}$
5 	Four active gages in unaxial stress field two aligned with maximum principal strain, two "Poisson" gages (column).	$\frac{E_o}{E} = \frac{F\epsilon(1+\nu) \times 10^{-3}}{2 + F\epsilon(1-\nu) \times 10^{-6}}$
6 	Four active gages in uniaxial stress field — two aligned with maximum principal strain, two "Poisson" gages (beam).	$\frac{E_o}{E} = \frac{F\epsilon(1+\nu) \times 10^{-3}}{2}$
7 	Four active gages with pairs subjected to equal and opposite strains (beam in bending or shaft in torsion).	$\frac{E_o}{E} = F\epsilon \times 10^{-3}$

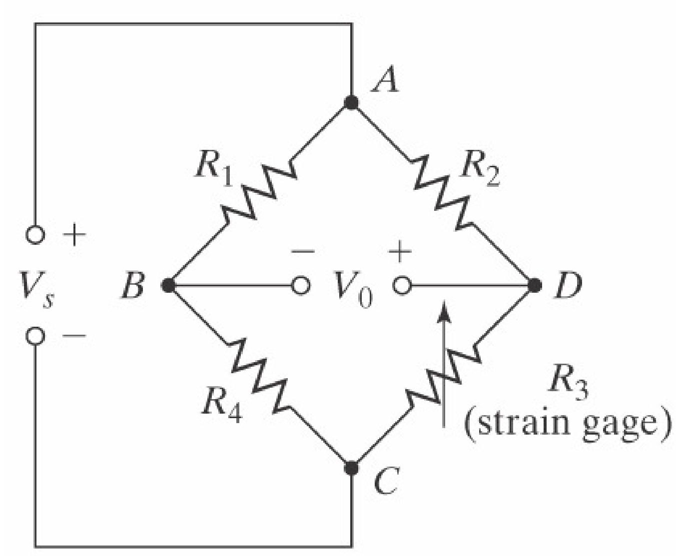
Notes: 1. $(R_1/R_4)_{nom} = 1$; $(R_2/R_3)_{nom} = 1$ when two or less active arms are used.
2. Constant voltage power supply is assumed.
3. ϵ and ϵ_i (strains) are expressed in microstrain units (in/in x 10^6).

Temperature Compensation

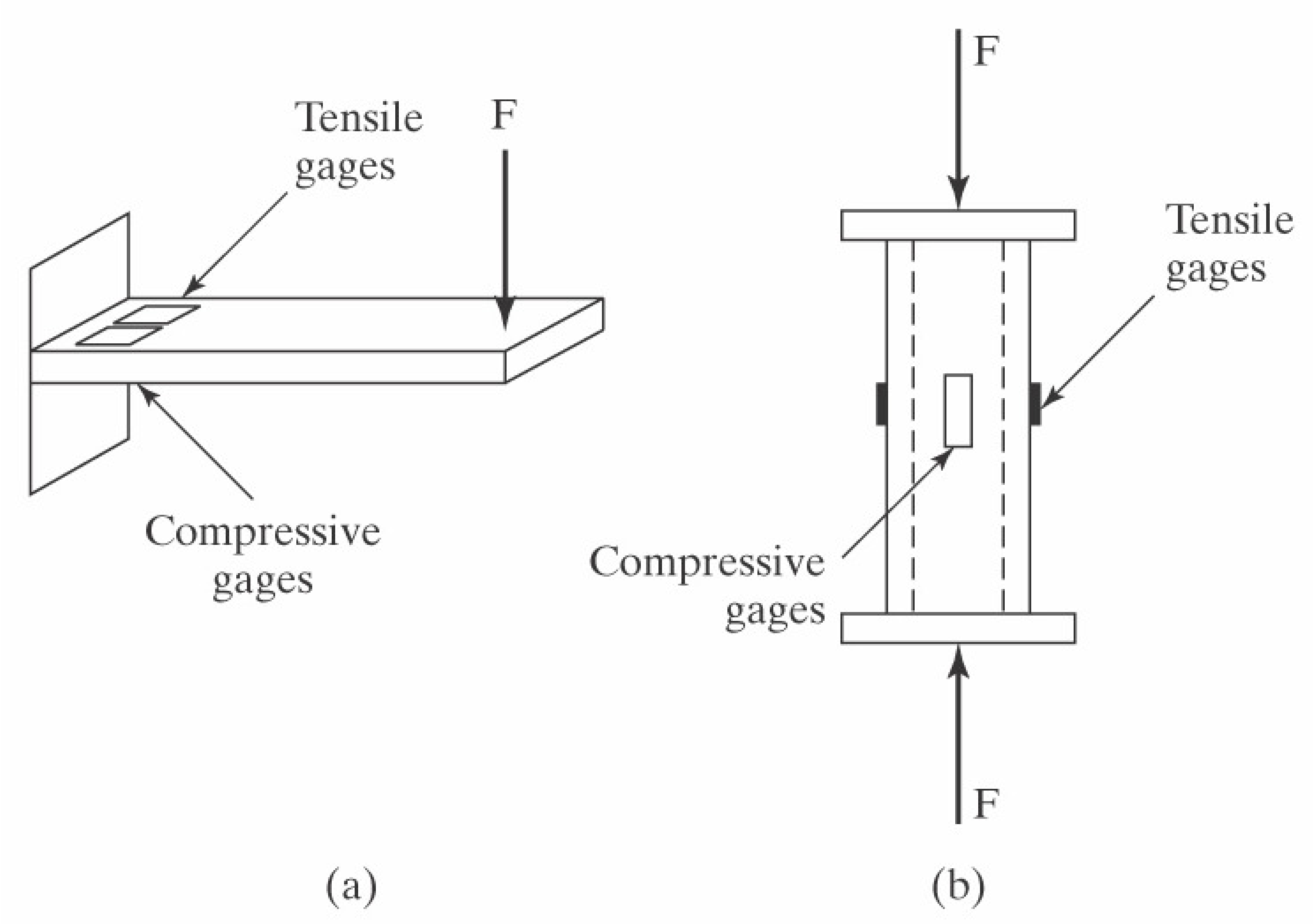
- There are two important factors to related to temperature T that need to be considered for strain gages:
  - Resistivity ρ is a temperature-dependent property, so gage factor is not really constant.
  - There is a difference in coefficient of thermal expansion between the gage and the structural material to which it is attached.
- One effective method of temperature compensation is to use twin resistors (e.g. R_2 and R_3) in the same location, such that in the bridge circuit the temperature effects "cancel out".

$$\rho = \rho\{T\}$$

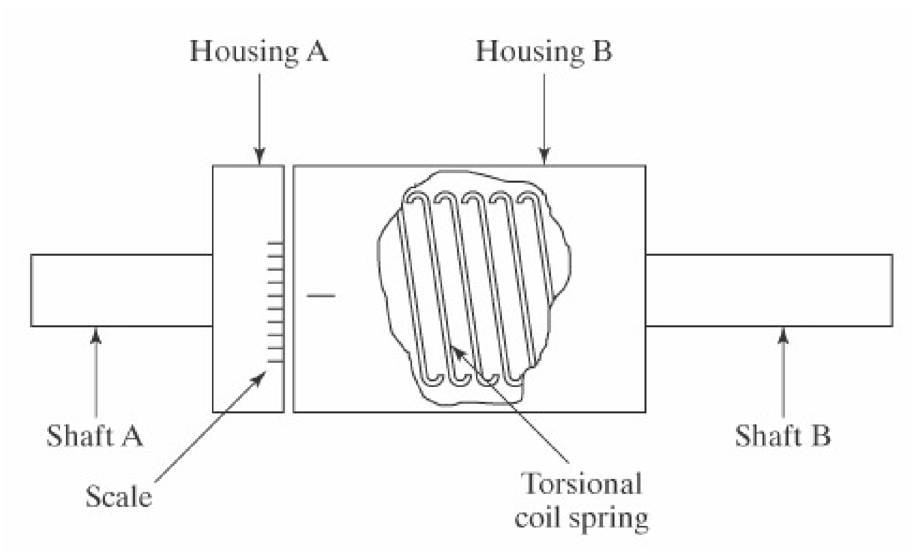
$$S = \frac{\Delta R / R}{\varepsilon}$$

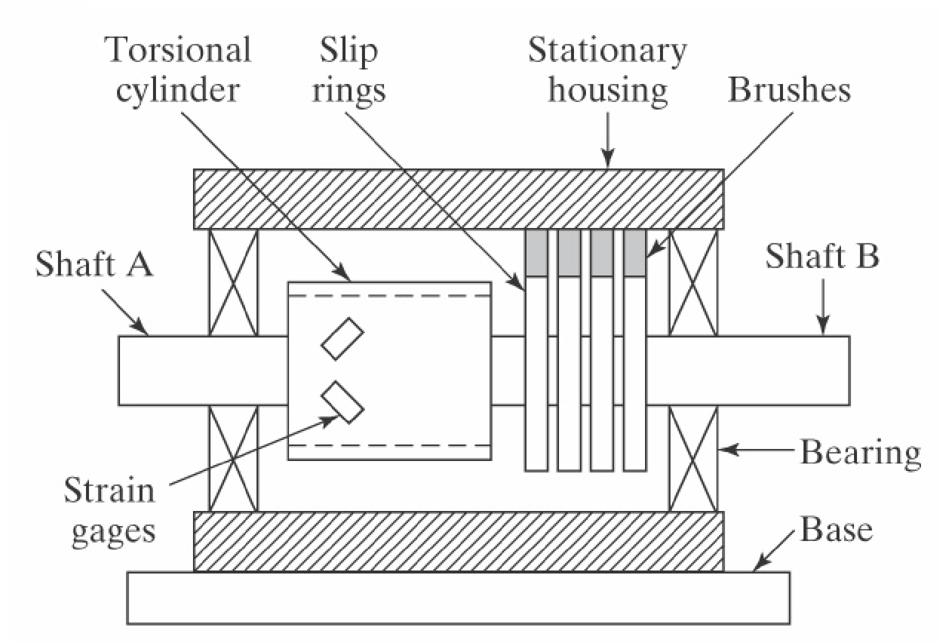


Load Cells


Tensile
gages
F
Compressive
gages
(a)
F
Tensile
gages
Compressive
gages
F
(b)

Torque Meters





Dynamometers