Dynamic Signals

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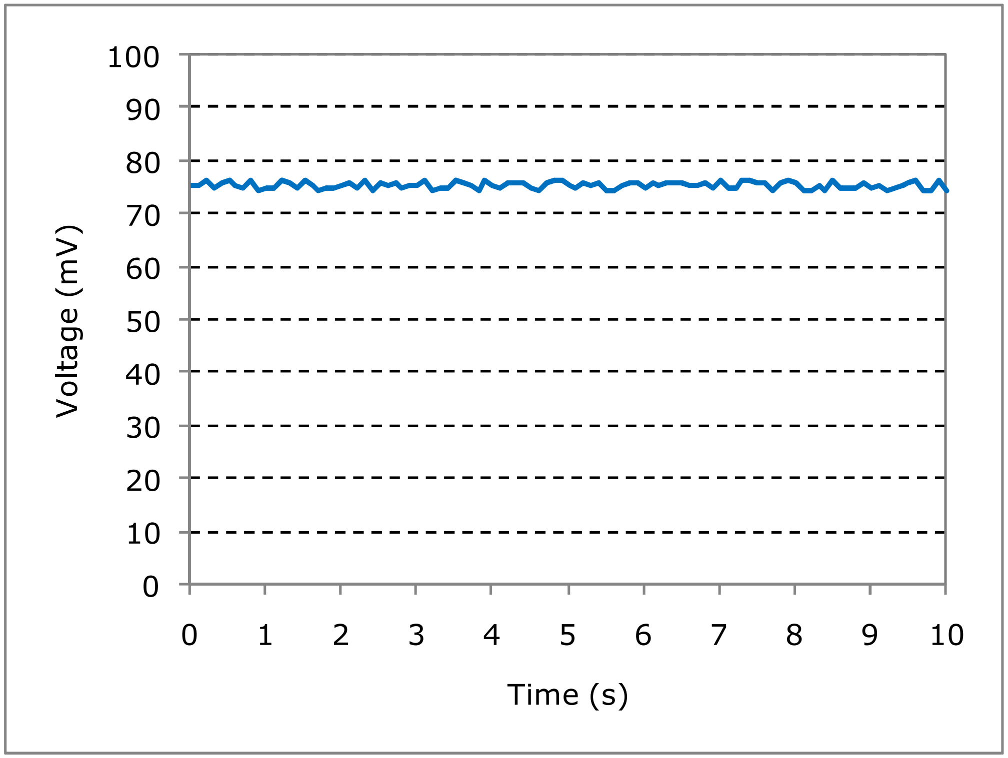
Static vs. Dynamic Signals

- In principle, all signals are dynamic; they do not have a perfectly constant value over time.
- **Static** signals are those for which changes over time are negligible for the intended measurement.
- **Dynamic** signals are those for which changes over time are relevant for the intended measurement. Examples include:
  - Impulses, steps, ramps
  - Periodic functions (e.g. sinusoidal)
  - Random noise

Repeatability Error and Time

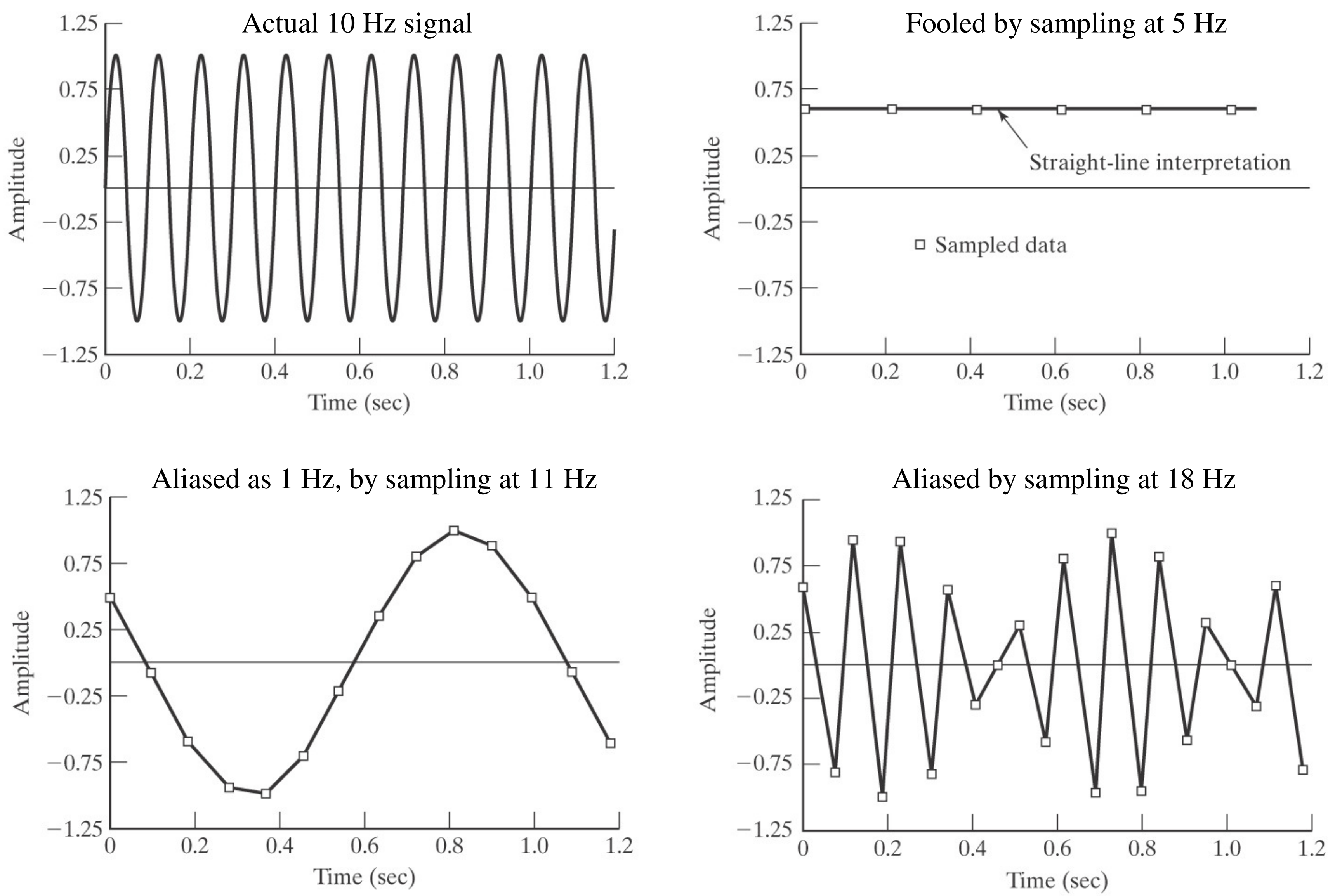


Root Mean Square:

$$\mathrm{RMS} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2}$$

- What is the value of this signal?

Discrete Sampling Rate Limitations



Sampling-Rate Theorem

- The sampling-rate theorem states that the sampling frequency f_s must be greater than twice the actual signal frequency f to correctly determine the frequency of the actual signal:

$$f_s > 2f$$

- For signals containing more than one characteristic frequency, it applies to the highest frequency.
- It is not unusual for actual signals to have frequencies too high to set sampling frequency greater than $2f$, in which case filters may be necessary.

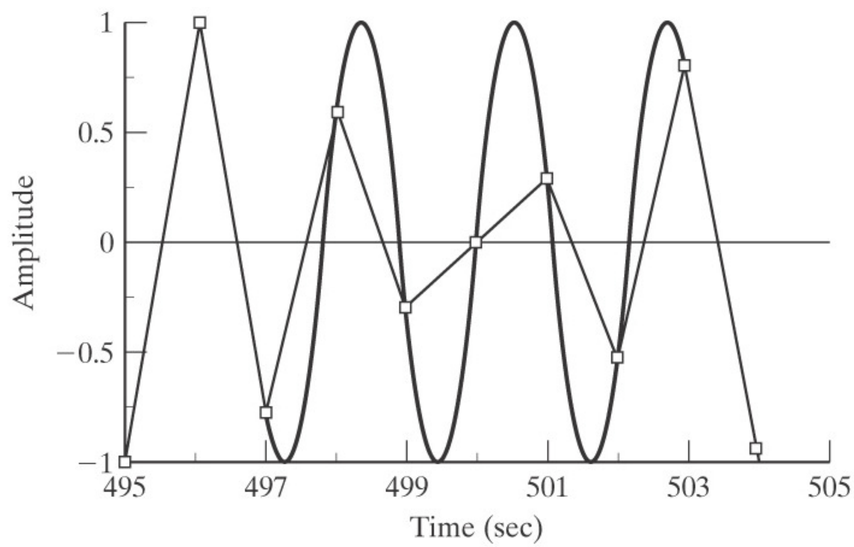
- Names associated with the sampling-rate theorem include Nyquist, Shannon, and Cauchy, but this theorem must not be confused with the "Nyquist Stability Criterion" from controls theory.

Reconstructing Waveforms

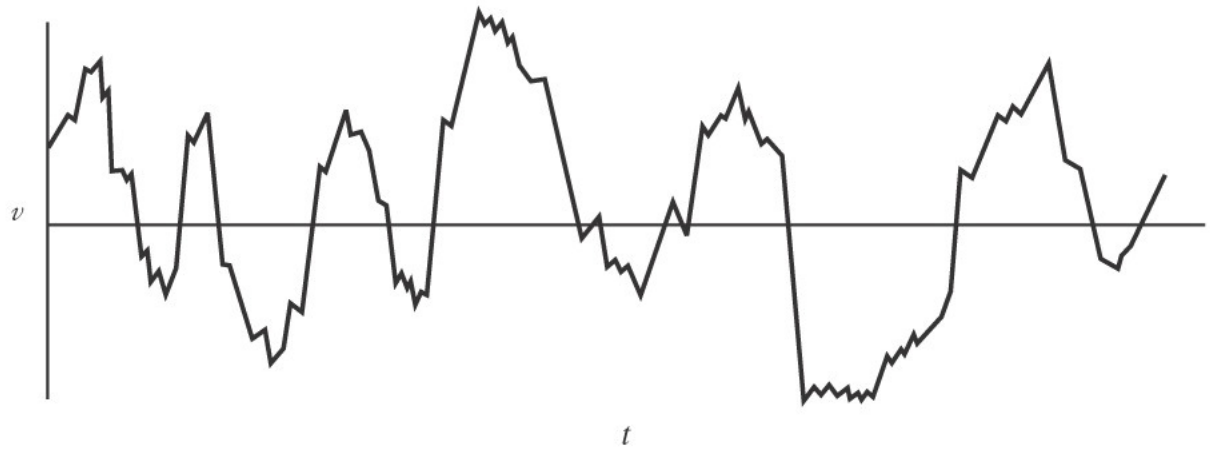
- Even if the sampling-rate theorem condition is met such that $f_s > 2f$, the sampled data does not necessarily reveal the same amplitude as the actual signal.
- However (amazingly) if the data set is sufficiently large, the original waveform can be reconstructed with high fidelity from the *n* available measurements by using the following series equation:

$$x\{t\} = \frac{1}{\pi} \sum_{n=-\infty}^{+\infty} x\{n\Delta T\} \frac{\sin\left[\pi(t/\Delta T - n)\right]}{t/\Delta T - n}$$

$$\Delta T = 1/f_s$$



Spectral Analysis



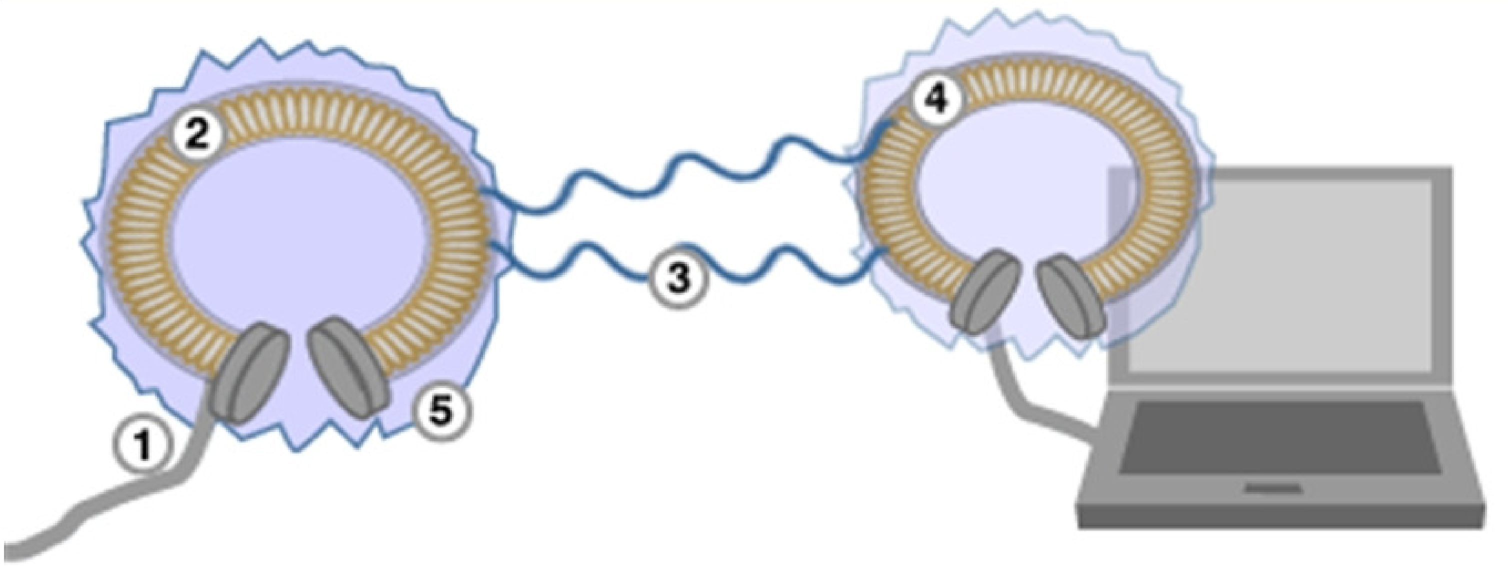
- A general Time Varying signal does not have the form of a simple sine wave
- Complicated waveforms can be considered to be a sum of set of sine or cosine waves of different frequencies
- Process of determining => Spectral analysis

Examples of Spectral Analysis Applications

- Vibration measurement
- Structural design & fatigue analysis
- Noise cancellation
- Sensitive resonance-based transducers
- Wireless power sources

Example of Applied Frequency Analysis

HOW WIRELESS POWER COULD WORK



- **1)** Power from mains to antenna, which is made of copper
- **2)** Antenna resonates at a frequency of 6.4MHz, emitting electromagnetic waves
- **3)** 'Tails' of energy from antenna 'tunnel' up to 5m (16.4ft)
- **4)** Electricity picked up by laptop's antenna, which must also be resonating at 6.4MHz. Energy used to re-charge device
- **5)** Energy not transferred to laptop re-absorbed by source antenna. People/other objects not affected as not resonating at 6.4MHz

Image retrieved December, 2008 from http://news.bbc.co.uk/2/hi/technology/6129460.stm

Fourier Series

- Any periodic function $x\{t\}$ can be represented by a Fourier series with period $T = 1/f$ and fundamental angular frequency $\omega_0 = 2\pi f_0$.

$$x\{t\} = A_0 + \sum_{n=1}^{\infty}\left[A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t)\right]$$

$$A_0 = \frac{1}{T}\int_0^T x\{t\}dt \qquad A_n = \frac{2}{T}\int_0^T x\{t\}\cos(n\omega_0 t)dt \qquad B_n = \frac{2}{T}\int_0^T x\{t\}\sin(n\omega_0 t)dt$$

- Furthermore, dynamic signals that are not periodic can be constructed by superposition of harmonics…

Examples of Summed Harmonics

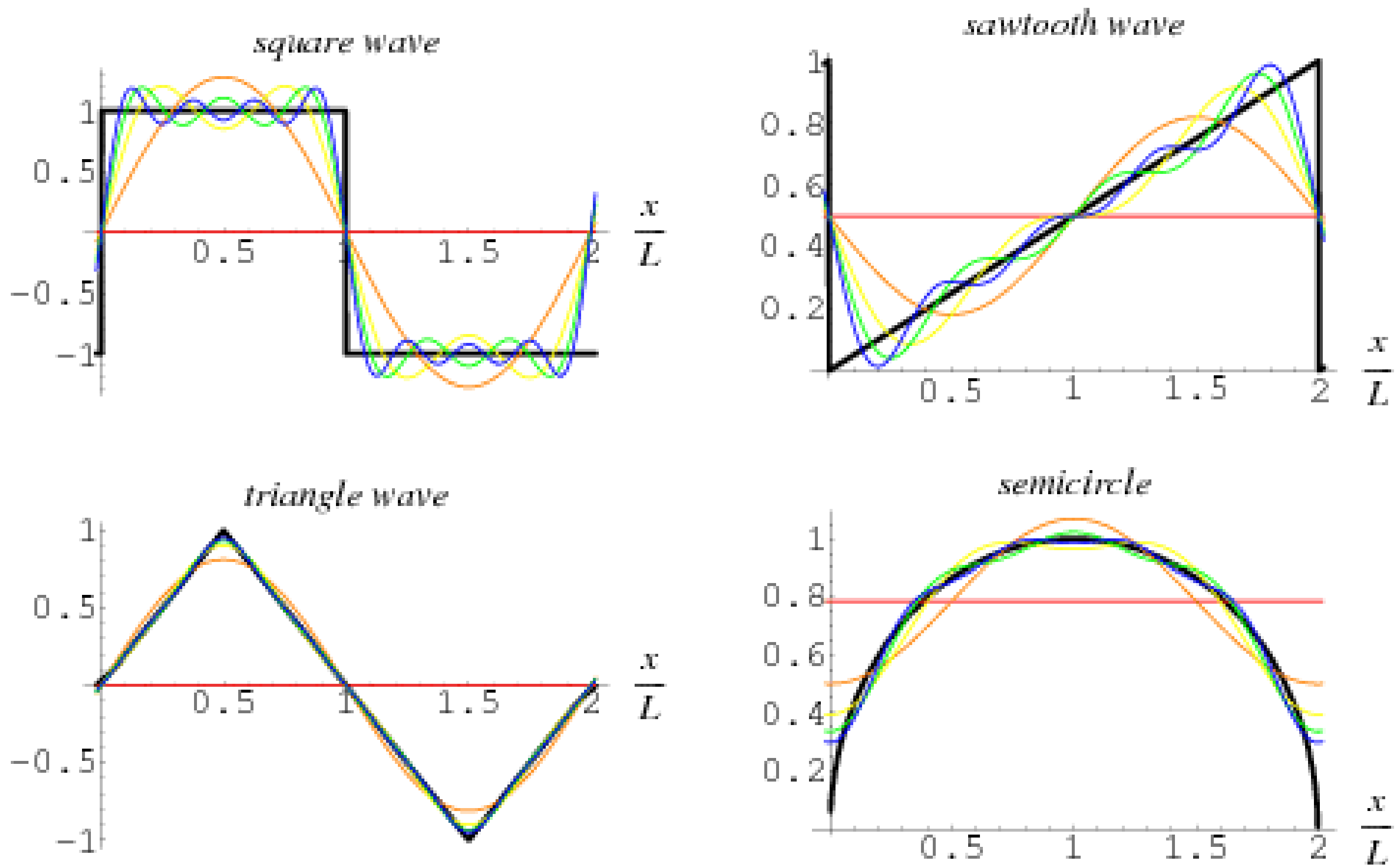


Image retrieved December, 2008 from http://mathworld.wolfram.com/

Fourier Transform

- As opposed to the limited case of summations in Fourier series, the Fourier transform is more useful because it treats frequency as a continuous function.

$$F\{\omega\} = \int_{-\infty}^{+\infty} f\{t\} e^{-j\omega t}\, dt$$

- The magnitude of $F\{\omega\}$ at any given frequency $f\{t\}$ along the spectrum reveals the relative contribution of that particular frequency to the original signal.
- Having a Fourier transform $F\{\omega\}$, the original function $f\{t\}$ can be recovered by the inverse Fourier transform:

$$f\{t\} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F\{\omega\} e^{j\omega t}\, d\omega$$

Discrete Fourier Transform

- Necessary for discrete sampling, the discrete Fourier transform is defined in terms of the finite number of N samples during an overall time period T, with time increment Δt and frequency increment $\Delta f = 1/T$.

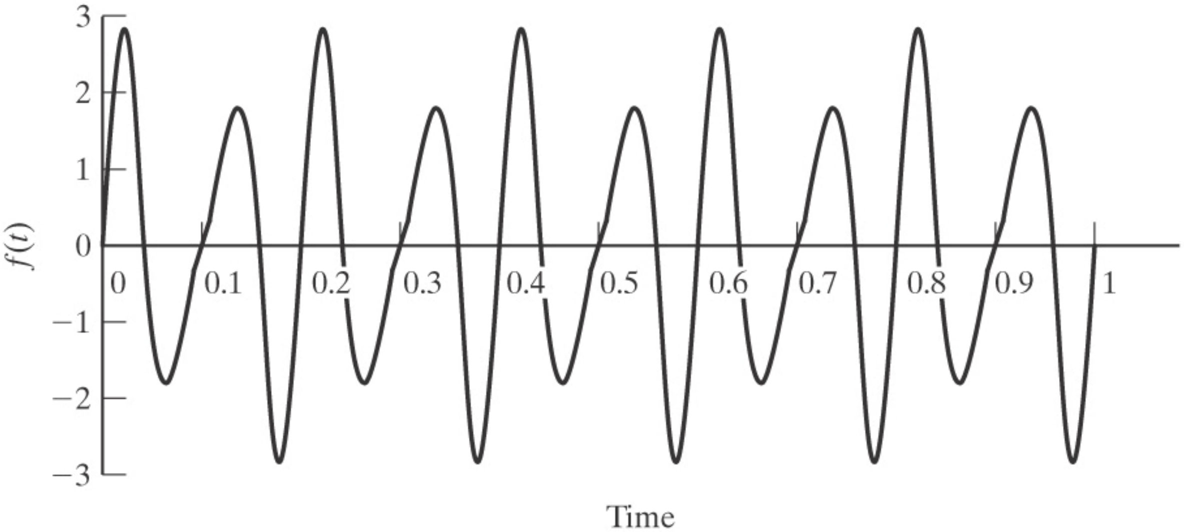
$$F\{k\Delta f\} = \sum_{n=0}^{N-1} f\{n\Delta t\} e^{-j(2\pi k\Delta f)(n\Delta t)} \qquad k = 0, 1, 2, \ldots N\text{-}1$$

- As in the case for (continuous) Fourier transforms, having $F\{k\Delta f\}$ provides a way to recover the original function $f\{n\Delta t\}$ using an inverse transform:

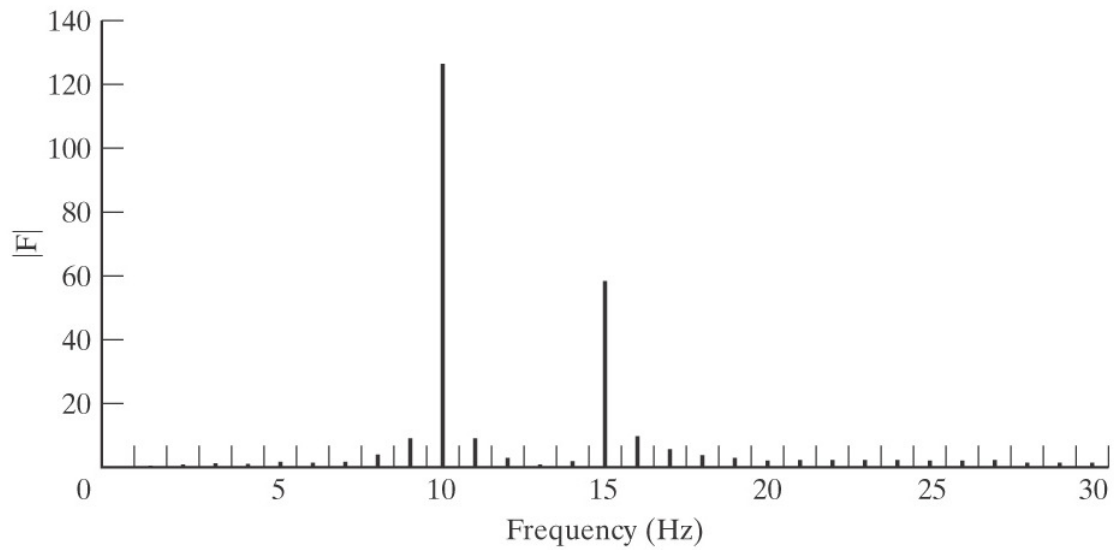
$$f\{n\Delta t\} = \frac{1}{N} \sum_{k=0}^{N-1} F\{k\Delta f\} e^{j(2\pi k\Delta f)(n\Delta t)} \qquad n = 0, 1, 2, \ldots N\text{-}1$$

- The Fast Fourier Transform (FFT) is a very effective and popular algorithm to compute discrete Fourier transforms.

FFT Example



3
2
1
0
−1
−2
−3
f(t)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time



140
120
100
80
60
40
20
|F|
0
5
10
15
20
25
30
Frequency (Hz)

Summary

- If a Fourier series is a summation, and a Fourier transform is superior because it treats frequency as continuous throughout a spectrum,

- And a discrete Fourier transform is necessary because of discrete (computerized) sampling,