METR 130: Assignment 2 (Spring 2011)
Due Date: March 10, 2011

Question 1

a. Derive an expression for the vertical profile of specific humidity \( q(z) \) in a neutral surface layer. Be sure to account for the surface specific humidity in your expression, and allow for the possibility that the roughness length for moisture can be different than that for wind.

b. Derive an expression for the turbulent exchange coefficient \( C_Q \) in a neutral surface layer based on the expression for \( q(z) \) derived in part ‘a’. Be sure to account for the possibility of different reference (i.e. measurement) heights for wind speed and specific humidity in the surface layer.

c. Calculate from your expression in part ‘b’ the value for \( C_Q \) for Question 1 of the “Take Home Assignment for Exam #1”. Assume \( z_{0h} = 0.1z_{0m} \) (a typically used relationship for bare or sparsely vegetated land surfaces). By how much (%) does this new calculation for \( C_Q \) differ from what was calculated from the expression originally assumed for this problem?

d. For inputs given in “Take Home Assignment for Exam #1”, estimate based on log-law expressions the wind speed at 50 meters above ground level.

Question 2

a. Using the log-law expression, calculate a value for friction velocity \( u^* \) for inputs given in “Take Home Assignment Exam #1”.

b. Using the value for \( u^* \) calculated in ‘part a’ and the surface sensible heat flux \( H_S \) calculated previously from “Take Home Assignment Exam #1”, calculate a value for Monin-Obukhov length, \( L \). From this, calculate the values for stability parameters for momentum (\( \zeta_m = z_{ref,m}/L \)) and heat (\( \zeta_h = z_{ref,h}/L \)).

c. Based on the calculated stability parameters in part ‘b’, recalculate a value for \( C_Q \) (= \( C_H \)) using the standard Monin-Obukhov expressions presented in class as well as in Chapter 11 of Arya. By how much (%) does this new calculation for \( C_Q \) differ from what was calculated from the expression originally assumed for this problem?

d. For inputs given in “Take Home Assignment for Exam #1”, estimate, but now based on Monin-Obukhov expressions, the wind speed at 50 meters above ground level. By how much does this 50-meter wind speed differ (%) from that calculated in Question 1 part ‘d’?
Question 3

The standard Monin-Obukhov stability function for the potential temperature profile in stably-stratified conditions is \( \phi_h = 1 + \beta z/L \). Based on this, derive an expression for \( d\theta/dz \) for large \( z/L \) (\( \gg 1 \)). Which surface layer scaling parameter cancels out in the course of deriving this equation, and thus does not appear in the final equation derived? As a result of this, what is the sole length scale appearing in the derived equation? Briefly discuss the physical significance of this, i.e. why it intuitively may make sense that the equation take the form you derived in such conditions (high \( z/L \))?