

# Chapter 24

## Wave Optics

# Wave Optics

- The wave nature of light is needed to explain various phenomena.
  - Interference
  - Diffraction
  - Polarization
- The particle nature of light was the basis for ray (geometric) optics.

# Interference

- Light waves interfere with each other much like mechanical waves do.
- All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

# Conditions for Interference

- For sustained interference between two sources of light to be observed, there are two conditions which must be met.
  - The sources must be *coherent*.
  - The waves they emit must maintain a constant phase with respect to each other.
  - The waves must have identical wavelengths.

# Producing Coherent Sources

- Light from a monochromatic source is allowed to pass through a narrow slit.
- The light from the single slit is allowed to fall on a screen containing two narrow slits.
- The first slit is needed to insure the light comes from a tiny region of the source which is coherent.
- Old method

# Producing Coherent Sources, Cont.

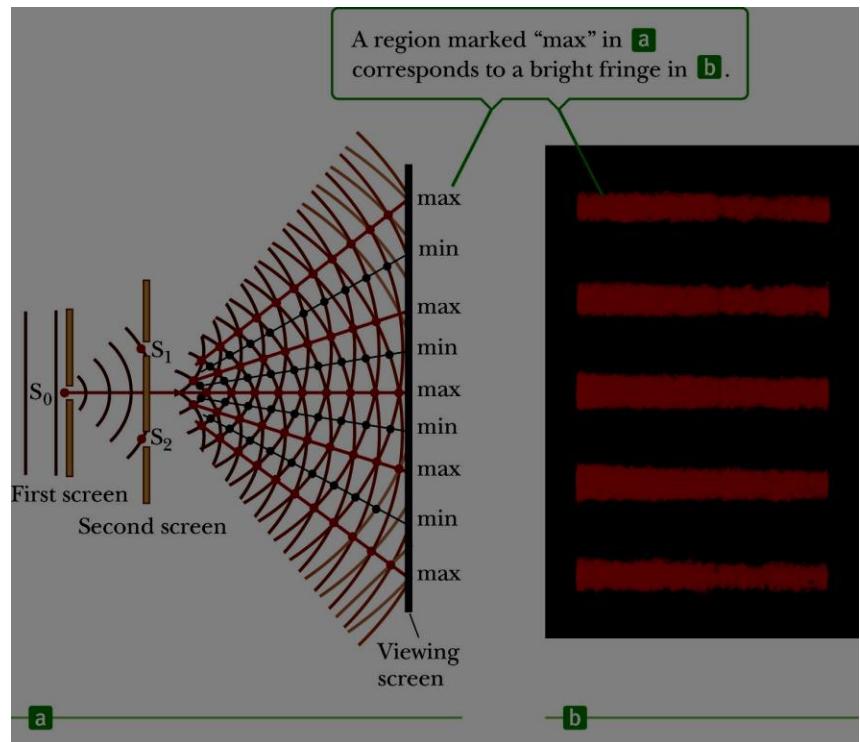
- Currently, it is much more common to use a laser as a coherent source.
- The laser produces an intense, coherent, monochromatic beam over a width of several millimeters.
- The laser light can be used to illuminate multiple slits directly.

# Young's Double Slit Experiment

- Thomas Young first demonstrated interference in light waves from two sources in 1801.
- Light is incident on a screen with a narrow slit,  $S_o$
- The light waves emerging from this slit arrive at a second screen that contains two narrow, parallel slits,  $S_1$  and  $S_2$

# Young's Double Slit Experiment, Diagram

- The narrow slits,  $S_1$  and  $S_2$  act as sources of waves.
- The waves emerging from the slits originate from the same wave front and therefore are always in phase.

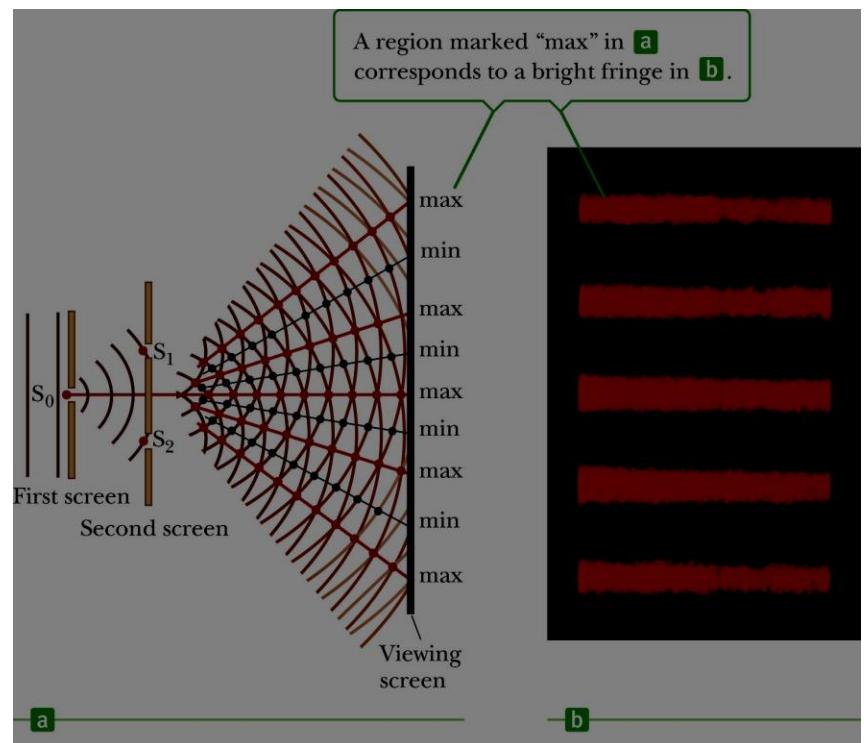


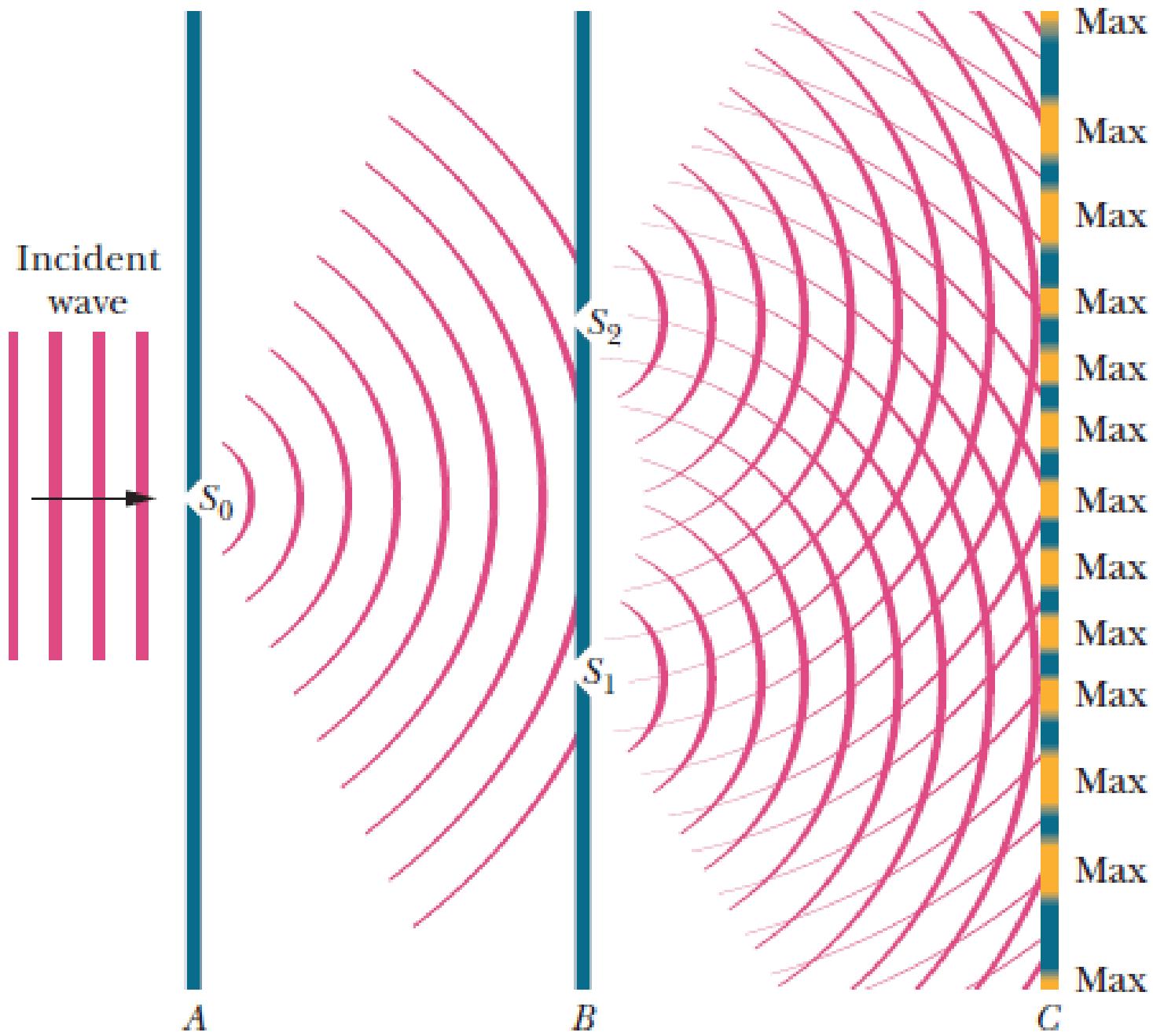
# Resulting Interference Pattern

- The light from the two slits form a visible pattern on a screen.
- The pattern consists of a series of bright and dark parallel bands called **fringes**.
- *Constructive interference* occurs where a bright fringe appears.
- *Destructive interference* results in a dark fringe.

# Fringe Pattern

- The fringe pattern formed from a Young's Double Slit Experiment would look like this.
- The bright areas represent constructive interference.
- The dark areas represent destructive interference.



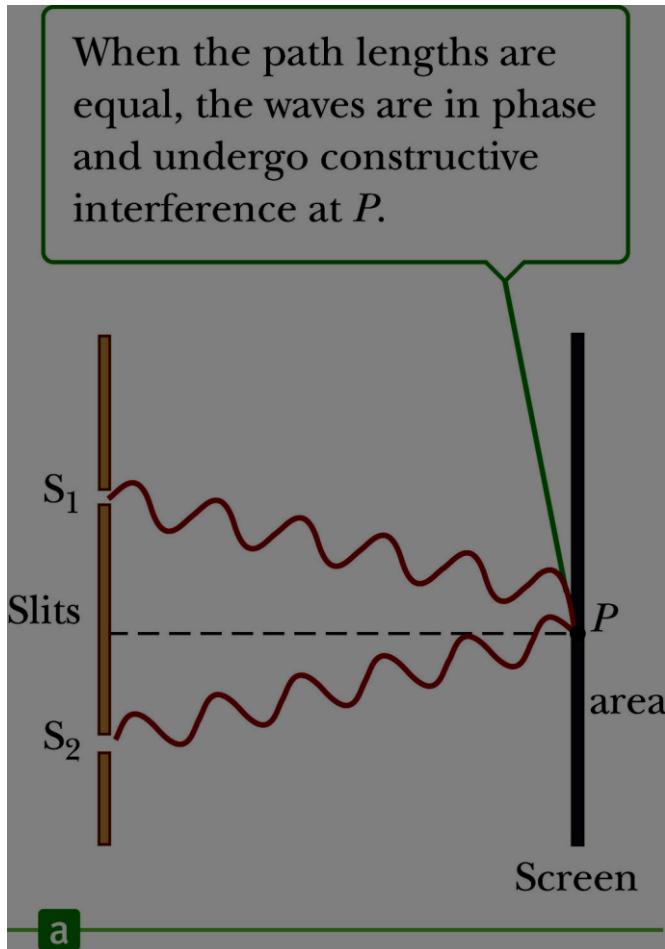




**Figure 24.2** An interference pattern involving water waves is produced by two vibrating sources at the water's surface. The pattern is analogous to that observed in Young's double-slit experiment. Note the regions of constructive and destructive interference.

# Interference Patterns

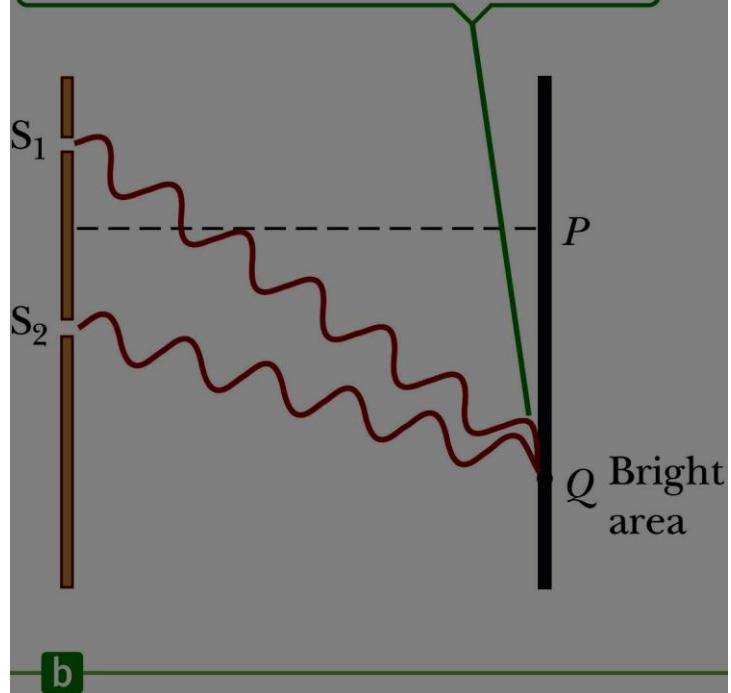
- Constructive interference occurs at the center point.
- The two waves travel the same distance.
  - Therefore, they arrive in phase.



# Interference Patterns, 2

- The upper wave has to travel farther than the lower wave.
- The upper wave travels one wavelength farther.
  - Therefore, the waves arrive in phase.
- A bright fringe occurs.

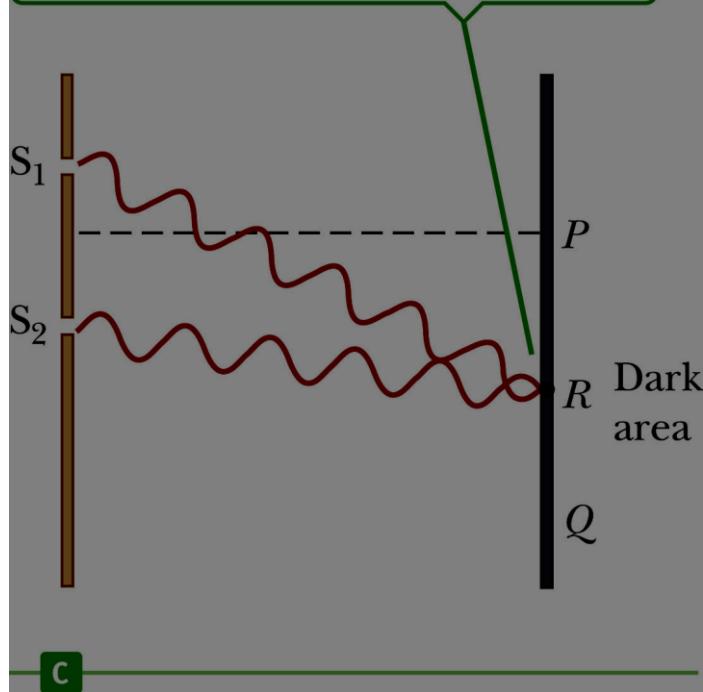
When the path lengths differ by a wavelength, the waves are in phase and undergo constructive interference at  $Q$ .



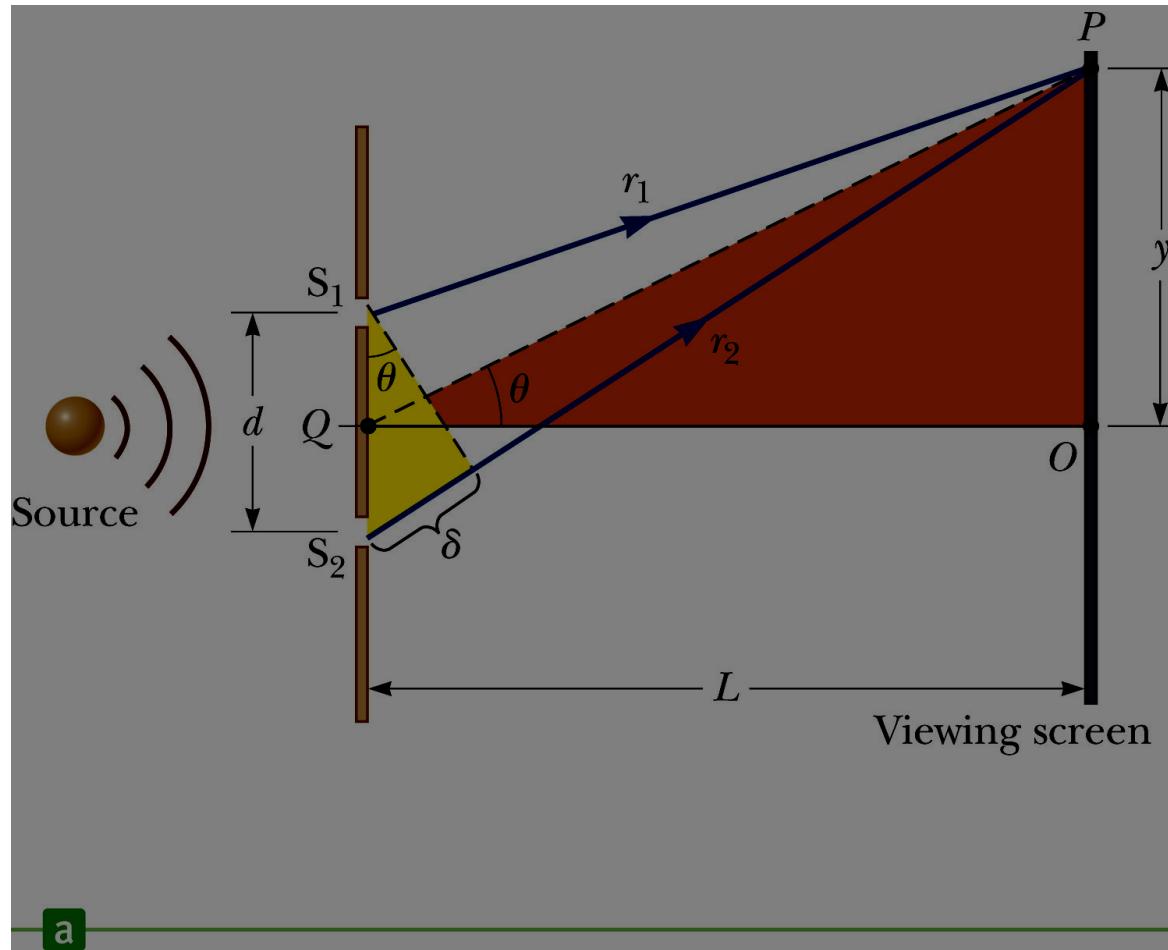
# Interference Patterns, 3

- The upper wave travels one-half of a wavelength farther than the lower wave.
- The trough of the bottom wave overlaps the crest of the upper wave.
- This is destructive interference.
  - A dark fringe occurs.

When the path lengths differ by half a wavelength, the waves are  $180^\circ$  out of phase and undergo destructive interference at  $R$ .

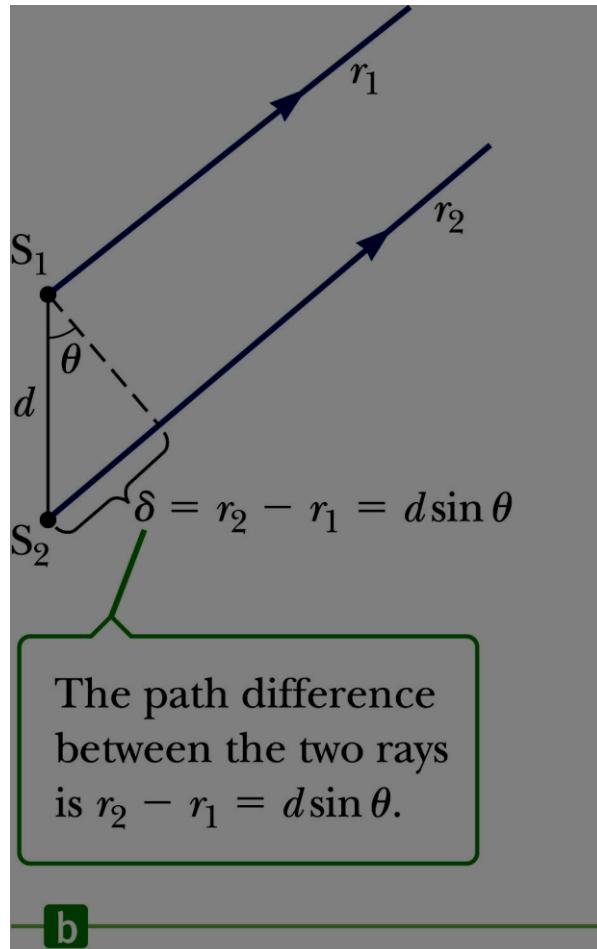


# Geometry of Young's Double Slit Experiment



# Interference Equations

- The path difference,  $\delta$ , is found from the small triangle.
- $\delta = r_2 - r_1 = d \sin \theta$ 
  - This assumes the paths are parallel.
  - Not exactly parallel, but a very good approximation since  $L$  is much greater than  $d$



# Interference Equations, 2

- For a bright fringe, produced by constructive interference, the path difference must be either zero or some integral multiple of the wavelength.
- $\delta = d \sin \theta_{\text{bright}} = m \lambda$ 
  - $m = 0, \pm 1, \pm 2, \dots$
  - $m$  is called the *order number*.
- When  $m = 0$ , it is the zeroth order maximum.
- When  $m = \pm 1$ , it is called the first order maximum.

# Interference Equations, 3

- When destructive interference occurs, a dark fringe is observed.
- This needs a path difference of an odd half wavelength.
- $\delta = d \sin \theta_{\text{dark}} = (m + \frac{1}{2}) \lambda$   
 $-m = 0, \pm 1, \pm 2, \dots$

# Interference Equations, 4

- The positions of the fringes can be measured vertically from the zeroth order maximum.

- $y = L \tan \theta \approx L \sin \theta$

- Assumptions

- $L \gg d$

- $d \gg \lambda$

- Approximation

- $\theta$  is small and therefore the approximation  $\tan \theta \approx \sin \theta$  can be used.

- The approximation is true to three-digit precision only for angles less than about  $4^\circ$

# Interference Equations, Final

- For bright fringes

$$y_{bright} = \frac{\lambda L}{d} m \quad m=0, \pm 1, \pm 2$$

$$\tan \theta \approx \theta \approx \frac{y_m}{L}$$

- For dark fringes

$$y_{dark} = \frac{\lambda L}{d} \left( m + \frac{1}{2} \right) \quad m=0, \pm 1, \pm 2$$

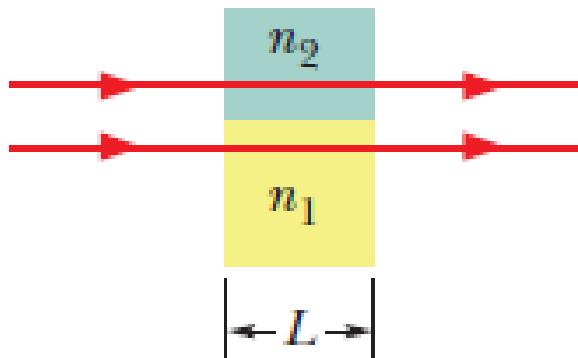
$$m\lambda = d \sin \theta \approx (d)(\theta); \quad \theta = \frac{m\lambda}{d} = \frac{y_m}{L}$$
$$\therefore y_m = \frac{\lambda L}{d} \cdot m.$$

# Uses for Young's Double Slit Experiment

- Young's Double Slit Experiment provides a method for measuring wavelength of the light.
- This experiment gave the wave model of light a great deal of credibility.
  - It is inconceivable that particles of light could cancel each other.

The phase difference between two light waves can change if the waves travel through different materials having different indexes of refraction

The difference in indexes causes a phase shift between the rays.



**Fig. 35-4** Two light rays travel through two media having different indexes of refraction.

$$v_1 = \frac{c}{n_1}; v_2 = \frac{c}{n_2} \quad (\text{same frequency})$$

$$\lambda_1 = \frac{v_1}{f} = \frac{c}{n_1 f} = \frac{\lambda}{n_1} \quad (c = f\lambda)$$

Similarly  $\lambda_2 = \frac{\lambda}{n_2}$ .

If light travels a thickness  $dt$ ,

The number of wavelengths in the two media are  $N_1 = \frac{t}{\lambda_1} = \frac{t n_1}{\lambda}$

$$N_2 = \frac{t n_2}{\lambda}, \quad N_2 - N_1 = \frac{t}{\lambda} (n_2 - n_1).$$

Problem : Calculate the phase difference when  $t = 2.600 \mu\text{m}$ ,  $n_1 = 1.000$ ,  $n_2 = 1.600$ ,  $\lambda = 550.0 \text{ nm}$ .

$$\begin{aligned}N_2 - N_1 &= \frac{t}{\lambda} (n_2 - n_1) \\&= \frac{(2.600 \times 10^{-6} \text{ m})(1.600 - 1.000)}{5.500 \times 10^{-7} \text{ m}} \\&= 2.84\end{aligned}$$

The path difference is 2.84 wavelengths.

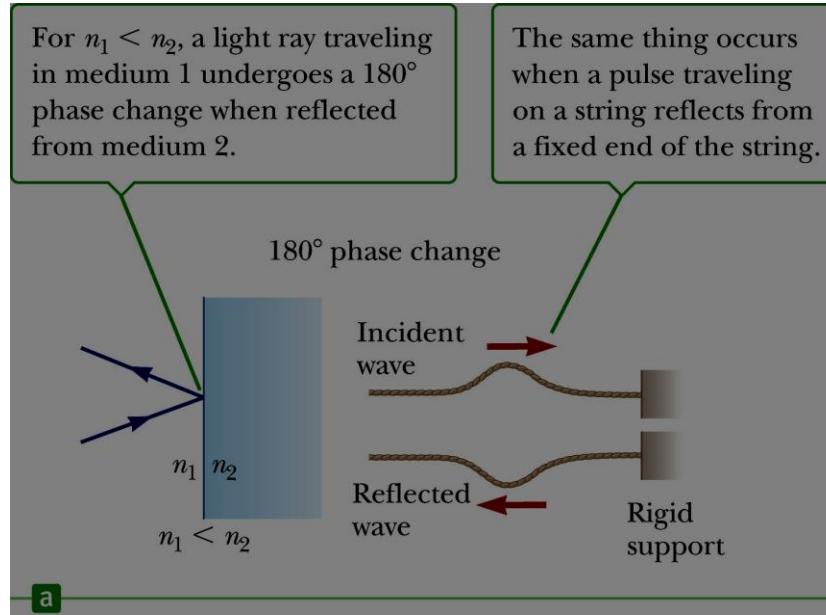
1.0 wavelength corresponds to  $360^\circ$  or  $2\pi$

2.84 wavelengths has 2 wavelengths  
plus 0.84 wavelengths. Integral  
wavelengths give zero phase shift  
effectively. Hence the phase  
difference is

$$0.84 \times 360^\circ = 302.4^\circ \text{ or}$$

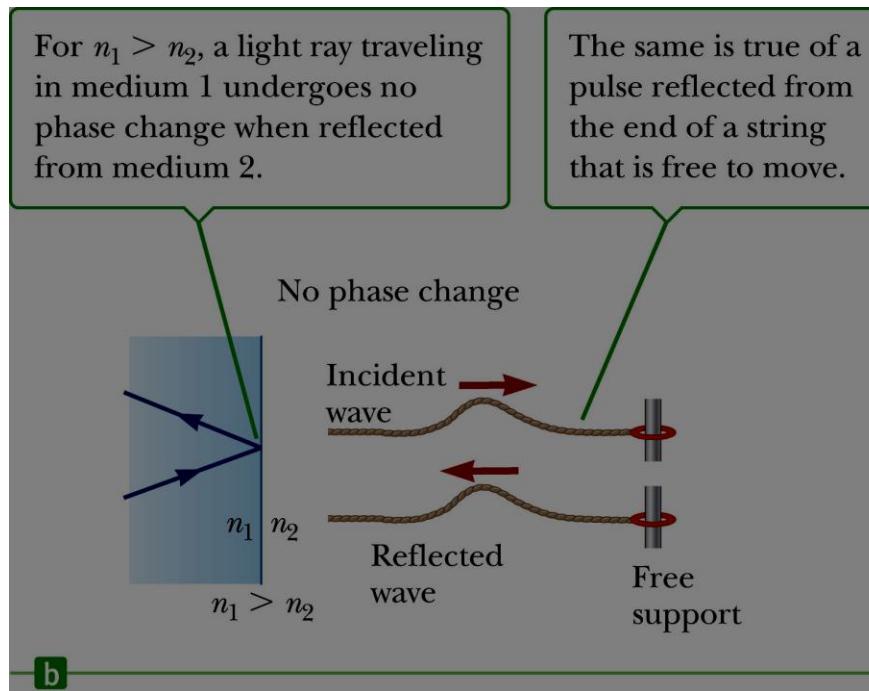
$$0.84 \times 2\pi = 5.27 \text{ radians -}$$

# Phase Changes Due To Reflection



- An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling.
  - Analogous to a reflected pulse on a string

# Phase Changes Due To Reflection, Cont.

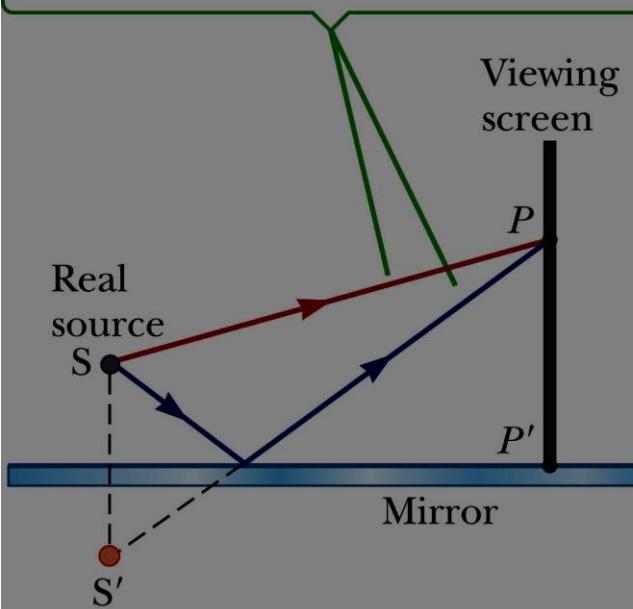


- There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction.
  - Analogous to a pulse in a string reflecting from a free support

# Lloyd's Mirror

- An arrangement for producing an interference pattern with a single light source
- Waves reach point P either by a direct path or by reflection.
- The reflected ray can be treated as a ray from the source S' behind the mirror.

An interference pattern is produced on a screen at P as a result of the combination of the direct ray (red) and the reflected ray (blue). The reflected ray undergoes a phase change of  $180^\circ$ .



# Interference Pattern from the Lloyd's Mirror

- An interference pattern is formed.
- The positions of the dark and bright fringes are *reversed* relative to pattern of two real sources.
- This is because there is a  $180^\circ$  phase change produced by the reflection.

# Interference in Thin Films

- Interference effects are commonly observed in thin films.
  - Examples are soap bubbles and oil on water
- The interference is due to the interaction of the waves reflected from both surfaces of the film.

Dr. Jeremy Burgess/Science Photo Library/  
Photo Researchers, Inc.

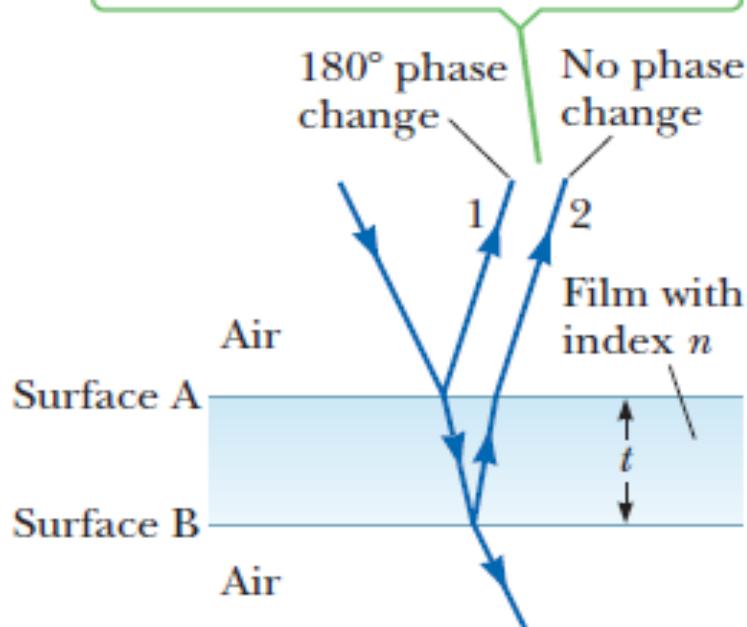


The colors observed in soap bubbles are due to interference between light rays reflected from the front and back of the thin film of soap making up the bubble. The color depends on the thickness of the film, ranging from black where the film is at its thinnest to magenta where it is thickest.

# Interference in Thin Films, 2

- Facts to remember
  - An electromagnetic wave traveling from a medium of index of refraction  $n_1$  toward a medium of index of refraction  $n_2$  undergoes a  $180^\circ$  phase change on reflection when  $n_2 > n_1$
  - There is no phase change in the reflected wave if  $n_2 < n_1$
  - The wavelength of light  $\lambda_n$  in a medium with index of refraction  $n$  is  $\lambda_n = \lambda/n$  where  $\lambda$  is the wavelength of light in vacuum.

Interference in light reflected from a thin film is due to a combination of rays 1 and 2 reflected from the upper and lower surfaces of the film.



- Ray 1 undergoes a phase change of  $180^\circ$  with respect to the incident ray.
- Ray 2, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave.

# Interference in Thin Films, 4

- Ray 2 also travels an additional distance of  $2t$  before the waves recombine.
- For constructive interference
  - $-2nt = (m + \frac{1}{2})\lambda$   $m = 0, 1, 2 \dots$
  - This takes into account both the difference in optical path length for the two rays and the  $180^\circ$  phase change
- For destructive interference
  - $-2nt = m\lambda$   $m = 0, 1, 2 \dots$

# Interference in Thin Films, 5

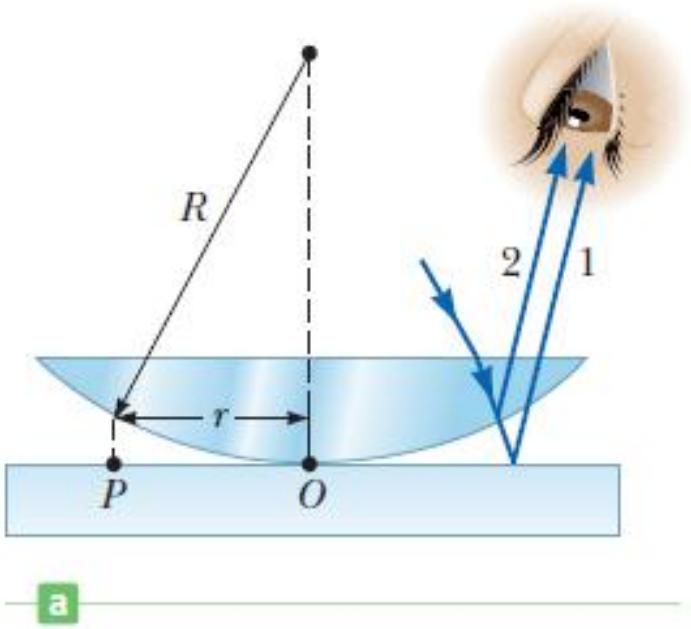
- Two factors influence interference.
  - Possible phase reversals on reflection
  - Differences in travel distance
- The conditions are valid if the medium above the top surface is the same as the medium below the bottom surface.
- If the thin film is between two different media, one of lower index than the film and one of higher index, the conditions for constructive and destructive interference are *reversed*.

# Interference in Thin Films, Final

- Be sure to include two effects when analyzing the interference pattern from a thin film.
  - Path length
  - Phase change

# Newton's Rings

- Another method for viewing interference is to place a planoconvex lens on top of a flat glass surface.
- The air film between the glass surfaces varies in thickness from zero at the point of contact to some thickness  $t$ .
- A pattern of light and dark rings is observed.
  - These rings are called *Newton's Rings*.
  - The particle model of light could not explain the origin of the rings.
- Newton's Rings can be used to test optical lenses.



a



b

Ray 1 undergoes a phase change of  $180^\circ$  on reflection because it is reflected from a boundary leading into a medium of higher refractive index, whereas ray 2 undergoes no phase change because it is reflected from a medium of lower refractive index.

$$2nt = (m + \frac{1}{2})\lambda \rightarrow \text{constructive Interf.}$$

# Problem Solving Strategy with Thin Films, 1

- Identify the thin film causing the interference.
- Determine the indices of refraction in the film and the media on either side of it.
- Determine the number of phase reversals: zero, one or two.

# Problem Solving with Thin Films, 2

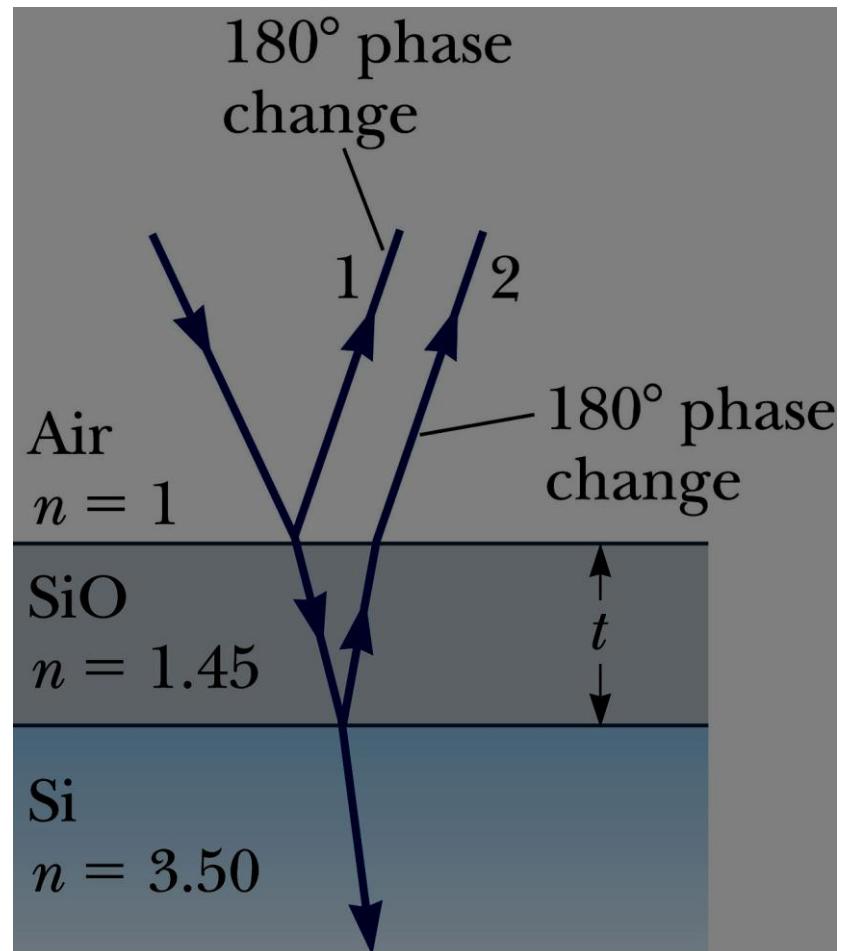
- The interference is constructive if the path difference is an integral multiple of  $\lambda$  and destructive if the path difference is an odd half multiple of  $\lambda$ .
  - The conditions are reversed if one of the waves undergoes a phase change on reflection.
- Substitute values in the appropriate equation.
- Solve and check.

# Problem Solving with Thin Films, 3

| Equation<br>$m = 0, 1, 2, \dots$  | 1 phase reversal | 0 or 2 phase reversals |
|-----------------------------------|------------------|------------------------|
| $2nt = (m + \frac{1}{2}) \lambda$ | constructive     | destructive            |
| $2nt = m \lambda$                 | destructive      | constructive           |

# Interference in Thin Films, Example

- An example of different indices of refraction
- A coating on a solar cell
- There are two phase changes



# CD's and DVD's

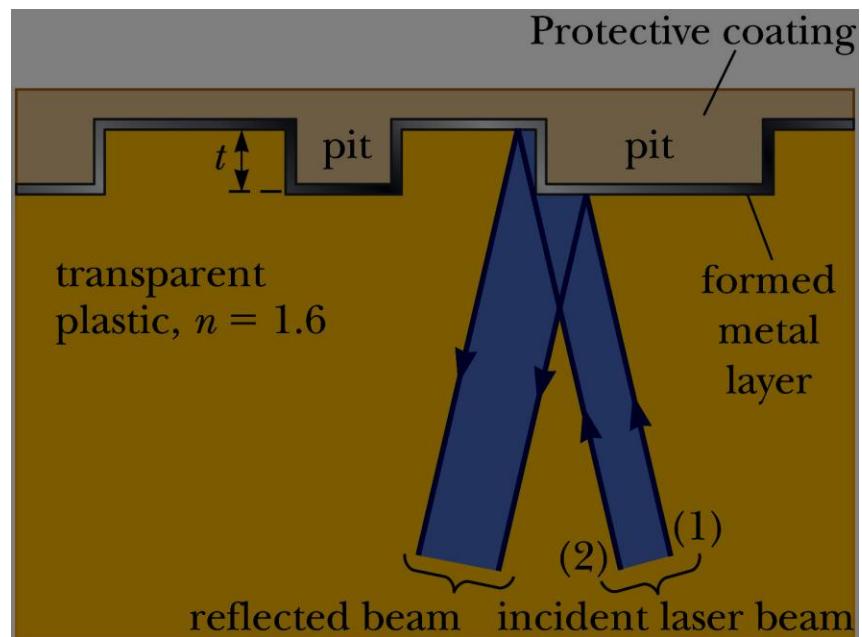
- Data is stored digitally.
  - A series of ones and zeros read by laser light reflected from the disk
- Strong reflections correspond to constructive interference.
  - These reflections are chosen to represent zeros.
- Weak reflections correspond to destructive interference.
  - These reflections are chosen to represent ones.

# CD's and Thin Film Interference

- A CD has multiple tracks.
  - The tracks consist of a sequence of pits of varying length formed in a reflecting information layer.
- The pits appear as bumps to the laser beam.
  - The laser beam shines on the metallic layer through a clear plastic coating.

# Reading a CD

- As the disk rotates, the laser reflects off the sequence of bumps and lower areas into a photodetector.
  - The photodetector converts the fluctuating reflected light intensity into an electrical string of zeros and ones.
- The pit depth is made equal to one-quarter of the wavelength of the light.



# Reading a CD, Cont.

- When the laser beam hits a rising or falling bump edge, part of the beam reflects from the top of the bump and part from the lower adjacent area.
  - This ensures destructive interference and very low intensity when the reflected beams combine at the detector.
- The bump edges are read as ones.
- The flat bump tops and intervening flat plains are read as zeros.

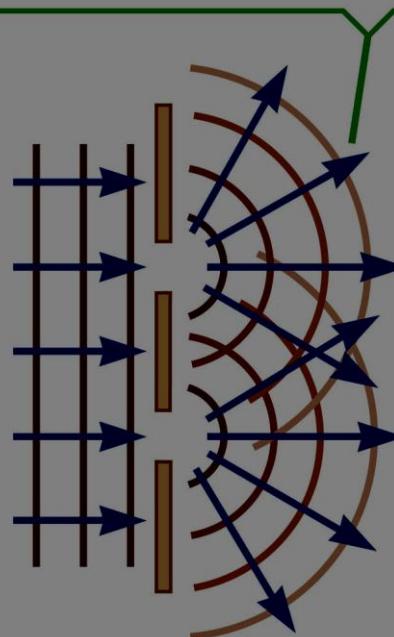
# DVD's

- DVD's use shorter wavelength lasers.
  - The track separation, pit depth and minimum pit length are all smaller.
  - Therefore, the DVD can store about 30 times more information than a CD.

# Diffraction

- Huygen's principle requires that the waves spread out after they pass through slits.
- This spreading out of light from its initial line of travel is called *diffraction*.
- In general, diffraction occurs when waves pass through small openings, around obstacles or by sharp edges.

Light passing through narrow slits *diffracts*.

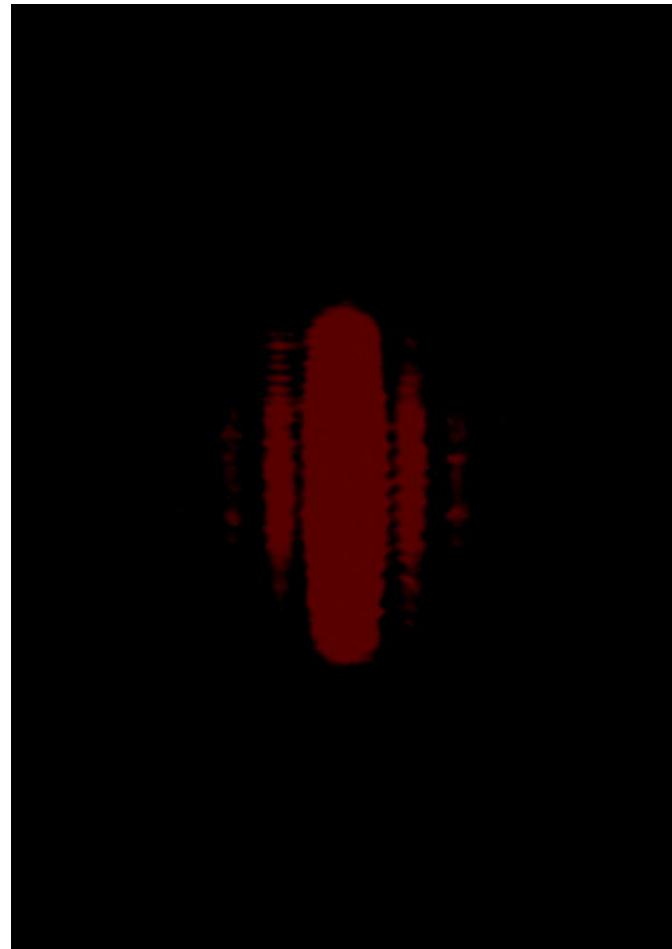


# Diffraction, 2

- A single slit placed between a distant light source and a screen produces a diffraction pattern.
  - It will have a broad, intense central band.
  - The central band will be flanked by a series of narrower, less intense secondary bands.
- Called secondary maxima
  - The central band will also be flanked by a series of dark bands.
- Called minima

# Diffraction, 3

- The results of the single slit cannot be explained by geometric optics.
  - Geometric optics would say that light rays traveling in straight lines should cast a sharp image of the slit on the screen.

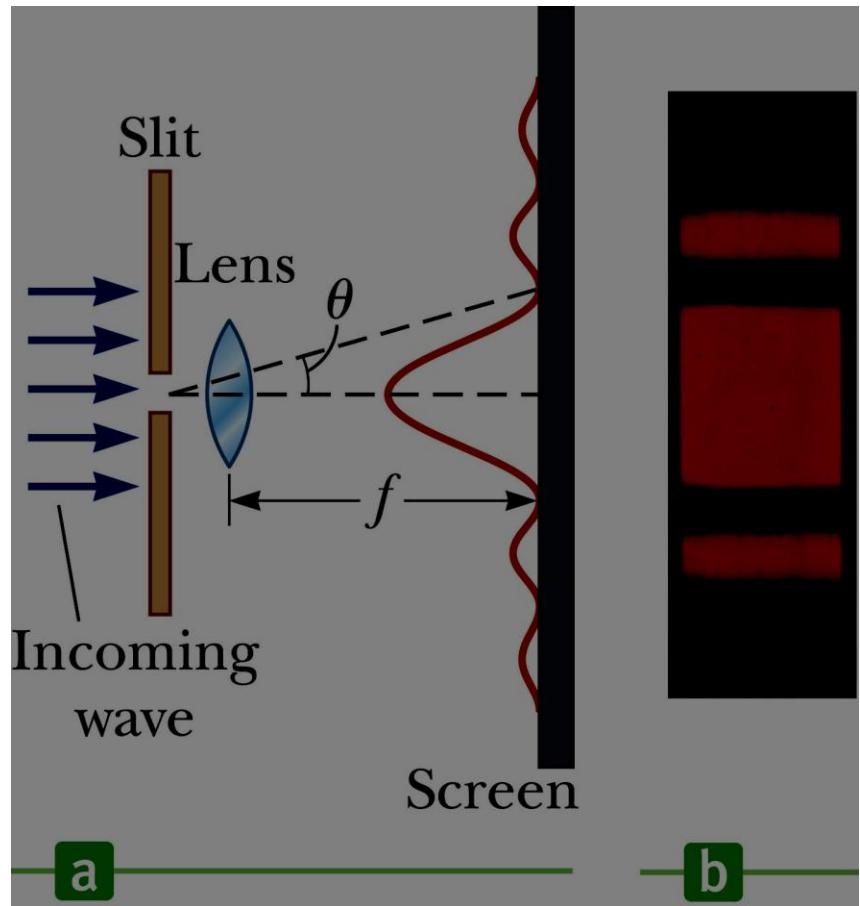


# Fraunhofer Diffraction

- *Fraunhofer Diffraction* occurs when the rays leave the diffracting object in parallel directions.

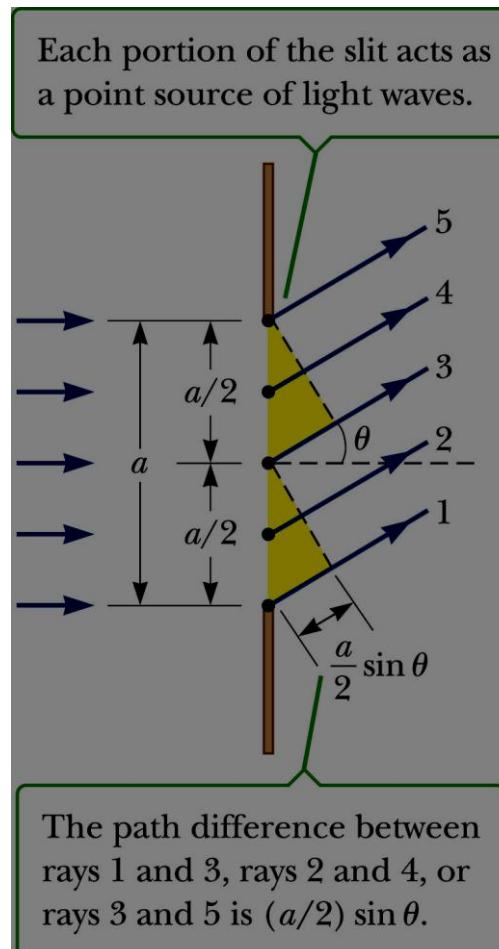
- Screen very far from the slit
- Converging lens (shown)

- A bright fringe is seen along the axis ( $\theta = 0$ ) with alternating bright and dark fringes on each side.



# Single Slit Diffraction

- According to Huygen's principle, each portion of the slit acts as a source of waves.
- The light from one portion of the slit can interfere with light from another portion.
- The resultant intensity on the screen depends on the direction  $\theta$



# Single Slit Diffraction, 2

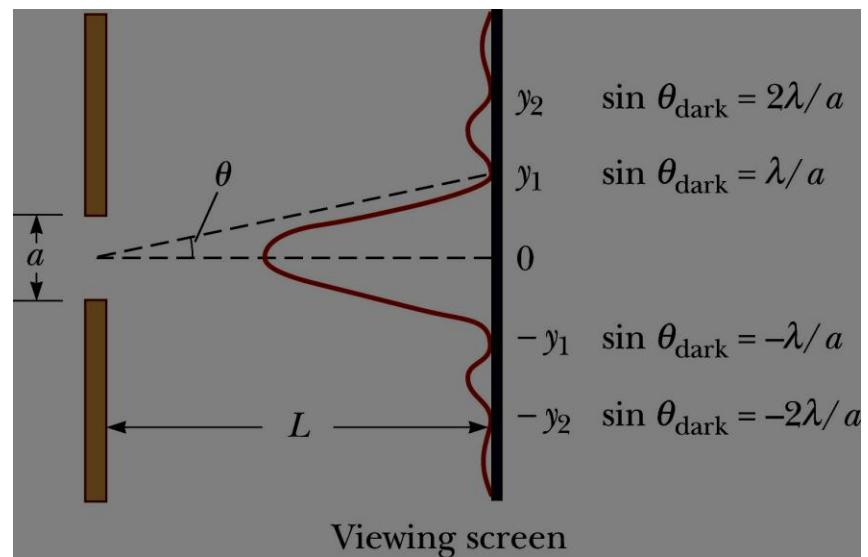
- All the waves that originate at the slit are in phase.
- Wave 1 travels farther than wave 3 by an amount equal to the path difference  $(a/2) \sin \theta$ 
  - $a$  is the width of the slit
- If this path difference is exactly half of a wavelength, the two waves cancel each other and destructive interference results.

# Single Slit Diffraction, 3

- In general, *destructive interference* occurs for a single slit of width  $a$  when  $\sin \theta_{\text{dark}} = m\lambda / a$   
–  $m = \pm 1, \pm 2, \pm 3, \dots$
- Doesn't give any information about the variations in intensity along the screen

# Single Slit Diffraction, 4

- The general features of the intensity distribution are shown.
- A broad central bright fringe is flanked by much weaker bright fringes alternating with dark fringes.
- The points of constructive interference lie approximately halfway between the dark fringes.

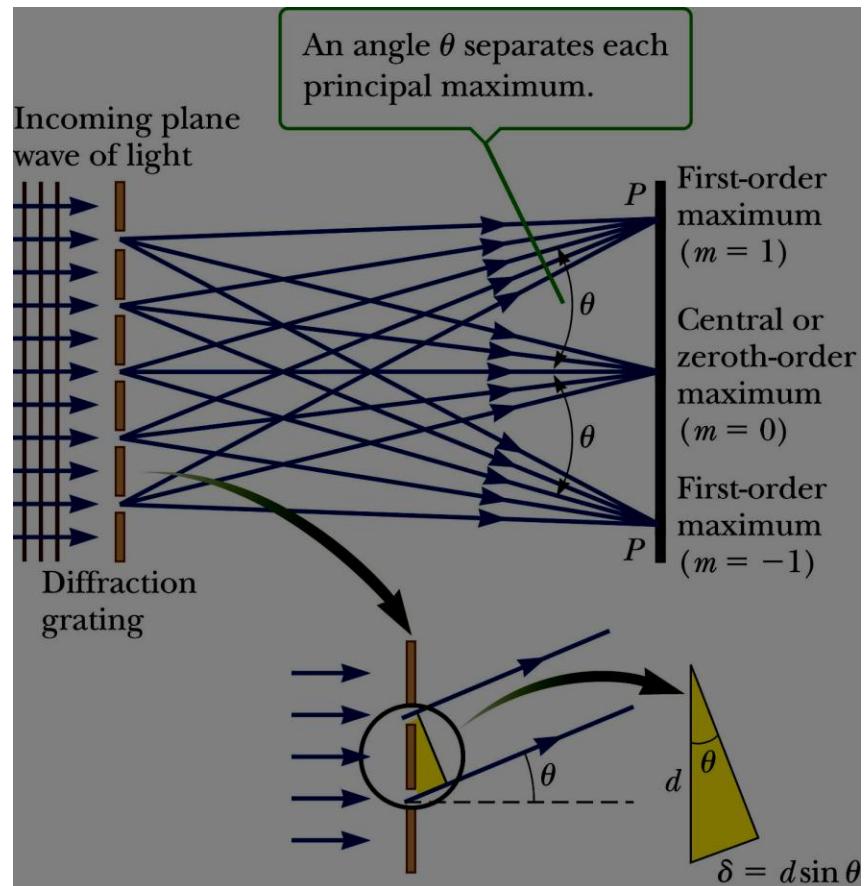


# Diffraction Grating

- The diffracting grating consists of many equally spaced parallel slits.
  - A typical grating contains several thousand lines per centimeter.
- The intensity of the pattern on the screen is the result of the combined effects of interference and diffraction.

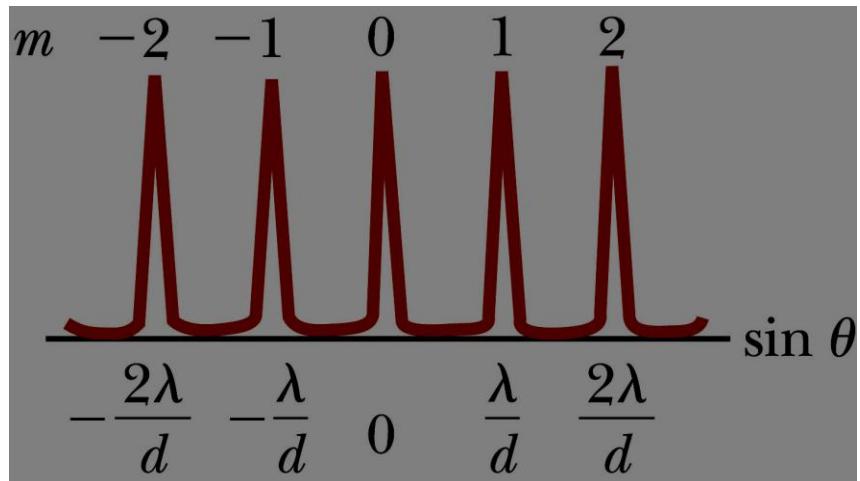
# Diffraction Grating, Cont.

- The condition for *maxima* is  
 $-d \sin \theta_{\text{bright}} = m \lambda$
- $m = 0, \pm 1, \pm 2, \dots$
- The integer  $m$  is the *order number* of the diffraction pattern.
- If the incident radiation contains several wavelengths, each wavelength deviates through a specific angle.



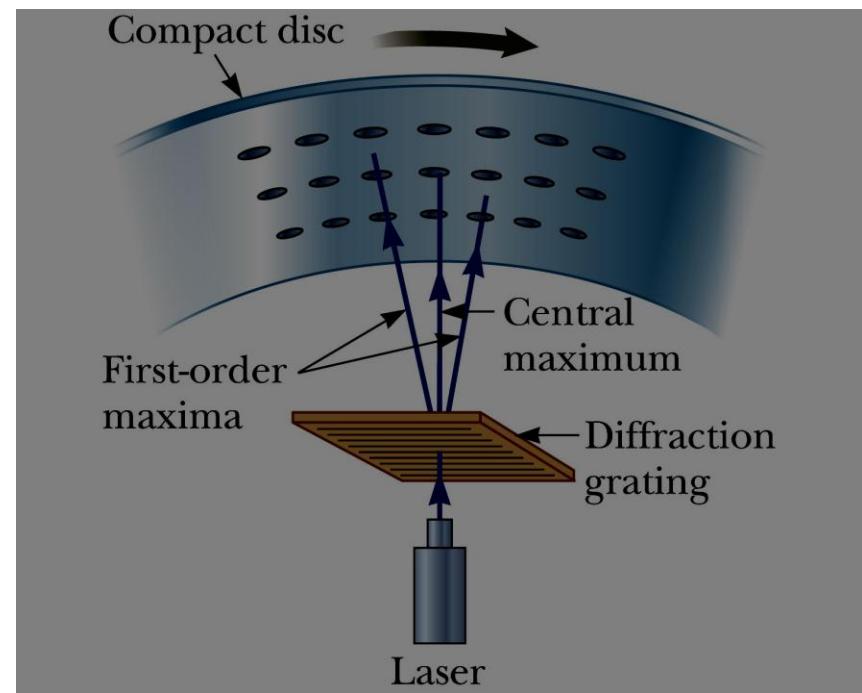
# Diffraction Grating, Final

- All the wavelengths are focused at  $m = 0$ 
  - This is called the zeroth order maximum
- The first order maximum corresponds to  $m = 1$
- Note the sharpness of the principle maxima and the broad range of the dark area.
  - This is in contrast to the broad, bright fringes characteristic of the two-slit interference pattern.



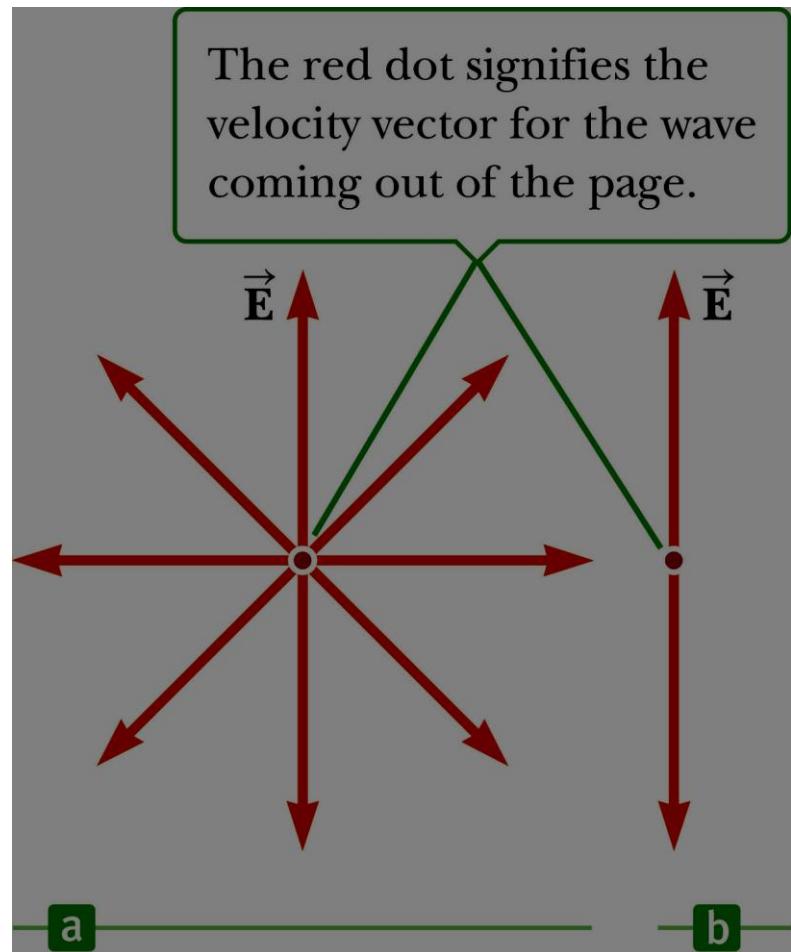
# Diffraction Grating in CD Tracking

- A diffraction grating can be used in a three-beam method to keep the beam on a CD on track.
- The central maximum of the diffraction pattern is used to read the information on the CD.
- The two first-order maxima are used for steering.



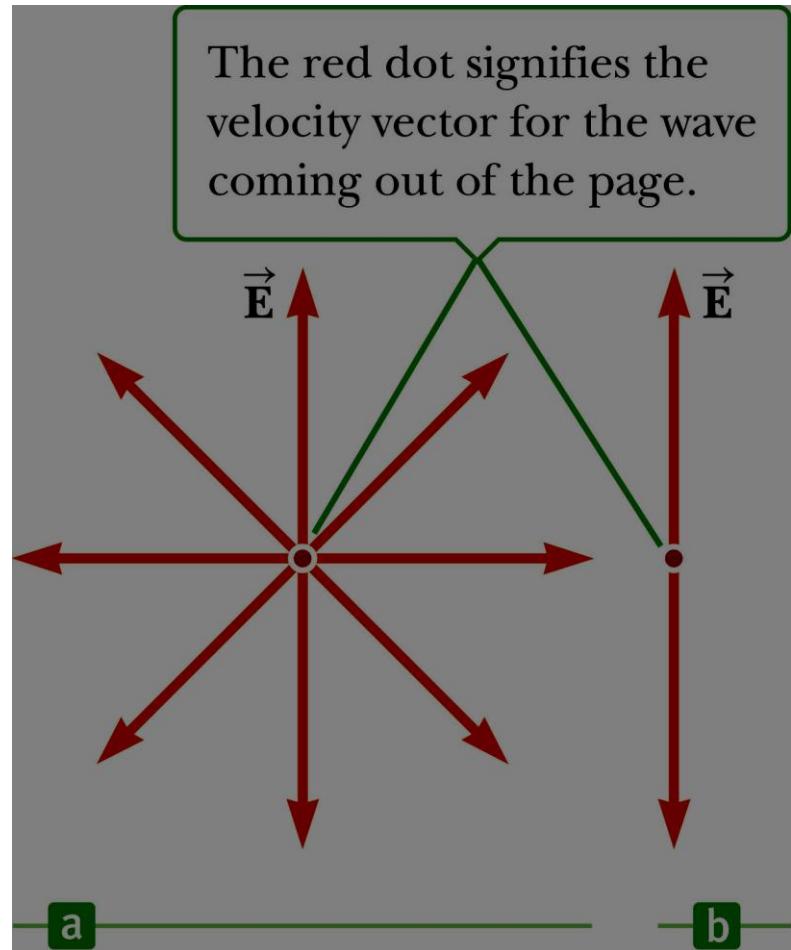
# Polarization of Light Waves

- Each atom produces a wave with its own orientation of  $\vec{E}$
- All directions of the electric field vector are equally possible and lie in a plane perpendicular to the direction of propagation.
- This is an unpolarized wave.

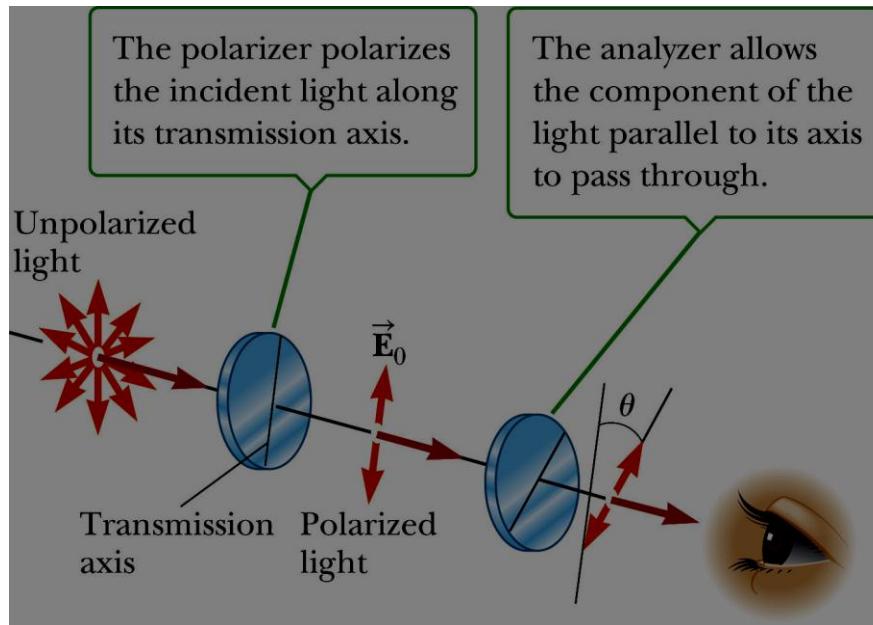


# Polarization of Light, Cont.

- A wave is said to be *linearly polarized* if the resultant electric field vibrates in the same direction at all times at a particular point.
- Polarization can be obtained from an unpolarized beam by
  - Selective absorption
  - Reflection
  - Scattering



# Polarization by Selective Absorption



- The most common technique for polarizing light
- Uses a material that transmits waves whose electric field vectors in the plane are parallel to a certain direction and absorbs waves whose electric field vectors are perpendicular to that direction

# Selective Absorption, Cont.

- E. H. Land discovered a material that polarizes light through selective absorption.
  - He called the material **Polaroid**.
  - The molecules readily absorb light whose electric field vector is parallel to their lengths and transmit light whose electric field vector is perpendicular to their lengths.

# Selective Absorption, Final

- The intensity of the polarized beam transmitted through the second polarizing sheet (the analyzer) varies as

$$I = I_0 \cos^2 \theta$$

- $I_0$  is the intensity of the polarized wave incident on the analyzer.
- This is known as **Malus' Law** and applies to any two polarizing materials whose transmission axes are at an angle of  $\theta$  to each other.

# Polarization by Reflection

- When an unpolarized light beam is reflected from a surface, the reflected light is
  - Completely polarized
  - Partially polarized
  - Unpolarized
- It depends on the angle of incidence.
  - If the angle is  $0^\circ$  or  $90^\circ$ , the reflected beam is unpolarized.
  - For angles between this, there is some degree of polarization.
  - For one particular angle, the beam is completely polarized.

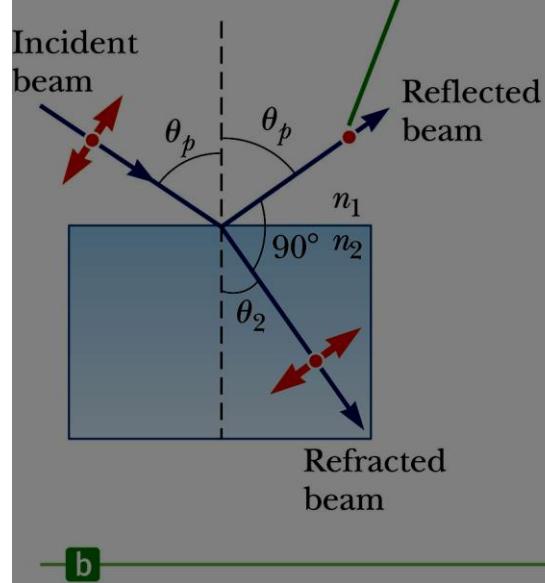
# Polarization by Reflection, Cont.

- The angle of incidence for which the reflected beam is completely polarized is called the *polarizing angle*,  $\theta_p$
- Brewster's Law relates the polarizing angle to the index of refraction for the material.

$$n = \frac{\sin \theta_p}{\cos \theta_p} = \tan \theta_p$$

- $\theta_p$  may also be called Brewster's Angle.

Electrons at the surface oscillating in the direction of the reflected ray (perpendicular to the dots and parallel to the blue arrow) send no energy in this direction.



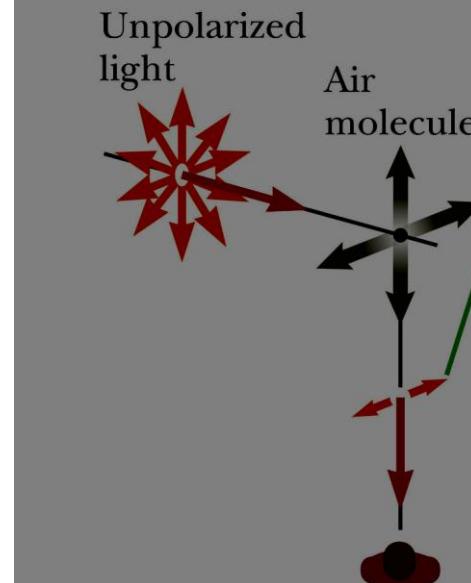
# Polarization by Scattering

- When light is incident on a system of particles, the electrons in the medium can absorb and reradiate part of the light.
  - This process is called **scattering**.
- An example of scattering is the sunlight reaching an observer on the earth becoming polarized.

# Polarization by Scattering, Cont.

- The horizontal part of the electric field vector in the incident wave causes the charges to vibrate horizontally.
- The vertical part of the vector simultaneously causes them to vibrate vertically.
- Horizontally and vertically polarized waves are emitted.

The scattered light traveling perpendicular to the incident light is plane-polarized because the vertical vibrations of the charges in the air molecule send no light in this direction.



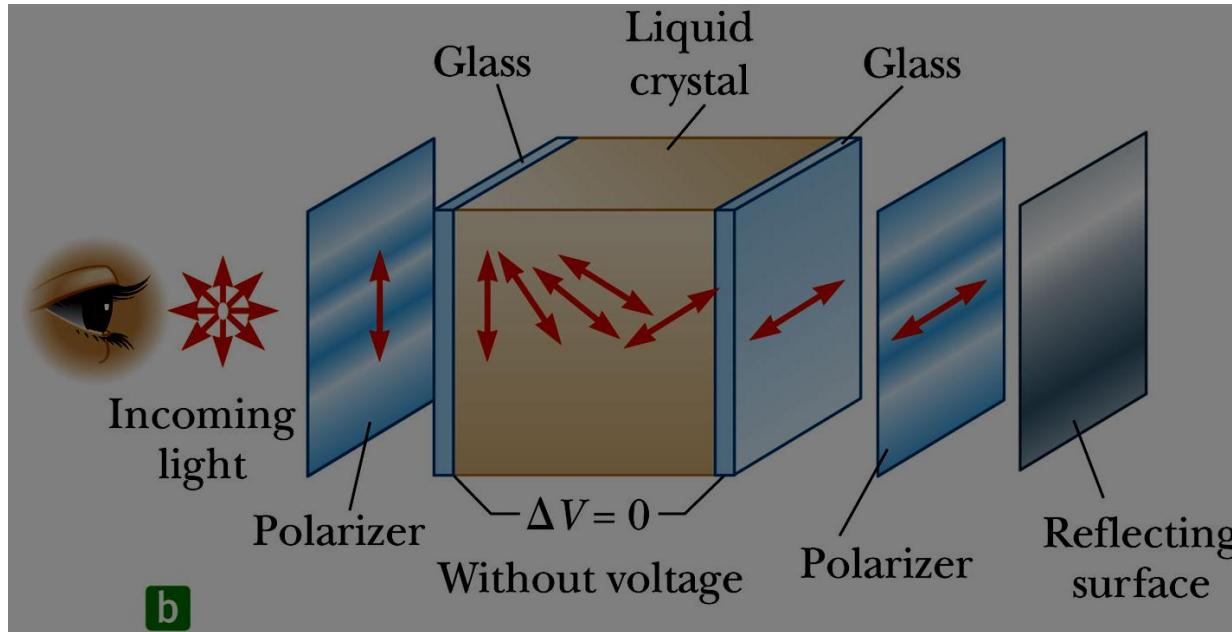
# Optical Activity

- Certain materials display the property of *optical activity*.
  - A substance is optically active if it rotates the plane of polarization of transmitted light.
  - Optical activity occurs in a material because of an asymmetry in the shape of its constituent materials.

# Liquid Crystals

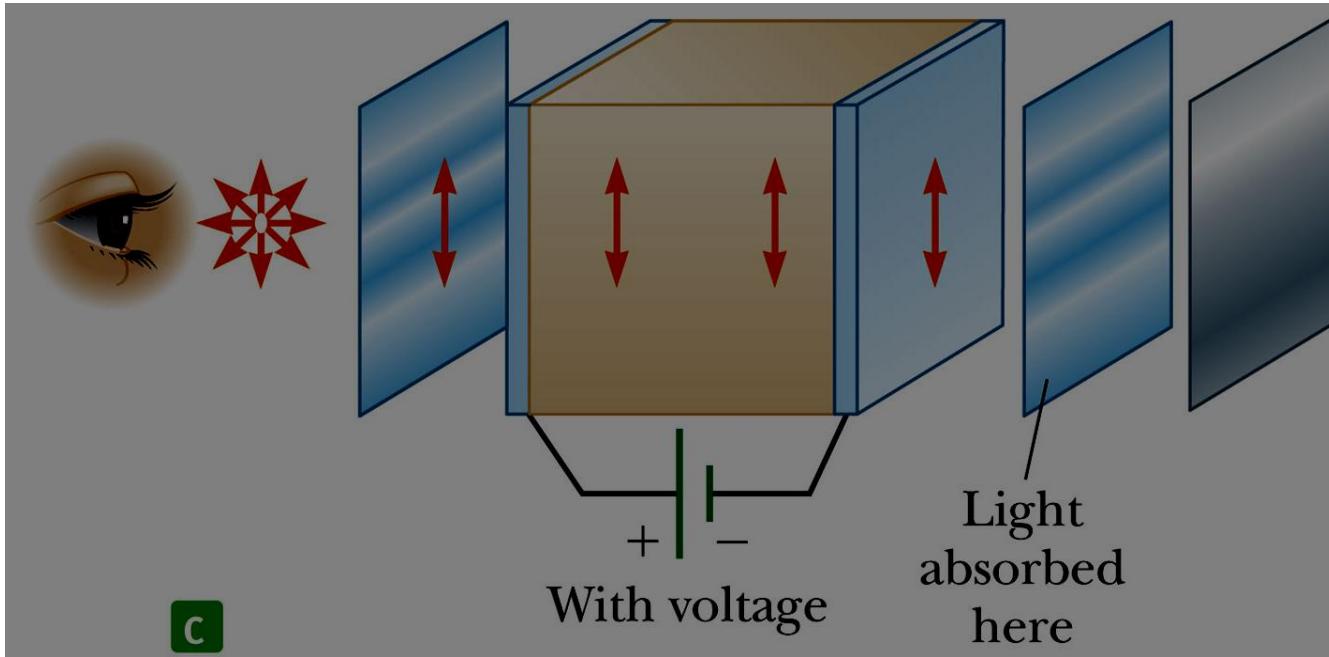
- A **liquid crystal** is a substance with properties intermediate between those of a crystalline solid and those of a liquid.
  - The molecules of the substance are more orderly than those of a liquid but less than those in a pure crystalline solid.
- To create a display, the liquid crystal is placed between two glass plates and electrical contacts are made to the liquid crystal.
  - A voltage is applied across any segment in the display and that segment turns on.

# Liquid Crystals, 2



- Rotation of a polarized light beam by a liquid crystal when the applied voltage is zero
- Light passes through the polarizer on the right and is reflected back to the observer, who sees the segment as being bright.

# Liquid Crystals, 3



- When a voltage is applied, the liquid crystal does not rotate the plane of polarization.
- The light is absorbed by the polarizer on the right and none is reflected back to the observer.
- The segment is dark.

# Liquid Crystals, Final

- Changing the applied voltage in a precise pattern can
  - Tick off the seconds on a watch
  - Display a letter on a computer display