Homework #2; Due Thursday 9/8

1) Compute the divergence of the following vector fields.

A) $\mathbf{v} = x^2 y\hat{\mathbf{x}} + \sqrt{\dfrac{1}{x^2 + z^3}}\hat{\mathbf{y}} + xyz^2\hat{\mathbf{z}}$

B) $\mathbf{v} = (r\cos\theta)\hat{\mathbf{r}} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}$

2) Check the divergence theorem for the vector field in part 1B), using as your volume a sphere of radius R, centered on the origin.

3) A cylindrical wire of radius R has a uniform charge density of ρ throughout its bulk. (The wire runs on the z-axis, and is infinitely long.)

A) What is the E-field at a radius r, where $r<R$?

B) What is the E-field at a radius r, where $r>R$?

C) Now imagine a defective spherical hole (with no charge) is in the middle of the wire, right at the origin. The hole has radius b ($b<R$). If the charge density in the rest of the wire is still uniform, calculate the electric field everywhere on the x-axis. (Hint: zero charge is the same as equal amounts of positive and negative charge. So you can still do this with Gauss's Law!)

4) You have three infinite planes, all with normal vectors in the x-direction. Plane 1 is at x=0, plane 2 is at x=1 cm, and plane 3 is at x=2 cm. Plane 1 and 3 have an areal charge density of σ_0, and plane 2 has an areal charge density of $-\sigma_1$.

A) Find the electric field at x=3cm, by considering the three planes, one at a time.

B) Now find a single Gaussian surface that would let you calculate the electric field at x=3cm, handling all three planes at once. (Hint: use symmetry!)