

Practice Final Exam Problems:

NOTE: Also see "practice Midterm #2 problems", already loaded on the website.

1. Consider how the Bohr model would be different if the force between the electron and the proton was  $A/r^4$  (where  $A$  is a known constant.)

[Hint: centripetal acceleration for circular motion is  $v^2/r$ .]

A) Solve for the allowed orbital radii, using the same logic as the Bohr model.

B) If the potential energy is  $-A/(3r^3)$ , find the allowed total energies of the system.

C) Explain (in a sentence or two) one major difference between the behavior of this system and the hydrogen atom.

**Problem 2: An electron (511keV rest energy) and a "positron" (also 511keV rest energy) can "annihilate" each other, turning mass into light.**

A. An electron moves to the right at  $0.7c$ , and a positron moves to the left at  $0.9c$ . Calculate the kinetic energy of the system AND the total energy of the system AND the total momentum of the system. (3 numbers!) Use any units you want.

B) Knowing how  $E$  and  $p$  are related, calculate the rest mass  $m$  of the SYSTEM. (This will NOT be twice the mass of the electron!) Use convenient units.

C) Show that if these two particles collide and "annihilate", becoming a single zero-mass object (one "particle" of light), then it is impossible for both energy and momentum to be conserved.

D) What actually happens is that they annihilate and turn into *two* zero-mass objects (two particles of light) moving in opposite directions. Determine the energy of each resulting object.

**Problem 3 (worked in class, when I was gone one day):**

A particle is shot from a "gun", and then hits a detector positioned 400 meters away from the "gun" (as measured in the lab frame).

A) In the particle's reference frame, it takes 1.0 microseconds to go from gun to detector. In the lab frame, how fast is the particle moving? (This is not trivial; you know the time in one frame, and the distance in another. You can use the invariant  $s^2$  to get the answer, or set up a system of equations.)

(You need the answer to part A to do the rest of this... )

B) The same experiment is observed by a tiny alien in a tiny spaceship. The spaceship is moving at  $0.9c$  relative to the lab frame, in the same direction as the particle. To the alien: 1) What is the apparent velocity of the particle? 2) What is the distance from the gun to the detector?

C) How long will the particle take to go from gun to detector (remember, the detector is moving; once you have the answers to B, this is a Physics-50 problem with a moving target.)

Discussion Question: Why can't you instead solve C) using an  $s^2$  invariant? (Hint: the answer to B2 is not the same as the distance travelled by the particle.)

4: Consider the following 3-energy-level structure in a certain molecule. A large number of these molecules form a lasing medium, after pumping it with \*another\* "pump" laser. (30 pts)

\_\_\_\_\_ 3.2 eV

**The Fermi energy is at 0.1 eV.**

\_\_\_\_\_ 0.2 eV

\_\_\_\_\_ 0.0 eV

Assume  $T=0$  in part A.

A useful fact: the product " $hc$ " is equal to  $1240 \text{ eV nm}$ .

The  $3.2\text{eV}$  state is relatively stable (low spontaneous decay rate), and the  $0.2 \text{ eV}$  is relatively unstable (high spontaneous decay rate).

A) Answer the following questions:

i) What laser wavelength should be used to **pump** this molecule, to create a laser?

ii) Once it is pumped, what **output** laser wavelength might result from this system?

iii) What is the quantum efficiency of this laser?

B) Explain qualitatively why this laser might fail to work if the temperature was too high.

(Hint: notice the location of the Fermi energy...)

C) If 10% of the molecules can be pumped to the upper state, calculate the temperature at which this material will stop lasing. Use Fermi-Dirac Statistics.

5. Suppose the normalized wavefunction of an electron in a hydrogen atom in  $R_{nl}Y_{lm}$  notation is:

$$\psi = \frac{1}{\sqrt{2}} R_{21}Y_{11} + \frac{1}{\sqrt{3}} R_{20}Y_{00} + \frac{1}{\sqrt{6}} R_{31}Y_{10}$$

A) The magnitude of the electron's orbital angular momentum is measured. What values might one measure, and which what probability for each? (Give me the value of the magnitude, not the square or mere quantum numbers.)

B) Suppose the above wavefunction is known to be the state of the electron at time  $t=0$ . Write the wavefunction at a future time  $t$ , adding only terms with " $t$ " and fundamental constants (including 13.6eV).

C) At  $t=0$ , the electron is measured to have an z-component of the orbital angular momentum equal to  $\hbar$ . Determine the newly "collapsed" wavefunction that would result from the measurement (make it normalized for full credit). Hint: You need to find a wavefunction for which a repeated, identical measurement would get the same result 100% of the time.