NO LATE HOMEWORK ACCEPTED DUE TO MIDTERM!!

1. Suppose a wavefunction of an electron in a Hydrogen atom is given by
Ψ = A [(|l = 1, m_l = -1, m_s = 1/2⟩ + 2i |l = 2, m_l = -1, m_s = 1/2⟩ - 2 |l = 1, m_l = 0, m_s = -1/2⟩].

(a) Normalize the wavefunction (find A).
(b) If you measure L^2, what do you find, and with what probabilities?
(c) If you measure L_z, what do you find, and with what probabilities?
(d) Use the Clebsch-Gordon table to rewrite this state in the |l, j, m_j⟩ basis. Combine like terms if (and only if!) all three of those numbers are identical.
(e) What values might result from a J^2 measurement (total angular momentum squared), and what are the probabilities of each possible result? (Give values, not quantum numbers!)

2) A two-particle spin state (both spin-1/2) can always be written as
a ↑↑ + b ↑↓ + c ↓↑ + d ↓↓. The coefficients a, b, c, d are all complex; assume they are normalized already. What is the probability that the combined angular momentum magnitude of the whole system will be measured to be zero? (Hint: Just use the Born rule!)

3) An electron is in a magnetic field of strength B, where B points in the NEGATIVE z-direction.
Construct a normalized spin state \( \begin{pmatrix} a \\ b \end{pmatrix} \) with an energy expectation value of \( \frac{eB\hbar}{6m} \).

Hint: Set this up in terms of two equations and two unknowns; then solve!

4) (Use the information on bottom of the next page to make this problem solvable.) A spin-1 particle with gyromagnetic ratio \( \gamma \) is in a magnetic field of strength B; the magnetic field points in the negative z-direction.

A) At time \( t=0 \), an experimenter measures the spin angular momentum in the y-direction and gets a result of \( -\hbar \). What is the wavefunction at a time \( t \)?
B) At time "t" the experimenter immediately measures the spin angular momentum in the x-direction. What values might she get, and what are the probabilities of each?

5. Two electrons in a singlet state (total spin =0) are prepared. Electron #1 is sent to Alice, while electron #2 is sent to Bob.

Alice measures $S_z$ for her electron. Bob measures $S_\theta$, where $\theta$ is at the angle $\theta$ from the z axis in the x-z plane. ($\phi=0$). (Use the results from problem 4.30 to figure out Bob's measured eigenstates.)

A) Find the 4 eigenstates of the *whole system* that correspond to all 4 possible measurement results. (Alice can get one of two results, Bob can get one of two results, so there are 4 total results in all.) Hint: tensor product each of Alice's eigenstates with each of Bob's eigenstates.

B) Find the probabilities of each of the four outcomes if $\theta =45^\circ$. (Use the Born rule!)

C) Find the probabilities of each of the four outcomes if $\theta =135^\circ$.

D) For each of the angles in parts B) and C), calculate the expectation value of the product of Alice's result and Bob's result.

**Spin Matrices and Normalized Eigenvectors**

3D Hilbert Space, in basis defined by a diagonal $S_z$ (highest eigenvalue on top).

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix} : \quad |+h\rangle_s = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad |0\rangle_s = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad |-h\rangle_s = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i\sqrt{2} & 0 \\ i\sqrt{2} & 0 & -i\sqrt{2} \\ 0 & i\sqrt{2} & 0 \end{pmatrix} : \quad |+h\rangle_s = \frac{1}{2} \begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix}, \quad |0\rangle_s = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad |-h\rangle_s = \frac{1}{2} \begin{pmatrix} 1 \\ -i\sqrt{2} \\ -1 \end{pmatrix}$$