Homework #11; Phys 163. Due Thurs 12/7 (last day of class)

Note: Final on 12/13, 9:30am.

This HW is worth 15 pts instead of 10.

1. (a) Find $<r>$ for the ground state of hydrogen. Be careful with 3D integrals in spherical coordinates! Don’t forget the $r^2 \sin(\theta)$ term in the volume element.
   (b) Find the most probable value of $r$ for the ground state of hydrogen. See hints in problem 4.14.

2. Calculate the Fermi energy for non-interacting electrons in a TWO-dimensional infinite square well. Give your answer as a function of $\sigma$, the number of free electrons per unit area.

3. Problem 5.35. Skip part B; the answer to that part is $U = -(3/5) G N^2 M^2 / R$. Hint: you will have a lot of complicated constants in your expressions for part C, so group them together when possible and give them simple definitions. (Plug in at the end to get numbers.)

(If you’re interested in the limit to white dwarf stability, see problem 5.36!)

Final-Like Problems:

4) Suppose a (unnormalized) wavefunction of the electron in a hydrogen atom, in $|n, \ell, m_\ell, m_s\rangle$ notation, is given by
   $$ |\psi\rangle = A|3, 2, -1, \uparrow\rangle - 2A|3, 2, 0, \downarrow\rangle. $$
   ("A" is the normalization constant; solve for this first.)

A) Give each possible outcome for each of the following measurements, and the probability of each outcome.
   a) Energy
   b) Magnitude of the orbital angular momentum
   c) $z$-component of the orbital angular momentum
   d) Square of the spin angular momentum
   e) $z$-component of the spin angular momentum

B) Give all possible outcomes and associated probabilities for a measurement of the magnitude of the *combined* angular momentum (spin +orbital).

(see next page for last problem)
5) Suppose that some operator $Q = A \begin{pmatrix} 2 & i & 0 \\ -i & 2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ corresponds to a Q-measurement made on a state $|\psi\rangle = B \begin{pmatrix} 1+i \\ 1-i \\ 2 \end{pmatrix}$.

(the constant $A$ is known, but the constant $B$ must be calculated.)

Calculate all possible results that you might find from the Q-measurement, and the probabilities of each of those results. (hint: the operator is of a special form that might simplify matters a bit.)