Problem #1. Suppose you pump a lasing medium to a small-signal gain of 1.00 m$^{-1}$. The inhomogeneously-broadened medium has the triangular line shape $g(\nu)$ given by the figure below, with a full-width at the base of the triangle $\Delta \nu = 210$ GHz. It's put in the cavity shown, and each time the beam enters or leaves the 11-cm-long lasing medium, there is a loss of 1%. The index of refraction of the medium is 1.55.

![G(\nu)](image)

How many modes can lase at once? (Don't ignore cavity losses!)

Problem 2. Consider the following ring laser cavity, where the laser is constrained to only travel in the counter-clockwise direction. There are no other losses in the cavity. A detector, as shown, measures the output laser intensity. The stimulated emission cross section of the homogeneously broadened lasing transition is $10^{-16}$ m$^2$, and the output wavelength is 600nm.

![Cavity Diagram](image)

a) What is the minimum population inversion $\Delta N$ needed to allow lasing?

b) At some unknown small-signal gain coefficient the detector measures an output intensity of 10 W/cm$^2$. But when pumped at double the original small-signal gain coefficient (twice the original pump strength), the output intensity rises to 25 W/cm$^2$. What is the small signal gain coefficient for the case of the 10 W/cm$^2$ output? (Set up two equations for two unknowns and solve them.)

c) What is the lifetime of the upper lasing state? (Hint: use the other unknown from part b to find a term which includes this value.)
3. Find an expression for the number of modes that can lase at once in terms of the peak small signal gain coefficient \( \gamma_0 \), the free spectral range \( \Delta \nu_{\text{fsr}} \), the survivability \( S \), the roundtrip distance through the laser medium \( L_{\text{rt}} \), and the FWHM of the lineshape, \( \Delta \nu_0 \). For the broadening shape, assume:

A) Doppler broadening (Gaussian); See Eqn 2.5.17.

B) Lifetime broadening (Lorentzian): see Eqn 2.4.8.

1) First solve this case for the question of how many laser modes can start getting amplified, at first.

2) Then explain how many modes end up lasing once steady-state is reached, and why.