Consider the differential equation for a harmonic oscillator
\[
\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + 10x = 5 \delta(t)
\]
subject to conditions \( x(0) = 0 \) \( x'(0) = 0 \).

a) Explain the physical significance of the term \( 5 \delta(t) \).

Give your answer with a simple calculation.

b) Calculate \( x(t) \) using Laplace Transform.

c) Calculate \( x(t) \) using Fourier Transform.

2. A particle with mass \( m \) is contained in a right circular cylinder of radius \( R \) and height \( H \). The particle is described by a wave function satisfying the Schrödinger wave equation:

\[
-\frac{\hbar^2}{2m} \nabla^2 \psi(p, y, z) = E \psi(p, y, z)
\]

Show that the wave function can be written as the product of three functions as \( \psi(p, y, z) = R(p) Q(y) Z(z) \).

Calculate each function and show one is a Bessel function.

3. Use Fourier Transform to calculate the solution to differential equation

\[
\frac{\partial^2 q(x, t)}{\partial x^2} = \frac{\partial q(x, t)}{\partial t}
\]

subject to the condition \( q(x, 0) = 5 \delta(x) \).
4. Calculate the following integral using Laplace Transform.
\[ \int_{-\infty}^{\infty} \frac{\sin^2 tx}{x^2} \, dx \]

5. a. Calculate the expression for electric potential at point M on Z axis due to the charges shown in the figure in series form for Z > a.

b. Use the expression for potential at M to write down the expression for potential at point N(r, \theta) in a series form containing Legendre function. Assume r > a.