Math 30P - Spr 13 Test 2

Name: Solutions

Problem 1. (Two parts). (a) If $f(x) = \sqrt{\frac{x}{x+1}}$ and $g(x) = e^{-x} + 1$, find $(f \circ g)(x)$. (b) Express the function $F(x) = \sqrt{\frac{x}{x+1}}$, in the form $f \circ g$ (i.e., find $f(x)$ and $g(x)$.)

Solution: (a) $(f\circ g)(x) = f(g(x)) = f(e^{-x}+1) = \sqrt{\frac{e^{-x}+1}{(e^{-x}+1)+1}}$

$= \boxed{\sqrt{\frac{e^{-x}+1}{e^{-x}+2}}}$

(b) $g(x)$ must be $\frac{x}{x+1}$, and $f(x) = \sqrt{x}$.

Problem 2. (Two parts). Starting with the graph of $y = e^x$, write the equation $y = ?$ of the graph that results from: (a) Shifting two units to the left. (b) Reflecting about the y-axis.

Solution:

(a) $\boxed{y = e^{x+2}}$

(b) $\boxed{y = e^{-x}}$

Problem 3. Find a formula for $f^{-1}(x)$, the inverse of $f(x) = 6x + 9$.

Solution:

$$y = 6x + 9$$

$$x = 6y + 9$$

$$x - 9 = 6y$$

$$\frac{x-9}{6} = y, \quad \boxed{\frac{x-9}{6} = f^{-1}(x)}$$

Problem 4. Solve for x: $e^{3x+2} = 7$. (Round to 3 significant figures).

Solution:

$$\ln e^{3x+2} = \ln 7$$

$$3x + 2 = \ln 7$$

$$x = \frac{\ln 7 - 2}{3}$$

$$= \boxed{-0.018}$$

Problem 5. For the graph of the function $y = \sqrt{x}$, what is the slope of the secant line connecting the points $(4, 2)$ and $(9, 3)$?

Solution:

$$\frac{3-2}{9-4} = \boxed{\frac{1}{5}}$$

Problem 6. Find the infinite limit: $\lim_{x \to -3^-} \frac{x+2}{x+3}$. Show your work to get credit.

Solution: use -3.5 as test number:

$$\frac{-3.5+2}{-3.5+3} = \frac{\ominus}{\ominus} = \oplus$$

$$\boxed{+\infty}$$

Problem 7. Evaluate the limit, if it exists: $\lim_{x \to 0} \frac{\sqrt{1+3x}-1}{x}$.

Solution:

$$\lim_{x \to 0} \frac{(\sqrt{1+3x}-1)}{x} \cdot \frac{(\sqrt{1+3x}+1)}{(\sqrt{1+3x}+1)}$$

$$= \lim_{x \to 0} \frac{(1+3x)-1}{x(\sqrt{1+3x}+1)} = \lim_{x \to 0} \frac{3}{\sqrt{1+3x}+1}$$

$$\stackrel{DS}{=} \frac{3}{1+1} = \boxed{\frac{3}{2}}$$

Problem 8. Evaluate the limit, if it exists: $\lim_{x \to 5} \frac{x^2-5x+6}{x-5}$.

Solution:

$$\lim_{x \to 5} \frac{x^2-5x+6}{x-5}$$

"DS" gives $\frac{b}{0}$, $b \neq 0$

So $\boxed{\text{no limit exists}}$

Problem 9. Evaluate the limit, if it exists: $\lim_{x \to 3} \tan(2x^2 + 1)$.

Solution:

$$\lim_{x \to 3} \tan(2x^2+1) \overset{DS}{=} \tan(19)$$

remember arguments of trig functions are in radians. Use radian mode.

$$\tan(19) = \boxed{0.152}$$

Problem 10. Evaluate $\lim_{x \to 3} \sqrt{2x^2 + x + 7}$ using appropriate Limit Laws (attached). Justify each step by showing which Laws you used.

Solution:

$$\lim_{x \to 3} \sqrt{2x^2+x+7} \overset{(11)}{=} \sqrt{\lim_{x \to 3} (2x^2+x+7)}$$

$$\overset{(1)}{=} \sqrt{\lim_{x \to 3} (2x^2) + \lim_{x \to 3} x + \lim_{x \to 3} 7}$$

$$\overset{(3),(8),(7)}{=} \sqrt{2 \lim_{x \to 3} x^2 + 3 + 7}$$

$$\overset{(9)}{=} \sqrt{2 \cdot 3^2 + 3 + 7}$$

$$= \sqrt{28}$$

$$= \boxed{5.29}$$

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

Law

6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer

In applying these six limit laws, we need to use two special limits:

7. $\lim_{x \to a} c = c$

8. $\lim_{x \to a} x = a$

These limits are obvious from an intuitive point of view (state them in words or draw graphs of $y = c$ and $y = x$), but proofs based on the precise definition are requested in the exercises for Section 2.4.

If we now put $f(x) = x$ in Law 6 and use Law 8, we get another useful special limit.

9. $\lim_{x \to a} x^n = a^n$ where n is a positive integer

A similar limit holds for roots as follows. (For square roots the proof is outlined in Exercise 37 in Section 2.4.)

10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer

(If n is even, we assume that $a > 0$.)

11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer

[If n is even, we assume that $\lim_{x \to a} f(x) > 0$.]