Math 30P - Spr 13 Test 3

Name: Solutions

Problem 1. Use the Intermediate Value Theorem to show that there is at least one root of the equation $\underbrace{x^5 - 3x + 3}_{f(x)} = 0$ in the interval $(-2, -1)$.

Solution:

$f(-2) = -23$

$f(-1) = 5$

$-23 < 0 < 5$

So by IVT, there is a root in $(-2,-1)$

(Although I discussed this class, no HW assigned, so everyone received at least 7/10)

Problem 2. **Using the limit definition of derivative** (not differentiation rules), find $\frac{d}{dx}\sqrt{6-2x}$.

Solution:

$$\frac{d}{dx}\sqrt{6-2x} \overset{def}{=} \lim_{h\to 0} \frac{\sqrt{6-2(x+h)} - \sqrt{6-2x}}{h} \cdot \frac{\left(\sqrt{6-2(x+h)}+\sqrt{6-2x}\right)}{\left(\sqrt{6-2(x+h)}+\sqrt{6-2x}\right)}$$

$$= \lim_{h\to 0} \frac{[6-2(x+h)]-[6-2x]}{h\left(\sqrt{6-2(x+h)}+\sqrt{6-2x}\right)}$$

$$= \lim_{h\to 0} \frac{-2h}{h\left(\sqrt{6-2(x+h)}+\sqrt{6-2x}\right)}$$

$$= \lim_{h\to 0} \frac{-2}{\sqrt{6-2(x+h)}+\sqrt{6-2x}}$$

$$= \frac{-\cancel{2}}{\cancel{2}\sqrt{6-2x}} = \boxed{\frac{-1}{\sqrt{6-2x}}}$$

Problem 3. Find (in any way) $f'(x)$ when $f(x) = \dfrac{4x^2 - x + 1}{x+2}$.

Solution:

$$f'(x) \overset{QR}{=} \frac{(x+2)(8x-1) - 1\cdot(4x^2-x+1)}{(x+2)^2}$$

$$= \frac{8x^2 - x + 16x - 2 - 4x^2 + x - 1}{(x+2)^2}$$

$$= \boxed{\frac{4x^2 + 16x - 3}{(x+2)^2}}$$

Problem 4. Differentiate $y = x^2(x^3 - 1)$.

Solution:

$$y = x^5 - x^2$$

$$\frac{dy}{dx} = \boxed{5x^4 - 2x}$$

Problem 5. Find $f'(x)$ for $f(x) = x^2 e^{x^2}$.

Solution:

$$f'(x) \overset{PrR}{=} x^2 \frac{d}{dx} e^{x^2} + e^{x^2} \cdot \frac{d}{dx} x^2$$

$$\overset{CR}{=} x^2 e^{x^2} \cdot 2x + e^{x^2} \cdot 2x$$

$$= \boxed{2xe^{x^2}(x^2+1)}$$

Problem 6. Differentiate $f(t) = e^{e^t}$. $\underbrace{e^t}_{g(t)}$

Solution:

$$f'(t) = \frac{d}{dt} e^{e^t}$$

$$\overset{CR}{=} \boxed{e^{e^t} \cdot e^t}$$

Problem 7. For what values of x does the graph of $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 19$ have a horizontal tangent line?

Solution:

$$f'(x) = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$\boxed{x = 3, 2}$$

Problem 8. Find: $\lim_{x\to\infty} \frac{-2x+3}{3x^2+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

Solution:

$$= \lim_{x\to\infty} \frac{-\frac{2}{x} + \frac{3}{x^2}}{3 + \frac{1}{x^2}} = \frac{0}{3} = \boxed{0}$$

Problem 9. Find an equation of the tangent line to the curve $y = 3x^2 + 6x - e^x$ that passes through the point $(0, -1)$.

Solution:

$$y - (-1) = m(x - 0)$$

$$m = y'(0)$$

$$y' = 6x + 6 - e^x$$

$$y'(0) = 6 - 1 = 5$$

$$y + 1 = 5x \quad \text{or} \quad \boxed{y = 5x - 1}$$

Problem 10. Find $\frac{dy}{dx}$ for $4y^2 - xy = 2$ by implicit differentiation.

Solution:

$$\frac{d}{dx}(4y^2 - xy) = \frac{d}{dx}2$$

$$8y \cdot \frac{dy}{dx} - \left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) = 0$$

$$(8y - x)\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \boxed{\frac{-y}{8y - x}}$$