Math 30P - Spr 13 Test 4

Name: 100 nice!

Problem 1. (Two parts.) If a rock is thrown vertically upward from the surface of Mars with velocity 15 m/s, its height after t seconds is: $h = 15t - 1.86t^2$. (a) What is the velocity of the rock after 2 s? (b) What is the velocity of the rock when its height is 25 m on its way up?

Solution: $h = 15t - 1.86t^2$

Part (a) When $t = 2$

$V(t) = -3.72t + 15$

$V(t) = -3.72(2) + 15$

$\boxed{V(t) = 7.56}$

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Part (b)

$25 = 15t - 1.86t^2$

$-1.86t^2 + 15t - 25 = 0$

$\frac{15 \pm \sqrt{39}}{-3.72} = 2.35$

$= \cancel{5.771}$

$h'(t) = -3.72t + 15$

$h'(2.35) = -3.72(2.35) + 15$

$= 6.258$ m/s

Problem 2. Differentiate: $f(x) = \ln(\frac{1}{\ln x})$.

Solution: $\ln\left(\frac{1}{\ln x}\right)$

$\overset{CR}{=} \frac{d}{dx}\left(\frac{1}{\ln x}\right)$

$= -\frac{\ln(x)\left(\frac{d}{dx}(\ln(x))\right)}{\ln^2 x}$

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$= -\frac{\frac{d}{dx}(\ln(x))}{\ln x}$

$= \boxed{-\frac{1}{x\ln(x)}}$

Problem 3. Differentiate: $g(x) = (\sin x)^x$.

Solution: $(\sin x)^x$

$\stackrel{CR}{=} x \sin^{x-1}(x)\left(\frac{d}{dx}(\sin(x))\right) + \sin^x(x)\ \ln \sin(x)\left(\frac{d}{dx}(x)\right)$

$= \sin^x(x) \cdot \ln(\sin(x))\left(\frac{d}{dx}(x)\right) + x \sin^{x-1}(x) \cos(x)$

$= \sin^x(x) \ln(\sin(x)) + x \sin^{x-1}(x) \cos(x)$

$\boxed{= \sin^x(x)\,(x \cot(x)) + \ln(\sin(x))}$

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Problem 4. Find the absolute maximum and absolute minimum values of $f(x) = x^4 - 4x^2 + 2$ in the interval $[-3, 2]$.

Solution: $f'(x) = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

$4x = 0 \qquad x^2 - 2 = 0$ 10

$x = 0 \qquad x^2 = 2$

$x = \pm\sqrt{2}$

$\left.\begin{array}{l} f(\sqrt{2}) = -2 \\ f(-\sqrt{2}) = -2 \end{array}\right\}$ both equal

$f(0) = 2$ $\Rightarrow$

$f(2) = 2$

$f(-3) = 47$

Abs Max = $(-3, 47)$
Abs Min = $(\pm\sqrt{2}, -2)$
abs max values : 47
abs min values : -2

Problem 5. The length of a rectangle is increasing at a rate of 5 cm/s and its width is increasing at a rate of 2 cm/s. How fast is the area of the rectangle increasing when its length is 16 cm and its width is 8 cm?

Solution: $A = l \cdot w$ $\frac{dl}{dt} = 5\,cm/s$ $\frac{dw}{dt} = 2\,cm/s$

$l = 16\,cm$ $w = 8\,cm$

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$$\frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt}$$

$$= 5 \cdot 8 + 16 \cdot 2$$

$$= 40 + 32$$

$$= 72$$

$$\boxed{\frac{dA}{dt} = 72\ cm/s}$$

Problem 6. Find the linearization $L(x)$ of the function $f(x) = \dfrac{1}{\sqrt{3+x}}$ at $a = 0$.

Solution:

$$f(0) = \frac{1}{\sqrt{3+0}} = \frac{1}{\sqrt{3}} = .577$$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = .577 + (-.0962)(x-0)$

$= .577 - .0962x$

$\boxed{= -.0962x + .577}$

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$$f'(x) = -\frac{1}{2(3+x)^{3/2}} = -\frac{1}{2}(x+3)^{-3/2}$$

$$f'(0) = -\frac{1}{2(3+x)^{3/2}} = -\frac{1}{2}(3)^{-3/2} = -.0962$$

Problem 7. Differentiate: $f(x) = \sinh(e^{\sinh x})$.

Solution:

$$\frac{d}{dx} \sinh(e^{\sinh x})$$

$$= \cosh(e^{\sinh(x)}) \left(\frac{d}{dx}(e^{\sinh(x)})\right)$$

$$= \cosh(e^{\sinh(x)}) (e^{\sinh(x)}) \left(\frac{d}{dx}(\sinh(x))\right)$$

$$= \boxed{e^{\sinh(x)} \cosh(x) \cosh(e^{\sinh(x)})}$$

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$T(0)$

Problem 8. A glass of water at 80°F is placed in a freezer whose temperature is 0°F. After 30 minutes, the water temperature is 45°F. How much time after being placed in the freezer will it take for the water to freeze, i.e., reach 32°F? (Hint: First find the appropriate heating/cooling model, then solve it to find the time.)

Solution: $T(0) = 80°F$ $\quad T_s = 0°F$

$\boxed{K = \frac{\ln\left(\frac{45}{80}\right)}{30}}$

$T(t) = 0 + (80 - 0)e^{rt}$

$T(t) = 0 + (80)e^{rt}$

$T(t) = 80e^{rt}$

$\frac{45}{80} = \frac{80e^{30t}}{80}$

$\ln\left(\frac{45}{80}\right) = 30K$

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$T(t) = 80e^{\frac{1}{30}\ln\left(\frac{45}{80}\right)t}$

$32° = 80e^{\frac{1}{30}\ln\left(\frac{45}{80}\right)t}$

$\ln\frac{32}{80} = \frac{1}{30}\ln\left(\frac{45}{80}\right)t$

$\frac{30\ln\left(\frac{32}{80}\right)}{\ln\left(\frac{45}{80}\right)} = \frac{\cancel{\ln\left(\frac{45}{80}\right)}t}{\cancel{\ln\left(\frac{45}{80}\right)}}$

$\frac{30\ln\left(\frac{32}{80}\right)}{\ln\left(\frac{45}{80}\right)} = t$

$t = \frac{30(-.9162907)}{-.5753641449}$

$t = \frac{-27.48872196}{-.5753641449}$

$\boxed{t = 47.77 \text{ min}}$

Problem 9. If $y = 5x^3 + 7x^2$ and $\frac{dx}{dt} = 7$, find $\frac{dy}{dt}$ when $x = 2$.

Solution: $\frac{dy}{dx} = 15x^2 + 14x$ $\quad a = 2$ $\quad F(a) + F'(a)(x-a)$

$\Rightarrow (15(2)^2 + 14(2)) \cdot \frac{dx}{dt}\,7$

$\Rightarrow (60 + 28) \cdot \frac{dx}{dt}\,7$

$\Rightarrow (88) \cdot \frac{dx}{dt}\,7$

$\Rightarrow 88 \cdot 7 = 616$

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$\boxed{\frac{dy}{dt} = 616}$

Problem 10. A certain radioactive isotope has a half-life of 17.5 years. Starting with 15 grams of the isotope, how much mass remains after 40 years?

Solution: $M(t) = M(0)e^{kt}$

$M(t) = 15e^{k17.5}$

$\frac{1}{2} = 15e^{k17.5}$

$\ln\frac{1}{2} = 17.5k$

$-\frac{\ln 2}{17.5} = K$

$M(t) = 15e^{-\frac{\ln 2}{17.5}t}$

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$M(40) = 15e^{-\frac{\ln 2}{17.5}(40)}$

$\boxed{= 3.07~~62576~~ \text{ grams remaining}}$