Math 30P - Spr 13 50 good Test 5

Name: ___ 5/9/2013

Problem 1. Find the intervals on which $f(x) = x^3 - x + 1$ is increasing or decreasing.

$f'(x) = 3x^2 - 1 = 0$ dom $(-\infty, \infty)$

Solution:

$3x^2 = 1$

crit. $x = \pm\sqrt{\frac{1}{3}}$

$\sqrt{\frac{1}{3}} = 0.5773502692$

$(-\infty, -\sqrt{\frac{1}{3}})$	I
$(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$	D
$(\sqrt{\frac{1}{3}}, \infty)$	I

$f'(-1) = 3(-1)^2 - 1$
$= 2$

$f'(1) = 3(1)^2 - 1$
$= 2$

$f'(\frac{1}{2}) = 3(\frac{1}{2})^2 - 1$
$= \frac{3}{4} - 1$
$= -\frac{1}{4}$

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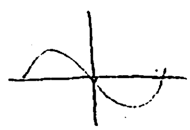

Problem 2. Find the intervals on which the function $f(x)$ from Problem 1 is concave up or down.

Solution: $f''(x) = 6x$

$x = 0$

	concave
$(-\infty, 0)$	D
$(0, \infty)$	U

$f''(-1) = 6(-1)$
$= -6$

$f''(1) = 6(1)$
$= 6$

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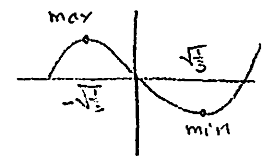
Problem 3. Find the local maximum and minimum values of the function $f(x)$ from Problem 1. critical # $= \pm\sqrt{\frac{1}{3}}$



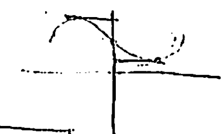
Solution: $f(x) = x^3 - x + 1$

$f(-\sqrt{\tfrac{1}{3}}) = (-\sqrt{\tfrac{1}{3}})^3 - (-\sqrt{\tfrac{1}{3}}) + 1$

$= -0.1924500897 + 0.5773502691 + 1$

$= 1.3849001794$

≈ 1.38



$f(\sqrt{\tfrac{1}{3}}) = (\sqrt{\tfrac{1}{3}})^3 - (\sqrt{\tfrac{1}{3}}) + 1$

$= 0.6150998205$

≈ 0.62

$\boxed{\begin{array}{l} \text{local max.} = 1.38 \\ \text{local min} = 0.62 \end{array}}$

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Problem 4. Find the limit: $\lim_{x\to 0} \frac{e^x - 1}{\sin x}$. $\overset{\text{'DS'}}{=} \lim_{x \to 0} \frac{e^0 - 1}{\sin 0} = \frac{0}{0}$

Solution: $\lim_{x \to 0} \frac{e^x - 1}{\sin x} \overset{L\cdot H}{=} \frac{e^x}{\cos x}$

$\overset{DS}{=} \frac{1}{1}$

$\boxed{= 1}$

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Problem 5. Find the limit: $\lim_{x\to\infty} x^{\frac{1}{x}}$. $\overset{\text{'DS'}}{=} \infty^{\frac{1}{\infty}} = \infty^0$

Solution: $\lim_{x \to \infty} \frac{1}{x} \ln x$

rewrite : $\lim_{x \to \infty} \frac{\ln x}{x} \overset{L\cdot H}{=} \frac{\frac{1}{x}}{1} \overset{\text{'DS'}}{=} \frac{1}{\infty} = 0 =$

$\boxed{e^0 = 1}$

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