Math 71 - Spring '13 Test 1

Name: Solutions

Problem 1. Consumer price inflation was 2.0% in 2010 and 2.2% in 2011. If the price of a typical item at the beginning of 2010 was \$20,000, what was its inflated price at the end of 2011?

Solution:

At end of 2010: $20000(1.02) = 20400$

At end of 2011: $20400(1.022) = \boxed{\$\,20848.80}$

See HW1, #7

Problem 2. You invest \$7000 in two stocks. In one year, the value of stock A increases by 5% and the value of stock B **decreases** by 2%. At the end of the year, your investment is still worth \$7000. How much was initially invested in both stocks?

Solution: Write an equation expressing "hidden equality":

See HW1, #10

$$x \cdot 1.05 + (7000 - x) \cdot 0.98 = 7000$$

$$0.07x + 6860 = 7000$$

$$0.07x = 140$$

$$x = 2000$$

$$7000 - x = 5000$$

$$\boxed{\begin{array}{l} A:\ \$2000 \\ B:\ \$5000 \end{array}}$$

Problem 3. (Two parts). Find the domains of the following functions:
(a) $f(x) = \sqrt{x-5} + \dfrac{2}{x-6}$. (b) $g(x) = \dfrac{1}{x^2 - 2x - 24}$.

Solutions:

$\ast$ See HW 2 #12-16

a) $\boxed{x \geq 5 \text{ and } x \neq 6}$

b) $x^2 - 2x - 24 \neq 0$

$(x-6)(x+4) \neq 0$

$\boxed{x \neq 6, \quad x \neq -4}$

Problem 4. If $\lim_{x \to 3} f(x) = 2$ and $\lim_{x \to 3} g(x) = 5$, evaluate $\lim_{x \to 3} \sqrt{[f(x)]^2 + 2g(x)}$.

Solution:

$$\lim_{x \to 3} \sqrt{[f(x)]^2 + 2g(x)} = \sqrt{\left[\lim_{x \to 3} f(x)\right]^2 + 2\lim_{x \to 3} g(x)}$$

$$= \sqrt{2^2 + 2 \cdot 5} = \boxed{\sqrt{14}}$$

$\ast$ See HW 3 #6-12

Problem 5. Find the limit, if it exists: $\lim_{x \to -2} \dfrac{x^2 + 4x + 4}{x^2 - 4}$.

Solution:

$$\lim_{x \to -2} \frac{x^2 + 4x + 4}{x^2 - 4} = \lim_{x \to -2} \frac{(x+2)^{\cancel{2}}}{\cancel{(x+2)}(x-2)}$$

$\ast$ See HW4 # 1-4

$$= \lim_{x \to -2} \frac{x+2}{x-2}$$

$$\overset{DS}{=} \frac{0}{-4} = \boxed{0}$$

Note: "0" is a real number: the limit exists!

Problem 6. Using the limit definition of the derivative, find the derivative $f'(x)$ when $f(x) = 2x^2 - 3$. Show your work and don't use differentiation rules.

Solution:

$$f'(x) \overset{\text{def}}{=} \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[2(x+\Delta x)^2 - 3] - [2x^2 - 3]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x \cdot \cancel{\Delta x} + 2(\Delta x)^{\cancel{2}} - \cancel{3} - \cancel{2x^2} + \cancel{3}}{\cancel{\Delta x}}$$

const.

$$= \lim_{\Delta x \to 0} (4x + 2\Delta x)$$

See HW 7 # 9-11

$$\overset{DS}{=} 4x + 2 \cdot 0 = \boxed{4x}$$

Problem 7. Find (in any way) the derivative of: $g(x) = \sqrt{7}x^{\sqrt{7}} + \pi x^{\pi}$.

Solution: Power + Sum Rule not responsible for !

$$\frac{d}{dx}\left(\sqrt{7}x^{\sqrt{7}} + \pi x^{\pi}\right) \overset{SR}{=} \sqrt{7}\frac{d}{dx}x^{\sqrt{7}} + \pi\frac{d}{dx}x^{\pi}$$

$$\overset{PR}{=} \sqrt{7} \cdot \sqrt{7}\, x^{\sqrt{7}-1} + \pi \cdot \pi \cdot x^{\pi - 1}$$

$$= \boxed{7x^{\sqrt{7}-1} + \pi^2 x^{\pi-1}}$$

Everyone received full credit

* See HW 7 # 4-9

Problem 8. Find (in any way) the derivative of: $h(x) = \dfrac{9}{\sqrt[3]{x^2}}$.

Solution:

$$h(x) = 9x^{-2/3}$$

$$h'(x) \overset{PR}{=} 9 \cdot \left(-\tfrac{2}{3}\right) \cdot x^{-5/3}$$

$$= \boxed{-6x^{-5/3}}$$

Problem 9. Find (in any way) an equation of the tangent line to the graph of $f(x) = 3x^4$ at the point $(2, 48)$.

Solution:

$$y - y_1 = m(x - x_1)$$

$$y - 48 = m(x - 2)$$

$$m = f'(x) = 12x^3$$ evaluate at $x = 2$!

$$f'(2) = 96$$

See comments posted on W/A

$$\boxed{y - 48 = 96(x - 2)}$$

* See HW 6 #4, 5, 8

Problem 10. (Two parts.) (a) If $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$, is $C = \{(d, 1), (b, 2), (a, 3), (d, 2)\}$ a function from A to B? If not, state why not. (b) Does the equation $2x + y = 6$ represent y as a function of x? If not, state why not.

Solution:

HW2 #1, 2, 3

(a) No, because a function cannot contain two ordered pairs with the same first coordinate but different second coordinates. Also, no ordered pair with "c" as first coord.

HW2 #4, 5, 6

(b) Yes; the equation can be solved for y with a unique solution

$$y = -2x + 6$$

shows y is a function of x.