SOLUTIONS

Math 71 - Spr 13 97 Test 3

Name: nice

**Problem 1.** Find the relative extrema of the function $f(x) = x^3 - 6x^2 + 15$. Identify which extremum is a maximum and which a minimum. (Remember extrema are *points* with two coordinates.)

**Solution:**

$f'(x) = 3x^2 - 12x$

$3x^2 - 12x = 0$

$x(3x - 12) = 0$

$x = 0$

$3x = 12$

$x = 4$

$f(0) = 0 - 0 + 15 = 15$

$f(4) = 4^3 - 6 \cdot 16 + 15 = -17$

$64 - 96 + 15$

$(0, 15)$ = Maximum

$(4, -17)$ = Minimum

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**Problem 2.** Find the absolute extrema of $g(x) = x^3 - 3x^2$ on the closed interval $[-1, 3]$.

**Solution:**

$g'(x) = 3x^2 - 6x$

$3x^2 - 6x = 0$

$x(3x - 6) = 0$

$x = 0$

$3x = 6$

$x = 2$

$g(0) = 0 - 0 = 0$ 7

$g(2) = 8 - 3 \cdot 4 = -4$

$-12$

$(0, 0)$ = Maximum $(3, 0)$

$(2, -4)$ = Minimum $(-1, -4)$

**Problem 3.** (Maximum Profit) A certain commodity has a demand (or price) function modeled by: $p(x) = 36 - 4x$ and a cost function modeled by: $C(x) = 2x^2 + 6$, where $x$ is the number of units. What price $p(x)$ yields a maximum profit? (Hint: first find $x$ that maximizes the profit $P$.)

**Solution:** $P = R - C$

$P = x(36-4x) - (2x^2+6)$

$P = 36x - 4x^2 - 2x^2 - 6$

$P = 36x - 6x^2 - 6$

$P' = 36 - 12x$

$36 - 12x = 0$

$12x = 36$

$x = 3$ units

$P(3) = 36 - 4(3)$

$= 36 - 12$

$P = 24$

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**Problem 4.** (Compound Interest) You deposit a lump sum $P$ in an account that earns 1.5% interest compounded *continuously*. What must $P$ be in order to have an account balance of \$200,000 after 10 years?

**Solution:**

$A = Pe^{rt}$

$200000 = P(e^{.015(10)})$

$P = \frac{200000}{1.16183} \approx \$172,141.60$

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**Problem 5.** (Two parts) Find the derivatives of: (a) $f(x) = e^{x^2}$; (b) $g(x) = \ln(x^2)$.

**Solution:**

a) $f'(x) = e^{x^2} \cdot 2x$

b) $g'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

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**Problem 6.** Find: $\int \frac{1-4x^4}{x^2}\,dx.$

**Solution:** Split

$$\int \frac{1}{x^2}\,dx - \int \frac{4x^4}{x^2}\,dx$$

$$\int x^{-2}\,dx - \int 4x^2\,dx$$

$$\frac{x^{-1}}{-1} - \frac{4x^3}{3} + C$$

$$\boxed{-\frac{1}{x} - \frac{4x^3}{3} + C}$$

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**Problem 7.** Find: $\int \frac{-3}{\sqrt{2t+3}}\,dt.$

**Solution:**

let $u = 2t+3$

$du = 2$

$$-3\int \frac{1}{\sqrt{u}}\,dt$$

$$\frac{-3}{2}\int \frac{2}{\sqrt{u}}\,dt$$

$$-\frac{3}{2}\int u^{-\frac{1}{2}}\,du$$

$$-\frac{3}{2}\cdot 2u^{\frac{1}{2}} + C = -3u^{\frac{1}{2}} + C$$

$$= \boxed{-3\left(\sqrt{2t+3}\right) + C}$$

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**Problem 8.** Find the profit function $P(x)$ given that the marginal profit is: $dP/dx = -24x + 805$ and that the initial condition is: $P(12) = \$8000$.

**Solution:**

$$\int \frac{dP}{dx} = \int -24x + 805\,dx$$

$$P = \frac{-24x^2}{2} + 805x + C$$

$$P = -12x^2 + 805x + C$$

$P(12)$

$$8000 = -12(12)^2 + 805(12) + C$$

$$8000 = -1728 + 9660 + C$$

$$8000 = 7932 + C$$

$$C = 68$$

$$\boxed{P = -12x^2 + 805x + 68}$$

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**Problem 9.** Use the Exponential Rule for Integration to find: $\int e^{-x-1}\,dx$.

**Solution:**

$u=-x-1$

$du=-1$

$$-1\int e^u \cdot -1\,dx$$

$$-\int e^u\,du$$

$$-e^u+C$$

$$\boxed{-e^{-x-1}+C}$$

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**Problem 10.** Use the Log Rule for Integration to find: $\int \frac{2}{3x+5}\,dx$.

**Solution:**

$u=3x+5$

$du=3$

$$2\int \frac{1}{3x+5}\,dx$$

$$\frac{2}{3}\int \frac{3}{3x+5}\,dx$$

$$\frac{2}{3}\int \frac{1}{u}\,du$$

$$\boxed{\frac{2}{3}\ln|3x+5|+C}$$

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