PROBLEM 10.6

KNOWN: Water boiling on a mechanically polished stainless steel surface maintained at an excess temperature of 15 °C; water is at 1 atm.

FIND: Boiling heat transfer coefficient.

SCHEMATIC:

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Water, 1 atm
Mechanically polished stainless steel
ΔT_e = 15°C
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ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling occurs.

PROPERTIES: Table A-5, Saturated water (1 atm): T_{sat} = 100 °C, \( \rho_e = 957.9 \text{ kg/m}^3 \), \( \rho_v = 0.596 \text{ kg/m}^3 \), \( c_p,e = 4217 \text{ J/kg·K} \), \( \mu_e = 279 \times 10^{-6} \text{ N·s/m}^2 \), \( Pr_e = 1.76 \), \( \sigma = 58.9 \times 10^{-3} \text{ N/m} \), \( h_{fg} = 2257 \text{ kJ/kg} \).

ANALYSIS: The heat transfer coefficient can be expressed as

\[
h = \frac{q''_s}{\Delta T_e}
\]

where the nucleate pool boiling heat flux can be estimated using the Rohsenow correlation.

\[
q''_s = \frac{\mu_e h_{fg}}{\rho_e} \left[ \frac{g (\rho_e - \rho_v)}{\sigma} \right]^{1/2} \left( \frac{c_p,e \Delta T_e}{C_{s,f} h_{fg} Pr_e^{3/4}} \right)^3
\]

From Table 10.1, find for this liquid-surface combination, \( C_{s,f} = 0.013 \) and \( n = 1 \), and substituting numerical values,

\[
q''_s = 279 \times 10^{-6} \text{ N·s/m}^2 \times 2257 \times 10^3 \text{ J/kg} \left[ \frac{9.8 \text{ m/s}^2 (957.9 - 0.596) \text{ kg/m}^3}{58.9 \times 10^{-3} \text{ N/m}} \right]^{1/2} \times \left( \frac{4217 \text{ J/kg·K} \times 15 °C}{0.013 \times 2257 \text{ kJ/kg} \times 1.76} \right)^3
\]

\[
q''_s = 461.9 \text{ kW/m}^2
\]

Hence, the heat transfer coefficient is

\[
h = 461.9 \times 10^3 \text{ W/m}^2/15 °C = 30.790 \text{ W/m}^2·K
\]

COMMENTS: Note that this value of \( q''_s \) for \( \Delta T_e = 15 °C \) is similar to the typical boiling curve, Fig. 10.4.
**PROBLEM 10.20**

**KNOWN:** Thickness and thermal conductivity of a silicon chip. Properties of saturated fluorocarbon liquid.

**FIND:** (a) Temperature at bottom surface of chip for a prescribed heat flux and 90% of CHF, (b) Effect of heat flux on chip surface temperatures; maximum allowable heat flux.

**SCHEMATIC:**

![Diagram](image)

\[ q''_o = 5 \times 10^6 \text{ W/m}^2 \text{ or } C_1 \ q''_{\text{max}} \]

N.B. Constants: \( C_{s,r} = 0.005, n = 1.7 \)

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform heat flux and adiabatic sides, hence one-dimensional conduction in chip, (3) Constant properties, (4) Nucleate boiling in liquid.

**PROPERTIES:** Saturated fluorocarbon (given): \( c_{p,r} = 1100 \text{ J/kg K}, h_g = 84,400 \text{ J/kg}, \rho_r = 1619.2 \text{ kg/m}^3, \rho_v = 13.4 \text{ kg/m}^3, \sigma = 8.1 \times 10^{-3} \text{ kg/s}^2, \mu_r = 440 \times 10^{-6} \text{ kg/m} \cdot \text{s}, Pr_r = 9.01 \).

**ANALYSIS:** (a) Energy balances at the top and bottom surfaces yield \[ q''_o = q''_{\text{cond}} = k_s \left( T_o - T_s \right) / L = q''_s \]

where \( T_s \) and \( q''_s \) are related by the Rohsenow correlation,

\[ T_s - T_{\text{sat}} = \frac{C_{s,r}h_g Pr_r}{c_{p,r}} \left( \frac{q''_s}{\mu_r h_g} \right)^{1/3} \frac{\sigma}{g(\rho_r - \rho_v)}^{1/6} \]

Hence, for \( q''_s = 5 \times 10^6 \text{ W/m}^2 \),

\[ T_s - T_{\text{sat}} = \frac{0.005(84,400 \text{ J/kg})9.01^{1.7}}{1100 \text{ J/kg K}} \left( \frac{5 \times 10^6 \text{ W/m}^2}{440 \times 10^{-6} \text{ kg/m} \cdot \text{s} \times 84,400 \text{ J/kg}} \right)^{1/3} \times \left[ \frac{8.1 \times 10^{-3} \text{ kg/s}^2}{9.807 \text{ m/s}^2 (1619.2 - 13.4) \text{ kg/m}^3} \right]^{7/6} = 15.9^\circ \text{C} \]

\[ T_s = (15.9 + 57)^\circ \text{C} = 72.9^\circ \text{C} \]

From the rate equation,

\[ T_o = T_s + \frac{q''_o L}{k_s} = 72.9^\circ \text{C} + \frac{5 \times 10^4 \text{ W/m}^2 \times 0.0025 \text{ m}}{135 \text{ W/m K}} = 73.8^\circ \text{C} \]

For a heat flux which is 90% of the critical heat flux \((C_1 = 0.9)\), it follows that

\[ q''_o = 0.9q''_{\text{max}} = 0.9 \times 0.149 h_g \rho_v \left[ \frac{\sigma g (\rho_r - \rho_v)}{\rho_v} \right]^{1/4} = 0.9 \times 0.149 \times 84,400 \text{ J/kg} \times 13.4 \text{ kg/m}^3 \]

Continued...
PROBLEM 10.20 (Cont.)

\[
q_0'' = 0.9 \times 15.5 \times 10^4 \text{ W/m}^2 = 13.9 \times 10^4 \text{ W/m}^2
\]

From the results of the previous calculation and the Rohsenow correlation, it follows that

\[
\Delta T_e = 15.9^\circ C \left( q_0'' / 5 \times 10^4 \text{ W/m}^2 \right)^{1/3} = 15.9^\circ C \left( 13.9 / 3 \right)^{1/3} = 22.4^\circ C
\]

Hence, \( T_s = 79.4^\circ C \) and

\[
T_e = 79.4^\circ C + \frac{13.9 \times 10^4 \text{ W/m}^2 \times 0.0025 \text{ m}}{135 \text{ W/m K}} = 82^\circ C
\]

(b) Using the energy balance equations with the Correlations Toolpad of IHT to perform the parametric calculations for \( 0.2 \leq C_t \leq 0.9 \), the following results are obtained.

The chip surface temperatures, as well as the difference between temperatures, increase with increasing heat flux. The maximum chip temperature is associated with the bottom surface, and \( T_e = 80^\circ C \) corresponds to

\[
q_{0,\text{max}}'' = 11.3 \times 10^4 \text{ W/m}^2
\]

which is 73% of CHF (\( q_{\text{max}}'' = 15.5 \times 10^4 \text{ W/m}^2 \)).

COMMENTS: Many of today's VLSI chip designs involve heat fluxes well in excess of 15 W/cm², in which case pool boiling in a fluorocarbon would not be an appropriate means of heat dissipation.
PROBLEM 10.33

KNOWN: Forced convection and boiling processes occur in a smooth tube with prescribed water velocity and surface temperature.

FIND: Heat transfer rate per unit length of the tube.

SCHEMATIC:

\[ T_s = 110^\circ C \]
\[ u_m = 1.5 \text{ m/s} \]
\[ T_m = 95^\circ C \]
\[ \text{Brass tube, } D = 15 \text{ mm} \]

ASSUMPTIONS: (1) Fully-developed flow, (2) Nucleate boiling conditions occur on inner wall of tube, (3) Forced convection and boiling effects can be separately estimated.

PROPERTIES: Table A-6, Water \((T_m = 95^\circ C = 368K): \rho_f = 1/\nu_f = 962 \text{ kg/m}^3, \rho_v = 1/\nu_v = 0.500 \text{ kg/m}^3, h_f = 2270 \text{ kJ/kg}, c_{p,f} = 4212 \text{ J/kg.K}, \mu_f = 296 \times 10^{-6} \text{ N.s/m}^2, k_f = 0.678 \text{ W/m.K}, Pr_f = 1.86, \sigma = 60 \times 10^{-3} \text{ N/m}, \nu_f = 3.08 \times 10^{-7} \text{ m}^2/\text{s}.\)

ANALYSIS: Experimentation has indicated that the heat transfer rate can be estimated as the sum of the separate effects due to forced convection and boiling. On a per unit length basis,

\[ q' = q'_{fc} + q'_{boil}. \]

For *forced convection*, \(Re_D = u_m D/\nu_f = 1.5 \text{ m/s} \times 0.015 \text{ m} / 3.08 \times 10^{-7} \text{ m}^2/\text{s} = 73,052.\) Since \(Re > 2300\), flow is turbulent and since fully developed, use the Dittus-Boelter correlation but with the 0.023 coefficient replaced by 0.019 and \(n = 0.4\),

\[ Nu_D = h D/k = 0.019 Re_D^{4/5} Pr_f^{0.4} \]
\[ h = k D / Nu_D = \frac{0.678 \text{ W/m.K}}{0.015 \text{ m}} \times 0.019 (73,052)^{4/5} (1.86)^{0.4} = 8563 \text{ W/m}^2\text{.K}. \]

\[ q'_{fc} = h \pi D (T_s - T_m) = 8,563 \text{ W/m}^2\text{.K} \pi (0.015 \text{ m}) (110 - 95) ^\circ \text{C} = 6052 \text{ W/m}. \]

For *boiling*, \(\Delta T_g = (110 - 100) ^\circ \text{C} = 10 ^\circ \text{C} \) and hence nucleate boiling occurs. From the Rohsenow equation, with \(C_{sf} = 0.006\) and \(n = 1.0\),

\[ q''_{boil} = \mu_f h_f \left[ \frac{g (\rho_f - \rho_v)}{\sigma} \right]^{1/2} \left[ \frac{c_{p,f} \Delta T_g}{C_{sf} h_f Pr_f^{0.4}} \right]^{3} \]
\[ q''_{boil} = 296 \times 10^{-4} \left[ \frac{9.8 \text{ m/s}^2 (962 - 0.5) \text{ kg/m}^3}{60 \times 10^{-3} \text{ N/m}} \right]^{1/2} \left[ \frac{4212 \text{ J/kg.K} \times 10\text{K}}{0.006 \times 2270 \times 10^{3} \text{ J/kg} \times (1.86)^{0.4}} \right]^{3} \]
\[ q''_{boil} = 1.22 \times 10^6 \text{ W/m}^2 \]
\[ q''_{boil} (\pi D) = 1.22 \times 10^6 \text{ W/m}^2 (\pi \times 0.015 \text{ m}) = 57,670 \text{ W/m}. \]

The total heat rate for both processes is

\[ q' = (6052 + 57,670) \text{ W/m} = 6.37 \times 10^4 \text{ W/m}. \]

COMMENTS: Recognize that this method provides only an estimate since the processes are surely coupled.