Review
Electrostatic
Dr. Ray Kwok
SJSU
Party Balloons
Coulomb’s Law

\[ F_e = k \frac{q_1 q_2}{r^2} \]  
Coulomb force or electrical force. \textit{(vector)}

Be careful on determining the “sign” & direction.

\[ k = 9 \cdot 10^9 \text{ (N} \cdot \text{m}^2 / \text{C}^2) \]

\[ k = \frac{1}{4 \pi \varepsilon_o} \]

\( k \) is the Coulomb’s constant, \( \varepsilon_o \) is called permittivity of vacuum, for now it’s just a constant.

\[ \varepsilon_o = \frac{1}{4 \pi k} = \frac{10^{-9}}{36 \pi} = 8.84 \cdot 10^{-12} \left( \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) \]
Electric Field Lines \((F = qE)\)

Think of the direction of coulomb force on a “+” test charge

generate from a “+” charge

terminate into a “-” charge
Two opposite charges
Other combinations
Conductor & E-field: Property 1

- The electric field is **zero** everywhere **inside** the conducting material
  - Consider if this were **not** true
    - If there were an electric field inside the conductor, the free charge there would move and there would be a flow of charge
    - If there were a movement of charge, the conductor would not be in equilibrium. (Electrostatic !!)
Property 2

- Any excess **charge** on an isolated conductor resides entirely **on its surface**
  - A direct result of the $1/r^2$ repulsion between like charges in Coulomb’s Law
  - If some excess of charge could be placed inside the conductor, the repulsive forces would push them as far apart as possible, causing them to migrate to the surface

So $E = 0$ inside.
Property 3

- The electric field just outside a charged conductor is perpendicular to the conductor’s surface.
  - Consider what would happen if this was not true.
  - The component along the surface would cause the charge to move.
  - It would not be in equilibrium.
Property 4

- On an irregularly shaped conductor, the charge accumulates at locations where the radius of curvature of the surface is smallest (that is, at sharp points)
e.g. **Metal shielding**

Place a point charge inside a conducting shell.

E field lines are perpendicular to the surface.

E = 0 inside conductor.

Charges are induced on the surface.
E field of a charged conductor

irregular shape...

E field lines are perpendicular to the surface.
E = 0 inside conductor.
Charges are cumulated on the surface.
E field is more intense at small radius (higher concentration of charges)
Charge distribution

A non-conducting thin wire of 2 m long, carrying a charge density of +10\(\mu\)C/m. What is the electric field at 10 cm from the center of the wire?

\[
|dE| = k \frac{|dq|}{r^2} = k \frac{(\lambda dy)}{x^2 + y^2} \\
|dE|_x = |dE| \cos \theta = k \frac{(\lambda dy)}{x^2 + y^2} \left( \frac{x}{r} \right) = \frac{k \lambda x(dy)}{(x^2 + y^2)^{3/2}}
\]

\[
E = \int_{-1}^{1} \frac{k \lambda x(dy)}{(x^2 + y^2)^{3/2}}
\]

\[
E = k \lambda x \int_{-1}^{1} \frac{dy}{r^3} = (9 \cdot 10^9)(10^{-5})(0.1)(199) = 1.8 \cdot 10^6 \hspace{1cm} \text{N/C, to the right}
\]

Notation used in most textbook

\(\rho = \text{volume charge density}\)
\(\sigma = \text{surface (area) charge density}\)
\(\lambda = \text{linear charge density}\)
Gauss

In cgs, gauss is the unit for magnetic field.

Many contributions in math and geophysics.

Most known for his electrostatic work....
Known as the Gauss’s Law.

Carl Friedrich Gauss
German mathematician & scientist
1777 - 1855
Electric Flux

- Field lines penetrating an area $A$ perpendicular to the field
- The product of $EA$ is the flux, $\Phi$
- In general:
  - $\Phi_E = E \cdot A \cos \theta$

$$\Phi_E \equiv \int \vec{E} \cdot d\vec{a}$$
Gauss’ Law

Gauss’ Law states that the electric flux through any closed surface is equal to the net charge $Q$ inside the surface divided by $\varepsilon_0$.

$$\Phi_E = \frac{Q_{\text{inside}}}{\varepsilon_0}$$

- $\varepsilon_0$ is the permittivity of free space and equals $8.85 \times 10^{-12}$ C$^2$/Nm$^2$
- The area in $\Phi$ is an imaginary surface, a Gaussian surface, it does not have to coincide with the surface of a physical object.

$$\oint \bar{E} \cdot d\bar{a} = \frac{Q_{\text{inside}}}{\varepsilon_0}$$
Total flux does not depend on the shape chosen.

(a) The outward normal to the surface makes an angle $\phi$ with the direction of $\vec{E}$.

(b) The projection of the area element $dA$ onto the spherical surface is $dA \cos \phi$. 
Gaussian Surface

- Have the same symmetry as the charge distribution, such that...
- E field is uniform on the surface
- E field is perpendicular to the surface

\[ \Phi_E \equiv \int \vec{E} \cdot d\vec{a} = EA \]

\[ EA = \frac{Q_{\text{inside}}}{\varepsilon_0} \]
e.g. Point Charge

Gaussian surface is a sphere

\[ EA = \frac{Q_{\text{inside}}}{\varepsilon_o} \]

\[ E \left(4\pi r^2\right) = \frac{q}{\varepsilon_o} \]

\[ E = \frac{1}{4\pi \varepsilon_o} \frac{q}{r^2} \]

which is the E field for a point charge
E field of a line or plane of charge

infinite line charge

infinite plane of charge
E field of a uniformly charged sphere

\[ E(R) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^2} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Qr}{R^3} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]
Divergence Theorem

or Gauss’s Theorem

\[ \int_{V} \nabla \cdot \vec{E} dV = \oint_{S} \vec{E} \cdot d\vec{a} \]

for any vector \( \vec{E} \).

Gauss’s Law

\[ \oint_{S} \vec{E} \cdot d\vec{a} = \frac{Q_{\text{inside}}}{\varepsilon_{o}} \]

\[ \int_{V} \nabla \cdot \vec{E} dV = \int_{V} \frac{\rho dV}{\varepsilon_{o}} \]

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_{o}} \]

Gauss’s Law in differential form
Stoke’s Theorem

\[ \int_S \nabla \times \vec{E} \cdot d\vec{a} = \int_L \vec{E} \cdot d\vec{\ell} \]

for any vector \( \vec{E} \).

In electrostatic, we defined electric potential (voltage): \( V = -\int_L \vec{E} \cdot d\vec{\ell} \)

and \( \int_L \vec{E} \cdot d\vec{\ell} = 0 \) Conservative field, Kirchhoff’s rule.

\[ \nabla \times \vec{E} = 0 \]