Lossy Medium

EE142

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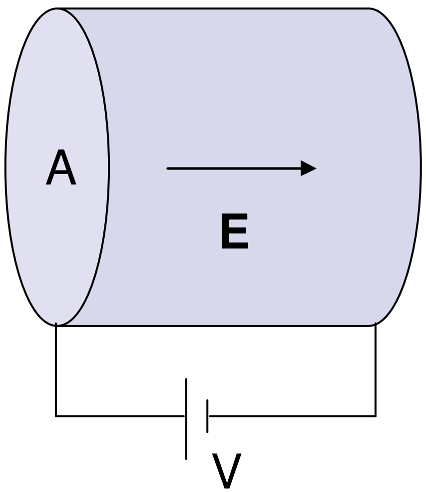
•reference:

Fundamentals of Engineering Electromagnetics, David K. Cheng (Addison-Wesley)

Electromagnetics for Engineers, Fawwaz T. Ulaby (Prentice Hall)

Ohm's Law



$$V = IR$$

$$E\ell = (JA)\left(\rho \frac{\ell}{A}\right)$$

$$\vec{E} = \vec{J}\rho$$ resisitivity

$$\vec{J} = \frac{1}{\rho}\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$ conductivity

- Low resistivity => "conductor" ~< 10^{-5} Ω cm (✪ T)
- High resistivity => "insulator" ~> 10^{10} Ω cm
- Intermediate resistivity => "semiconductor"
  typical ~ 10^{-3} to 10^{5} Ω cm (✪ $e^{E_g/kT}$)
- unit of conductivity = S/m = Siemens/meter = mho/m = $(\Omega\ m)^{-1}$

EM Wave through medium

$$\varepsilon\nabla\cdot\vec{E}=\rho_f$$

$$\nabla\times\vec{E}=-\mu\frac{\partial\vec{H}}{\partial t}$$

$$\nabla\cdot\vec{H}=0$$

$$\nabla\times\vec{H}=\vec{J}_f+\varepsilon\frac{\partial\vec{E}}{\partial t}$$

(homogeneous, linear, isotropic)

$\rho\approx 0,\ J\neq 0$ →

$$\nabla\cdot\vec{E}=0$$

$$\nabla\times\vec{E}=-j\omega\mu\vec{H}$$

$$\nabla\cdot\vec{H}=0$$

$$\nabla\times\vec{H}=\vec{J}_f+j\omega\varepsilon\vec{E}$$

$$\nabla\times\vec{H}=(\sigma+j\omega\varepsilon)\vec{E}=j\omega\left(\varepsilon+\frac{\sigma}{j\omega}\right)\vec{E}=j\omega\varepsilon_c\vec{E}$$

$$\varepsilon_c\equiv\varepsilon-j\frac{\sigma}{\omega}\equiv\varepsilon'-j\varepsilon''$$

finite σ means complex ε

Loss Tangent

$$\tan\delta \equiv \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}$$

good conductor $\sigma >> \omega\varepsilon$
good insulator $\sigma << \omega\varepsilon$

Low tanδ → low dielectric loss

the smaller, the better !!!

DIELECTRIC CONSTANTS AND LOSS TANGENTS FOR SOME MATERIALS

Material	Frequency	ϵ_r	$\tan\delta$ (25°C)
Alumina (99.5%)	10 GHz	9.5–10.	0.0003
Barium tetratitanate	6 GHz	37 ± 5%	0.0005
Beeswax	10 GHz	2.35	0.005
Beryllia	10 GHz	6.4	0.0003
Ceramic (A-35)	3 GHz	5.60	0.0041
Fused quartz	10 GHz	3.78	0.0001
Gallium arsenide	10 GHz	13.	0.006
Glass (pyrex)	3 GHz	4.82	0.0054
Glazed ceramic	10 GHz	7.2	0.008
Lucite	10 GHz	2.56	0.005
Nylon (610)	3 GHz	2.84	0.012
Parafin	10 GHz	2.24	0.0002
Plexiglass	3 GHz	2.60	0.0057
Polyethylene	10 GHz	2.25	0.0004
Polystyrene	10 GHz	2.54	0.00033
Porcelain (dry process)	100 MHz	5.04	0.0078
Rexolite (1422)	3 GHz	2.54	0.00048
Silicon	10 GHz	11.9	0.004
Styrofoam (103.7)	3 GHz	1.03	0.0001
Teflon	10 GHz	2.08	0.0004
Titania (D-100)	6 GHz	96 ± 5%	0.001
Vaseline	10 GHz	2.16	0.001
Water (distilled)	3 GHz	76.7	0.157

Example

A sinusoidal E-field with amplitude of 250 V/m and frequency 1 GHz exists in a lossy dielectric medium that has a ε_r = 2.5 and loss tangent of 0.001. Find the average power dissipated in the medium per cubic meter.

$$\tan\delta = 0.001 = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_o\varepsilon_r} = \frac{\sigma}{\left(2\pi\cdot 10^9\right)\left(\frac{10^{-9}}{36\pi}\right)(2.5)}$$

$$\sigma = 1.39\cdot 10^{-4} \quad \text{S/m}$$

The average power dissipated per unit volume is

$$\frac{P_{ave}}{V} = \frac{1}{2}\vec{J}\cdot\vec{E} = \frac{1}{2}\sigma E^2 = \frac{1}{2}\left(1.39\cdot 10^{-4}\right)(250)^2$$

$$\frac{P_{ave}}{V} = 4.34 \quad \text{W/m}^2$$

Note:

$$P_{ave} = \frac{1}{2}\frac{V^2}{R} = \frac{1}{2}\frac{(E\ell)^2}{\rho\ell/A} = \frac{1}{2}\sigma E^2(\ell A)$$

Wave Equation

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon_c \frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} = \mu \varepsilon_c \frac{\partial^2 \vec{E}}{\partial t^2}$$

plane wave equation still holds with modification of ε

$$\vec{E}(r,t) = \vec{E}_o e^{j(\omega t - k_c r)} \equiv \vec{E}_o e^{j\omega t} e^{-\gamma r}$$

allow k be complex since ε is

$$\gamma \equiv jk_c = j\omega\sqrt{\mu\varepsilon_c} \equiv \alpha + j\beta$$

phase constant (β)

attenuation constant (α)

propagation constant (γ)

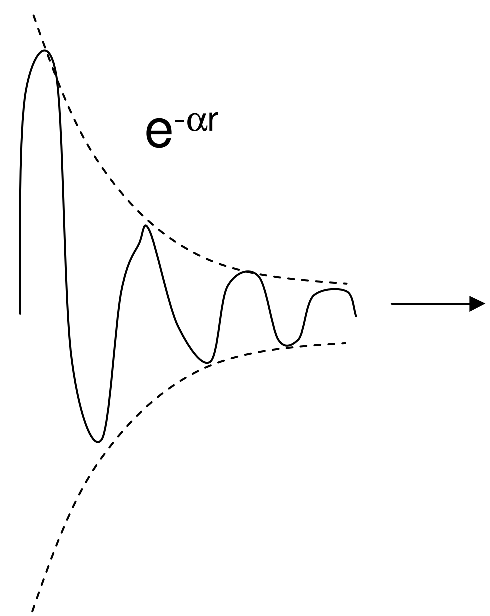
Complex Propagation Constant

$$\gamma = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu\varepsilon\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)} = j\omega\sqrt{\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)} = j\omega\sqrt{\mu\varepsilon(1 - j\tan\delta)} = \alpha + j\beta$$

The phasor

$$\vec{E}(r) = \vec{E}_o e^{-\gamma r} = \vec{E}_o e^{-\alpha r} e^{-j\beta r}$$

attenuation



dB scale

power intensity ratio in log scale, not a unit !!

$$(\mathrm{dB}) = 10\log\left(\frac{I}{I_o}\right) = 10\log\left(\frac{P}{P_o}\right) = 20\log\left(\frac{V}{V_o}\right)$$

I ↑ sound intensity; P ↑ power; V ↑ voltage

> 0 gain
< 0 loss

10 log(2) ≈ 3, 3 dB = double
10 log(1/2)≈ -3, -3 dB = half
10 log(10) = 10, 10 dB = 10x
10 log(100) = 20, 20 dB = 100x
10 log(0.1) = -10, -10 dB = 1/10

What is 6 dB? –9 dB? 7 dB? –44 dB?

4x 1/8 5x 4×10^{-5}

dBm & dBW

$$dBW \equiv 10\log\left(\frac{P}{1W}\right)$$

$$dBm \equiv 10\log\left(\frac{P}{1mW}\right)$$

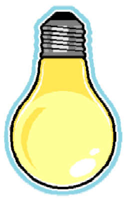
become real units

0 dBm = 1 mW
30 dBW = 1 kW
-30 dBm = 1 μW

What is	40 dBW?	-7 dBm?	-26 dBm?	21 dBm?
	10 kW	0.2 mW	2.5 μW	1/8 W

Example

Isotropic
100 W



How much electricity generated by the solar cell?
What if a 40 W bulb is used? 200 W bulb?

Intensity = power/area $= \frac{100}{4\pi R^2} = \frac{100}{4\pi(1)^2} = 7.96\,\frac{\text{W}}{\text{m}^2}$

1 m

Power generated in solar cell

$$= \left(7.96\,\frac{\text{W}}{\text{m}^2}\right)\left(100\text{cm}^2\right)\left(40\%\right) = 31.8\text{mW}$$

solar cell
10 x 10 cm^2
40% efficiency

In terms of dB $= 10\log\left(\frac{0.0318\text{W}}{100\text{W}}\right) = -35\text{dB}$

system "gain"

40 W bulb? $-35 = 10\log\left(\frac{P}{40}\right)$

Power of electricity generated = 12.6 mW

200 W bulb? $-35 = 10\log\left(\frac{P}{200}\right)$

Power of electricity generated = 63.2 mW

Attenuation

$$\vec{E}(r) = \vec{E}_o e^{-\gamma r} = \vec{E}_o e^{-\alpha r} e^{-j\beta r}$$

$$A(r)[dB] = 20\log\left|\frac{\vec{E}(r)}{\vec{E}(0)}\right| = 20\log\left|e^{-\alpha r}\right|$$

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

$$A(r)[dB] = \frac{-20\alpha r}{\ln(10)} = -8.686\alpha r$$

For example, if the electric field intensity going through a medium attenuates at a rate of 0.4 dB/m, what is α ?

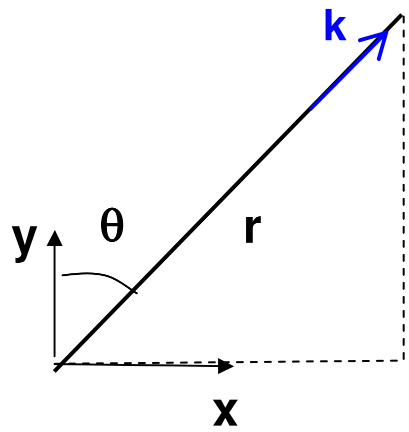
-0.4 dB = -8.686 α (1 m)
α = 0.4/8.686 = 0.046 nepers/m

Note: nepers (np) is not a real unit.
similar to radians !!!

Note also α is a positive number for attenuation.

α[dB/m] = 8.686 α[np/m]

Attenuation term



How to express $e^{-\alpha r}$ term??

$$\hat{k} = \hat{x}\sin\theta + \hat{y}\cos\theta$$

$$e^{-\alpha r} = e^{-\alpha(x\sin\theta + y\cos\theta)} = e^{-\alpha\sqrt{x^2+y^2}}$$

Same ?

$$x\sin\theta + y\cos\theta = x\left(\frac{x}{r}\right) + y\left(\frac{y}{r}\right) = \frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2}$$

Yes, Q.E.D.

Can think of :

$$\vec{\alpha} \equiv \alpha\hat{k}$$

$$\vec{\alpha}\cdot\vec{r} = \alpha(x\sin\theta + y\cos\theta)$$

$$e^{-\vec{\alpha}\cdot\vec{r}} = e^{-\alpha(x\sin\theta + y\cos\theta)} = e^{-\alpha\sqrt{x^2+y^2}}$$

Low-loss dielectric (ε"<<ε') or (σ<<ωε)

$$\gamma = j\omega\sqrt{\mu\varepsilon\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)} = j\omega\sqrt{\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)} = j\omega\sqrt{\mu\varepsilon(1 - j\tan\delta)}$$

$$(1+x)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^{n-k} = 1 + nx + \frac{n(n-1)}{2} x^2 + \ldots..$$

$$(1+x)^n \approx 1 + nx$$

for small x

$$\gamma = j\omega\sqrt{\mu\varepsilon}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{1/2} \approx j\omega\sqrt{\mu\varepsilon}\left(1 - \frac{1}{2} j\frac{\varepsilon''}{\varepsilon'}\right) \equiv \alpha + j\beta$$

$$\alpha = \frac{\omega\sqrt{\mu\varepsilon}}{2}\frac{\varepsilon''}{\varepsilon'} = \frac{\omega\sqrt{\mu\varepsilon}}{2}\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

small

$$\beta = \omega\sqrt{\mu\varepsilon} \approx \omega / v$$

$$\eta_c \equiv \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon(1 - j\varepsilon''/\varepsilon')}} = \sqrt{\frac{\mu}{\varepsilon}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \approx \sqrt{\frac{\mu}{\varepsilon}}\left(1 + \frac{j}{2}\frac{\varepsilon''}{\varepsilon'}\right) \approx \sqrt{\frac{\mu}{\varepsilon}}$$

Good Conductor (σ>>ωε)

$$\gamma = j\omega\sqrt{\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)} \approx j\omega\sqrt{-j\frac{\sigma}{\omega\varepsilon}\mu\varepsilon}$$

$$\sqrt{-j} = \sqrt{e^{-j\pi/2}} = e^{-j\pi/4} = \frac{1-j}{\sqrt{2}}$$

$$\gamma = j\omega\frac{1-j}{\sqrt{2}}\sqrt{\frac{\mu\sigma}{\omega}} = (j+1)\sqrt{\frac{\mu\sigma\omega}{2}} \equiv \alpha + j\beta$$

$$\alpha = \beta = \boxed{\sqrt{\frac{\mu\sigma\omega}{2}}}$$

$$\eta_c \equiv \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon(1 - j\sigma/\omega\varepsilon)}} \approx \sqrt{j\frac{\mu\omega\varepsilon}{\varepsilon\sigma}} \approx \frac{1+j}{\sqrt{2}}\sqrt{\frac{\mu\omega}{\sigma}} = (1+j)\sqrt{\frac{\mu\omega}{2\sigma}} = \boxed{(1+j)\frac{\alpha}{\sigma}}$$

Skin Depth δ

$$\delta \equiv \frac{1}{\alpha} = \sqrt{\frac{2}{\mu\sigma\omega}} = \delta_s$$

(NOT loss tangent δ !!!!!)

$$\vec{E}(r) = \vec{E}_o e^{-\alpha r} e^{-j\beta r}$$

At r = δ, |E| decreases to 1/e (or 63% drop).

$$A(r)[dB] = 20\log\left|\frac{\vec{E}(r)}{\vec{E}(0)}\right| = 20\log\left|e^{-\alpha r}\right| = -8.686\alpha r$$

At r = δ, |E| decreases by -8.7 dB.
At r = 2δ, |E| decreases by -17.3 dB….

General Material

$$\gamma = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} \equiv \alpha + j\beta$$

$$\gamma^2 = -\omega^2\mu(\varepsilon' - j\varepsilon'') = \alpha^2 - \beta^2 + 2j\alpha\beta$$

$$-\omega^2\mu\varepsilon' = \alpha^2 - \beta^2 \quad \text{real}$$

$$j\omega^2\mu\varepsilon'' = j2\alpha\beta \quad \text{imaginary}$$

$$\alpha^2 = \beta^2 - \omega^2\mu\varepsilon' = \left(\frac{\omega^2\mu\varepsilon''}{2\alpha}\right)^2 - \omega^2\mu\varepsilon'$$

$$4\alpha^4 + 4\alpha^2\omega^2\mu\varepsilon' - (\omega^2\mu\varepsilon'')^2 = 0$$

$$\alpha^2 = \frac{-4\omega^2\mu\varepsilon' \pm \sqrt{(4\omega^2\mu\varepsilon')^2 + 16(\omega^2\mu\varepsilon'')^2}}{8}$$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon'}{2}\left(-1 \pm \sqrt{1 + \tan^2\delta}\right)$$

$$\boxed{\alpha^2 = \frac{\omega^2\mu\varepsilon'}{2}\left(\sqrt{1 + \tan^2\delta} - 1\right)}$$

$$\beta^2 = \left(\frac{\omega^2\mu\varepsilon''}{2\alpha}\right)^2 = \frac{\omega^2\mu\varepsilon'\tan^2\delta}{2\left(\sqrt{1 + \tan^2\delta} - 1\right)}$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon'\tan^2\delta}{2\left(\sqrt{1 + \tan^2\delta} - 1\right)} \frac{\left(\sqrt{1 + \tan^2\delta} + 1\right)}{\left(\sqrt{1 + \tan^2\delta} + 1\right)}$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon'\tan^2\delta\left(\sqrt{1 + \tan^2\delta} + 1\right)}{2\left(1 + \tan^2\delta - 1\right)}$$

$$\boxed{\beta^2 = \frac{\omega^2\mu\varepsilon'}{2}\left(\sqrt{1 + \tan^2\delta} + 1\right)}$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'(1 - j\tan\delta)}}$$

$$\boxed{\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}(1 - j\tan\delta)^{-1/2}}$$

Summary

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega \left[\frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right] \right]^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f \mu \sigma}$	(Np/m)
$\beta =$	$\omega \left[\frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} + 1 \right] \right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f \mu \sigma}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'} \right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1 + j)\frac{\alpha}{\sigma}$	(Ω)
$u_p =$	ω/β	$1/\sqrt{\mu\varepsilon}$	$1/\sqrt{\mu\varepsilon}$	$\sqrt{4\pi f/\mu\sigma}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Notes: $\varepsilon' = \varepsilon$; $\varepsilon'' = \sigma/\omega$; in free space, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$; in practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' > 100$.

Example - The skin depth of a certain nonmagnetic conducting material is 2μm at 5 GHz. Determine the phase velocity in the material. What is the attenuation (in dB) when the wave penetrates 10 μm into the material?

phase velocity $v = \omega/\beta$

for conductor, $\alpha = \beta = 1/\delta$

$v = \omega\delta = (2\pi)(5 \times 10^9)(2 \times 10^{-6}) = 6.28 \times 10^4$ m/s

$$A(r)[dB] = 20\log\left|\frac{\vec{E}(r)}{\vec{E}(0)}\right| = 20\log\left|e^{-\alpha r}\right| = -8.686\alpha r$$

$$A(r)[dB] = -8.686 r/\delta = -8.686(10/2) = -43.4 dB$$

in just 5 skin depth.
(– 43 dB = 1 / 20,000 !!!)
Only surface current on conductors.

Example – (a) Calculate the dielectric loss (in dB) of an EM wave propagating through 100 m of teflon at 1 MHz. (b) at 10 GHz ?

Teflon: ε_r = 2.08, tanδ = 0.0004 at 25°C assuming frequency independence.

(a) $$\tan\delta = \frac{\sigma}{\omega\varepsilon}$$

$$\sigma = \omega\varepsilon_o\varepsilon_r\tan\delta = \left(2\pi\cdot 10^6\right)\left(\frac{10^{-9}}{36\pi}\right)(2.08)(0.0004) = 4.6\cdot 10^{-8}$$ S/m

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma\eta_o}{2\sqrt{\varepsilon_r}} = \frac{\left(4.6\cdot 10^{-8}\right)(377)}{2\sqrt{2.08}} = 6.04\cdot 10^{-6}$$ np/m

$$A(\mathrm{dB}) = -8.686\alpha r = -8.686\left(6.04\cdot 10^{-6}\right)(100) = -0.005$$ dB

(b) $$\sigma = \omega\varepsilon_o\varepsilon_r\tan\delta = \left(2\pi\cdot 10^{10}\right)\left(\frac{10^{-9}}{36\pi}\right)(2.08)(0.0004) = 4.6\cdot 10^{-4}$$ S/m

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma\eta_o}{2\sqrt{\varepsilon_r}} = \frac{\left(4.6\cdot 10^{-4}\right)(377)}{2\sqrt{2.08}} = 6.04\cdot 10^{-2}$$ np/m

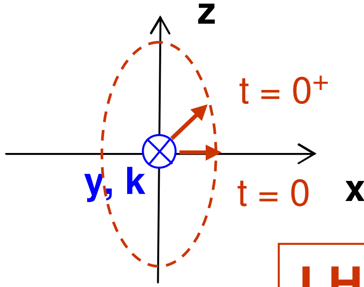
$$A(\mathrm{dB}) = -8.686\alpha r = -8.686\left(6.04\cdot 10^{-2}\right)(100) = -50$$ dB

Coaxial cable works well at low freq (TV to antenna) but not so well at high freq. !!

Example – In a nonmagnetic, lossy, dielectric medium, a 300-MHz plane wave is characterized by the magnetic field phasor $\vec{H} = (\hat{x} - j4\hat{z})e^{-2y}e^{-j9y}$ A/m. Obtain time-domain expressions for the electric and magnetic field vectors. What is the polarization state of this wave?

$$\vec{H}(\vec{r},t) = \Re e\left\{(\hat{x} - j4\hat{z})e^{-2y}e^{j(\omega t - 9y)}\right\}$$

$$\vec{H}(\vec{r},t) = \hat{x}e^{-2y}\cos(\omega t - 9y) + \hat{z}4e^{-2y}\sin(\omega t - 9y)$$



LHEP

$\alpha = 2,\ \beta = 9$

$$-\omega^2\mu\varepsilon' = \alpha^2 - \beta^2$$

$$\omega^2\mu\varepsilon'' = 2\alpha\beta$$

$$\frac{\varepsilon''}{\varepsilon'} = \frac{2\alpha\beta}{\beta^2 - \alpha^2} = \frac{2(2)(9)}{9^2 - 2^2} = 0.468 = \tan\delta$$

$$\varepsilon_r = \frac{\varepsilon'}{\varepsilon_o} = \frac{\beta^2 - \alpha^2}{\omega^2\mu_o\varepsilon_o} = \frac{77c^2}{\omega^2} = \frac{77(3\cdot 10^8)^2}{(2\pi\cdot 300\cdot 10^6)^2} = 1.95$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}(1 - j\tan\delta)^{-1/2} = \frac{\eta_o}{\sqrt{\varepsilon_r}}(1 - j\tan\delta)^{-1/2} = \frac{377}{\sqrt{1.95}}(1 - j0.468)^{-1/2}$$

$$\eta_c = 257\angle 12.5^\circ = 257e^{j0.22}$$

$$\vec{E} = \hat{z}\eta_c e^{-2y}\cos(\omega t - 9y) - \hat{x}4\eta_c e^{-2y}\sin(\omega t - 9y)$$

$$\vec{E} = \hat{z}257e^{-2y}\cos(\omega t - 9y + 0.22) - \hat{x}1028e^{-2y}\sin(\omega t - 9y + 0.22)$$

Exercise (how to write the attenuation?)

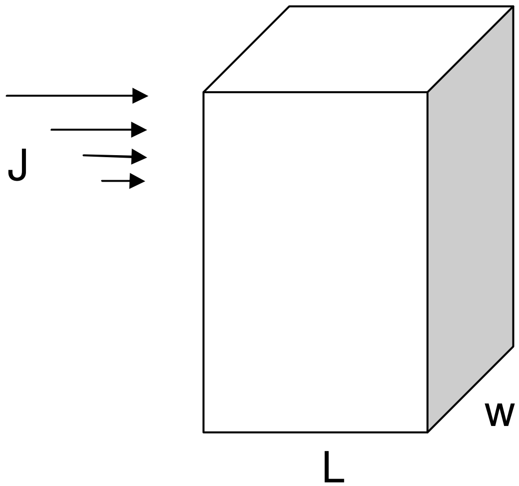
Given E_o at the origin has a amplitude of 1 V/m along the y-axis in a non-magnetic medium, with the propagation given by:

$$\sim \sin(\omega t + 30x - 15z)$$

$$\varepsilon = (4 - j0.02)\varepsilon_o$$

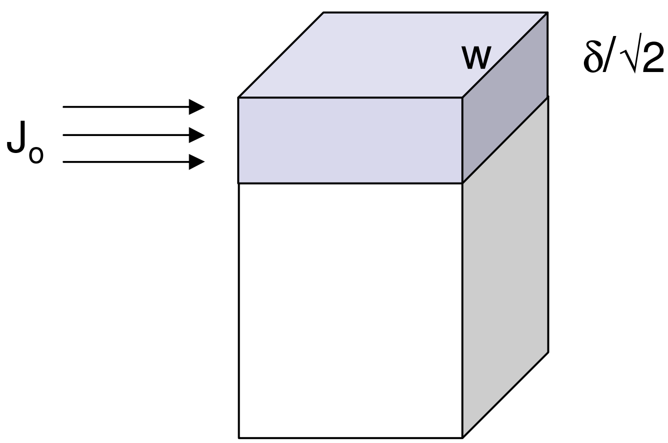
Write $\vec{H}(\vec{r}, t) = ?$

Surface resistance for conductors



$$J = \sigma E = J_o e^{-\alpha z} e^{-j\beta z} = J_o e^{-(1+j)z/\delta}$$

$$I = \int \vec{J} \cdot d\vec{a} = J_o w \int_0^\infty e^{-(1+j)z/\delta} dz = \frac{J_o w \delta}{(1+j)} = \frac{J_o w \delta}{\sqrt{2}} e^{-j\pi/4}$$



$$V = E_o L = \frac{J_o}{\sigma} L$$ on surface

$$Z = \frac{V}{I} = \frac{1+j}{\sigma\delta}\frac{L}{w} \equiv Z_s \frac{L}{w}$$ similar to R & ρ

$$Z_s = \frac{1+j}{\sigma\delta}$$ surface impedance (Ω)

Homework

1. Determine the frequency at which a time-harmonic electric field intensity causes a conduction current density and a displacement current density of equal magnitude in
   (a) seawater with $\varepsilon_r = 72$ and $\sigma = 4$ S/m, and
   (b) moist soil with $\varepsilon_r = 2.5$ and $\sigma = 10^{-3}$ S/m.

2. Calculations concerning the electromagnetic effect of currents in a good conductor usually neglect the displacement current even at microwave frequencies.
   (a) Assuming $\varepsilon_r = 1$ and $\sigma = 5.7 \times 10^7$ S/m for copper, compare the magnitude of the displacement current density with that of the conduction current density at 100 GHz.
   (b) Write the differential equation in phasor form for magnetic field intensity H in a source-free good conductor.