Chapter 17

Temperature and Heat

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Goals for Chapter 17

- To understand thermal equilibrium and what thermometers actually measure

- To learn how thermometers function

- To understand the absolute (Kelvin) temperature scale

- To learn how a temperature change affects the dimensions of an object

- To understand the meaning of heat and how it differs from temperature

- To do calculations involving heat flow, temperature changes, and phase changes

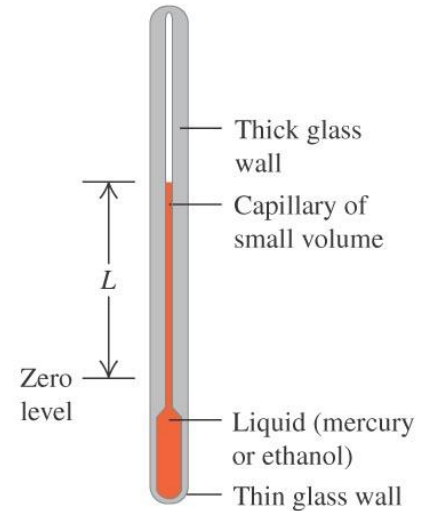- To understand conduction, convection, and radiation of heat

Introduction

- Does molten iron at 1500°C contain heat?

- The terms "temperature" and "heat" have very different meanings, even though most people use them interchangeably.

- In this chapter, we'll focus on large-scale, or *macroscopic*, objects, but in the next chapter we'll look at the *microscopic* scale.
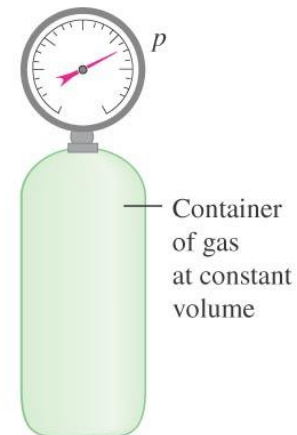
Temperature and thermal equilibrium

- We use a *thermometer* to measure *temperature*. Figure 17.1 at the right shows two types of thermometers.

- Two systems are in *thermal equilibrium* if and only if they have the same temperature.

(a) Changes in temperature cause the liquid's volume to change.

- Thick glass wall
- Capillary of small volume
- L
- Zero level
- Liquid (mercury or ethanol)
- Thin glass wall

(b) Changes in temperature cause the pressure of the gas to change.
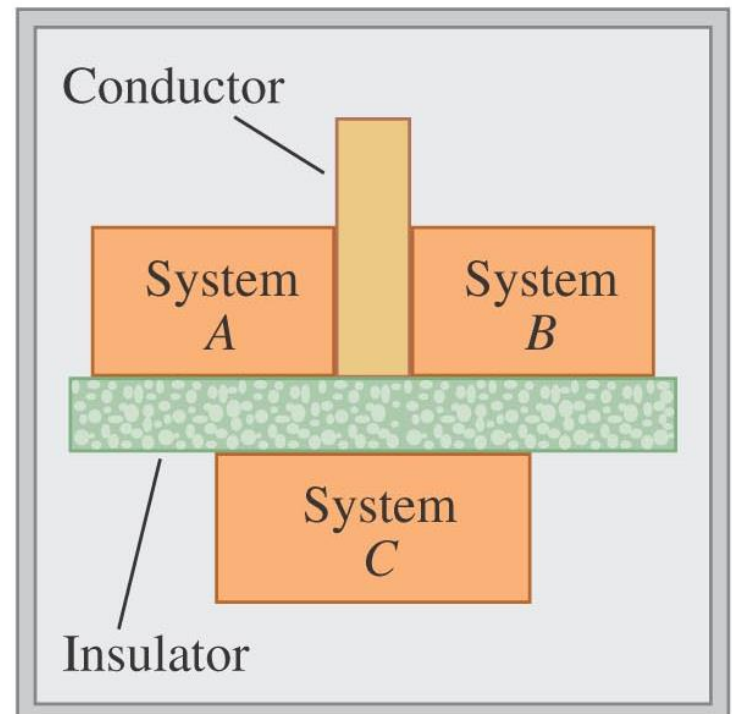
p

- Container of gas at constant volume

The zeroth law of thermodynamics

- *Zeroth law of thermodynamics*: If *C* is initially in thermal equilibrium with both *A* and *B*, then *A* and *B* are in thermal equilibium with each other. (See Figure 17.2 below.)

(a) If systems *A* and *B* are each in thermal equilibrium with system *C* …
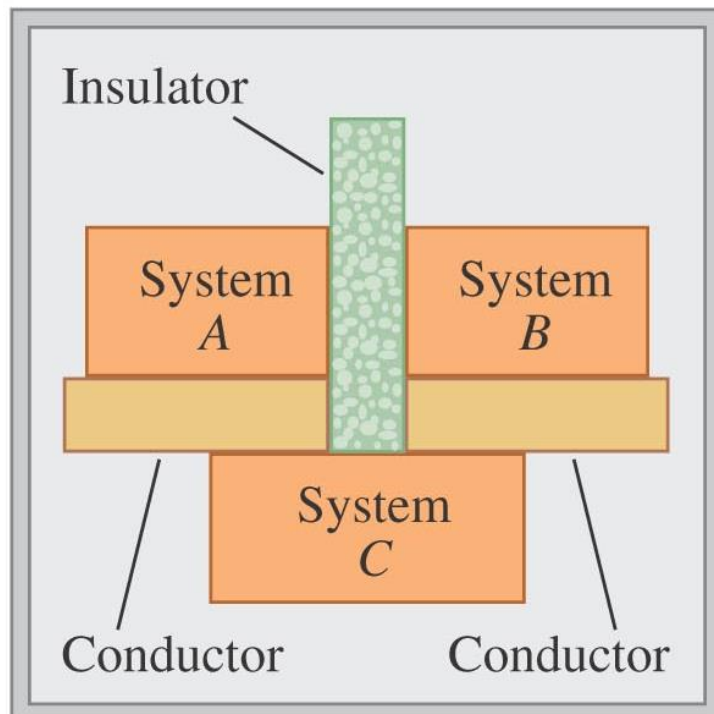
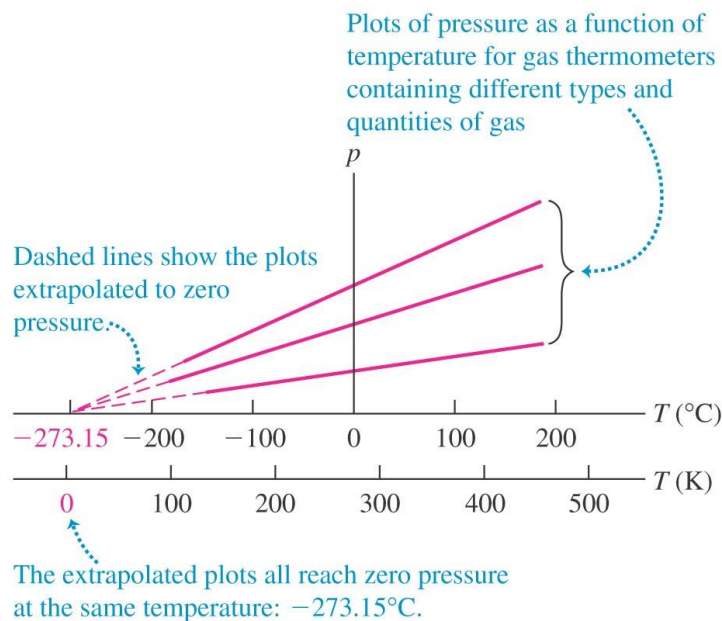(b) … then systems *A* and *B* are in thermal equilibrium with each other.

Temperature scales

- On the *Celsius* (or *centigrade*) *temperature scale*, 0°C is the freezing point of pure water and 100°C is its boiling point.

- On the *Fahrenheit temperature scale*, 32°F is the freezing point of pure water and 212°F is its boiling point.

- On the *Kelvin* (or *absolute*) *temperature scale*, 0 K is the extrapolated temperature at which a gas would exert no pressure. (See Figure 17.5 below.)

(a) A constant-volume gas thermometer

(b) Graphs of pressure versus temperature at constant volume for three different types and quantities of gas

Plots of pressure as a function of temperature for gas thermometers containing different types and quantities of gas

Dashed lines show the plots extrapolated to zero pressure.

The extrapolated plots all reach zero pressure at the same temperature: −273.15°C.

The Kelvin Scale and Absolute Temperature

The Celsius scale has two fixed points: the normal freezing and boiling temperatures of water. But we can define the Kelvin scale using a gas thermometer with only a single reference temperature. We define the ratio of any two temperatures T_1 and T_2 on the Kelvin scale as the ratio of the corresponding gas-thermometer pressures p_1 and p_2:

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \qquad \text{(constant-volume gas thermometer, } T \text{ in kelvins)} \quad (17.4)$$

The pressure p is directly proportional to the Kelvin temperature, as shown in Fig. 17.5b. To complete the definition of T, we need only specify the Kelvin temperature of a single specific state. For reasons of precision and reproducibility, the state chosen is the *triple point* of water. This is the unique combination of temperature and pressure at which solid water (ice), liquid water, and water vapor can all coexist. It occurs at a temperature of 0.01°C and a water-vapor pressure of 610 Pa (about 0.006 atm). (This is the pressure of the *water*; it has nothing to do directly with the gas pressure in the *thermometer*.) The triple-point temperature T_{triple} of water is *defined* to have the value T_{triple} = 273.16 K, corresponding to 0.01°C. From Eq. (17.4), if p_{triple} is the pressure in a gas thermometer at temperature T_{triple} and p is the pressure at some other temperature T, then T is given on the Kelvin scale by

$$T = T_{triple} \frac{p}{p_{triple}} = (273.16 \text{ K}) \frac{p}{p_{triple}} \qquad (17.5)$$

Low-pressure gas thermometers using various gases are found to agree very closely, but they are large, bulky, and very slow to come to thermal equilibrium. They are used principally to establish high-precision standards and to calibrate other thermometers.

Temperature conversions

- The following conversion equations are useful:

$$T_F = 9/5\ T_C + 32° \qquad T_C = 5/9\ (T_F - 32°) \qquad T_K = T_C + 273.15$$

- The chart in Figure 17.7 below shows the relationship between temperature scales, rounded to the nearest degree.

|	K	C	F
Water boils	373	100°	212°
	100 K	100 C°	180 F°
Water freezes	273	0°	32°
CO_2 solidifies	195	−78°	−109°
Oxygen liquefies	90	−183°	−298°
Absolute zero	0	−273°	−460°

Figure 17.7 shows the relationships among the three temperature scales we have discussed. The Kelvin scale is called an **absolute temperature scale**, and its zero point ($T = 0$ K $= -273.15°$C, the temperature at which $p = 0$ in Eq. (17.5)) is called **absolute zero**. At absolute zero a system of molecules (such as a quantity of a gas, a liquid, or a solid) has its *minimum* possible total energy (kinetic plus potential); because of quantum effects, however, it is *not* correct to say that all molecular motion ceases at absolute zero. To define more completely what we mean by absolute zero, we need to use the thermodynamic principles developed in the next several chapters. We will return to this concept in Chapter 20.
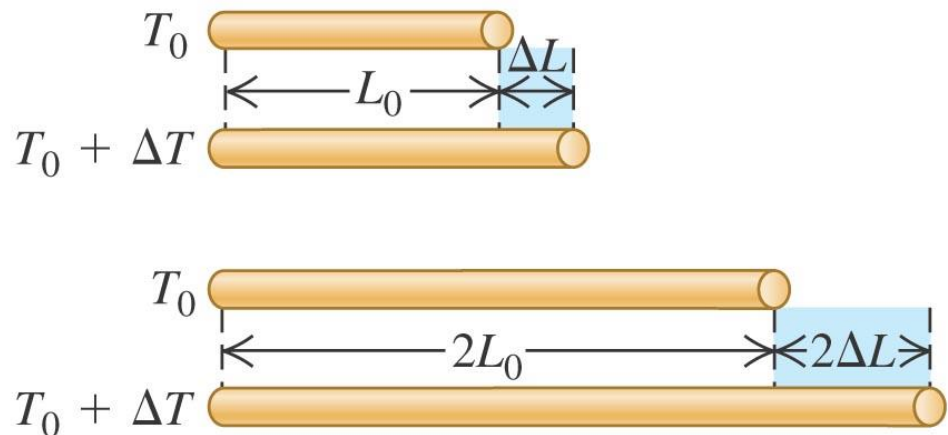
Linear thermal expansion

- Increasing the temp-
erature of a rod causes
it to expand (see Figure
17.8 at the right).

- The change in length is
given by $\Delta L = \alpha L_0 \Delta T$,
where α is the *coeffic-
ient of linear expansion*
of the material.

(a) For moderate temperature changes, ΔL is
directly proportional to ΔT.

T_0

$\longleftarrow L_0 \longrightarrow \Vert \Delta L \Vert$

$T_0 + \Delta T$

$\longleftarrow 2\Delta L \longrightarrow$

$T_0 + 2\Delta T$
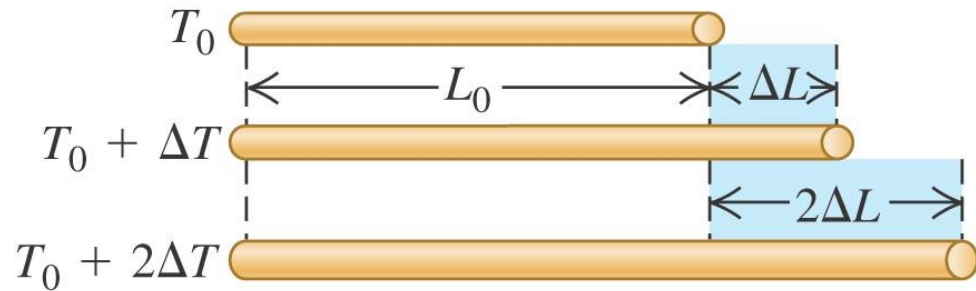
(b) ΔL is also directly proportional to L_0.

T_0

$\longleftarrow L_0 \longrightarrow \Vert \Delta L \Vert$

$T_0 + \Delta T$

T_0

$\longleftarrow 2L_0 \longrightarrow \Vert 2\Delta L \Vert$
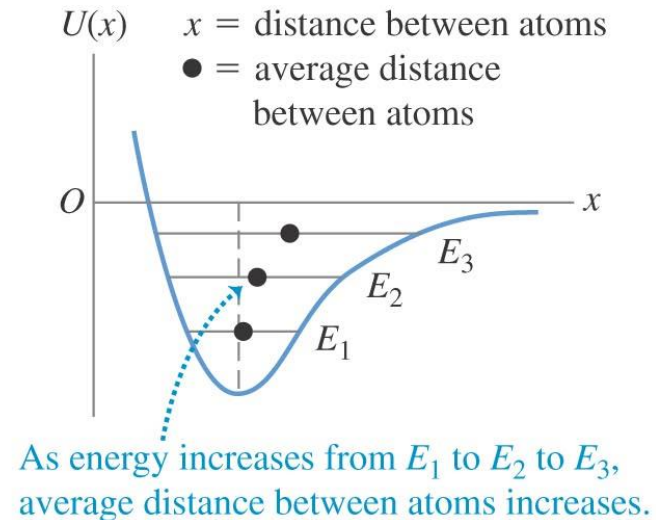
$T_0 + \Delta T$

Molecular basis for thermal expansion

- We can understand linear expansion if we model the atoms as being held together by springs. (See Figure 17.9 below.)

(a) A model of the forces between neighboring atoms in a solid
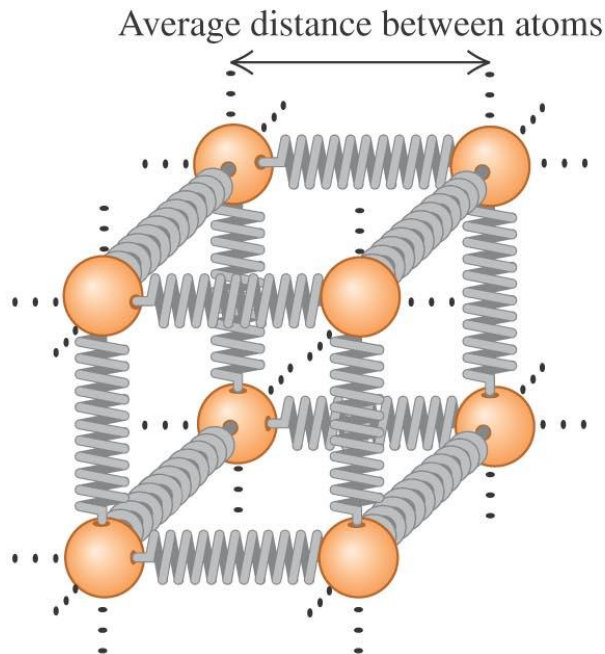
(b) A graph of the "spring" potential energy $U(x)$

Average distance between atoms

$U(x)$ x = distance between atoms
● = average distance between atoms

O x

E_3

E_2

E_1

As energy increases from E_1 to E_2 to E_3, average distance between atoms increases.

Expanding holes and volume expansion

- If an object has a hole in it, the hole also expands with the object, as shown in Figure 17.10 at the right. The hole does *not shrink*.

- The change in volume due to thermal expansion is given by $\Delta V = \beta V_0 \Delta T$, where β is the *coefficient of volume expansion* and is equal to 3α.

COLD

HOT

A plate expands when heated …

… so a hole cut out of the plate must expand, too.
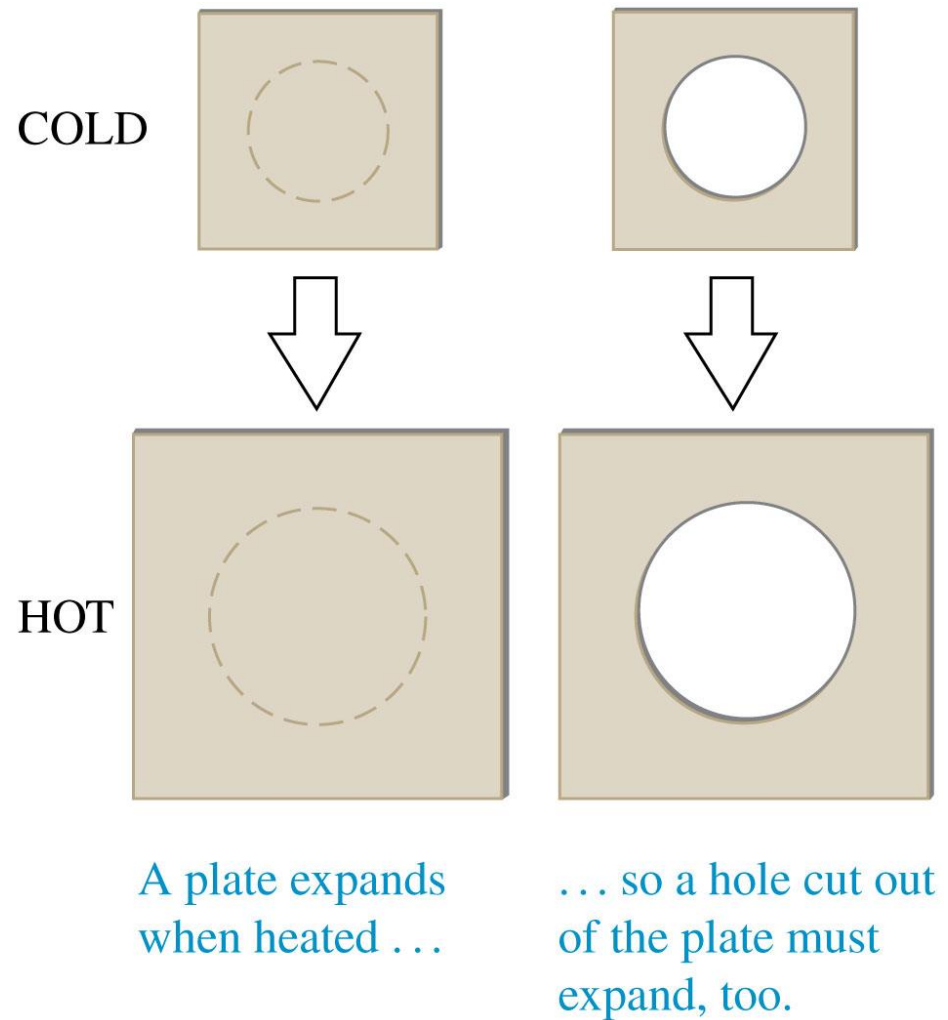
For solid materials there is a simple relationship between the volume expansion coefficient β and the linear expansion coefficient α. To derive this relationship, we consider a cube of material with side length L and volume $V = L^3$. At the initial temperature the values are L_0 and V_0. When the temperature increases by dT, the side length increases by dL and the volume increases by an amount dV given by

$$dV = \frac{dV}{dL} dL = 3L^2 \, dL$$

Now we replace L and V by the initial values L_0 and V_0. From Eq. (17.6), dL is

$$dL = \alpha L_0 \, dT$$

Since $V_0 = L_0^3$, this means that dV can also be expressed as

$$dV = 3L_0^2 \alpha L_0 \, dT = 3\alpha V_0 \, dT$$

This is consistent with the infinitesimal form of Eq. (17.8), $dV = \beta V_0 \, dT$, only if

$$\beta = 3\alpha \qquad (17.9)$$

Coefficients of expansion

- Tables 17.1 and 17.2 show the coefficients of linear and volume expansion for some materials.

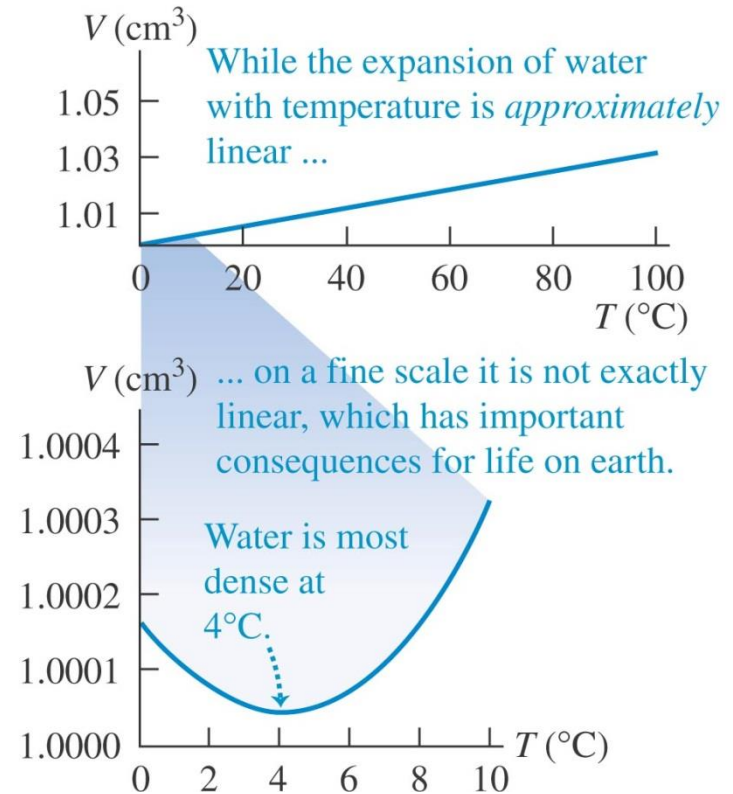**Table 17.1** Coefficients of Linear Expansion

Material	$\alpha \left[\text{K}^{-1} \text{ or } (\text{C}°)^{-1} \right]$
Aluminum	2.4×10^{-5}
Brass	2.0×10^{-5}
Copper	1.7×10^{-5}
Glass	$0.4–0.9 \times 10^{-5}$
Invar (nickel–iron alloy)	0.09×10^{-5}
Quartz (fused)	0.04×10^{-5}
Steel	1.2×10^{-5}

Table 17.2 Coefficients of Volume Expansion

| Solids	$\beta \left[\text{K}^{-1} \text{ or } (\text{C}°)^{-1} \right]$	Liquids	$\beta \left[\text{K}^{-1} \text{ or } (\text{C}°)^{-1} \right]$
Aluminum	7.2×10^{-5}	Ethanol	75×10^{-5}
Brass	6.0×10^{-5}	Carbon disulfide	115×10^{-5}
Copper	5.1×10^{-5}	Glycerin	49×10^{-5}
Glass	$1.2–2.7 \times 10^{-5}$	Mercury	18×10^{-5}
Invar	0.27×10^{-5}		
Quartz (fused)	0.12×10^{-5}		
Steel	3.6×10^{-5}		

Thermal expansion of water

- Between 0°C and 4°C, water *decreases* in volume with increasing temperature.

- Because of this anomalous behavior, lakes freeze from the top down instead of from the bottom up.

- Figure 17.12 at the right illustrates this behavior.

V (cm^3)

While the expansion of water with temperature is *approximately* linear ...

1.05
1.03
1.01

0   20   40   60   80   100
T (°C)

V (cm^3)   ... on a fine scale it is not exactly linear, which has important consequences for life on earth.

1.0004
1.0003
1.0002   Water is most dense at 4°C.
1.0001
1.0000

0   2   4   6   8   10 T (°C)

Thermal stress

- If we change the temperature of a rod but prevent it from expanding or contracting, *thermal stress* develops.

- The thermal stress is $F/A = -Y\alpha\Delta T$.

- Figure 17.13 at the right shows expansion joints on a bridge needed to prevent thermal stress.

- Follow Example 17.4 using Figure 17.14 below.

$A = 20\,cm^2$

Y

$10\,cm$
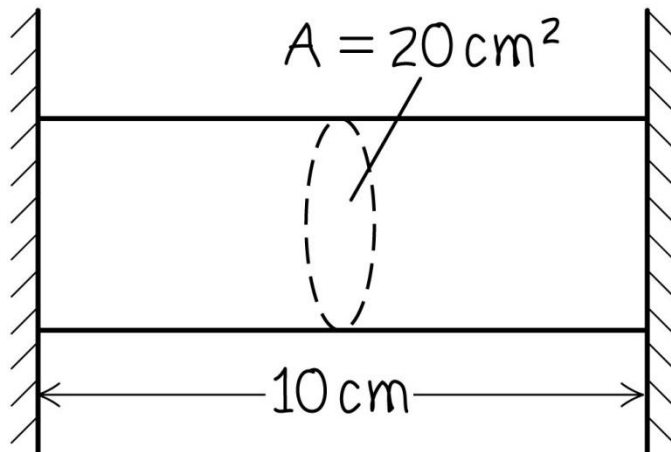
To calculate the thermal stress in a clamped rod, we compute the amount the rod *would* expand (or contract) if not held and then find the stress needed to compress (or stretch) it back to its original length. Suppose that a rod with length L_0 and cross-sectional area A is held at constant length while the temperature is reduced (negative ΔT), causing a tensile stress. The fractional change in length if the rod were free to contract would be

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} = \alpha \, \Delta T \qquad (17.10)$$

Both ΔL and ΔT are negative. The tension must increase by an amount F that is just enough to produce an equal and opposite fractional change in length $(\Delta L/L_0)_{\text{tension}}$. From the definition of Young's modulus, Eq. (11.10),

$$Y = \frac{F/A}{\Delta L/L_0} \qquad \text{so} \qquad \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \frac{F}{AY} \qquad (17.11)$$

If the length is to be constant, the *total* fractional change in length must be zero. From Eqs. (17.10) and (17.11), this means that

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} + \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \alpha\,\Delta T + \frac{F}{AY} = 0$$

Solving for the tensile stress F/A required to keep the rod's length constant, we find

$$\frac{F}{A} = -Y\alpha\,\Delta T \qquad \text{(thermal stress)} \qquad (17.12)$$

For a decrease in temperature, ΔT is negative, so F and F/A are positive; this means that a *tensile* force and stress are needed to maintain the length. If ΔT is positive, F and F/A are negative, and the required force and stress are *compressive*.

If there are temperature differences within a body, nonuniform expansion or contraction will result and thermal stresses can be induced. You can break a glass bowl by pouring very hot water into it; the thermal stress between the hot and cold parts of the bowl exceeds the breaking stress of the glass, causing cracks. The same phenomenon makes ice cubes crack when dropped into warm water. Heat-resistant glasses such as Pyrex™ have exceptionally low expansion coefficients and high strength.

An aluminum cylinder 10 cm long, with a cross-sectional area of 20 cm^2, is used as a spacer between two steel walls. At 17.2°C it just slips between the walls. Calculate the stress in the cylinder and the total force it exerts on each wall when it warms to 22.3°C, assuming that the walls are perfectly rigid and a constant distance apart.

$Y_{Al} = 7.0 * 10^{10}$ Pa $\alpha_{Al} = 2.4 * 10^{-5}$ K^{-1}

$$\frac{F}{A} = -Y_{Al}\alpha_{Al}\Delta T$$

$$= -(7.0 \times 10^{10} \text{ Pa})(2.4 \times 10^{-5} \text{ K}^{-1})(5.1 \text{ K})$$

$$= -8.6 \times 10^6 \text{ Pa} = -1200 \text{ lb/in.}^2$$

The total force is the cross-sectional area times the stress:

$$F = A\left(\frac{F}{A}\right) = (20 \times 10^{-4} \text{ m}^2)(-8.6 \times 10^6 \text{ Pa})$$

$$= -1.7 \times 10^4 \text{ N} = 1.9 \text{ tons}$$

1 pascal is equal to   0.000145037738007 pound/square inch

1 newton is equal to 0.000101971621298 ton-force

Energy transfer that takes place solely because of a temperature difference is called *heat flow* or *heat transfer,* and energy transferred in this way is called **heat.**

In physics the term "heat" always refers to energy in transit from one body or system to another because of a temperature difference, never to the amount of energy contained within a particular system.
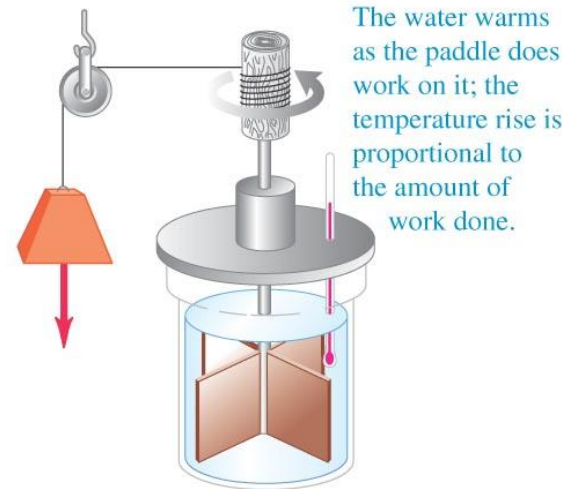
A **temperature** is a numerical measure of hot or cold. Its measurement is by detection of heat radiation or particle velocity or kinetic energy, or by the bulk behavior of a thermometric material

A convenient operational definition of temperature is that it is a measure of the average translational kinetic energy associated with the disordered microscopic motion of atoms and molecules. Definition involving thermodynamics will be discussed later.
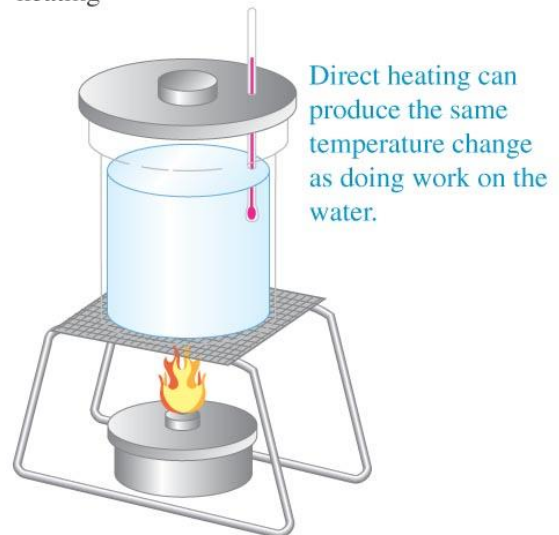
Quantity of heat

- Figure 17.15 at the right illustrates that we can change the temperature of a body by doing work on it or by adding heat to it. This means that heat is a form of energy.

- The heat to cause a temperature change is $Q = mc\Delta T$, where c is the *specific heat* of the material.

- Follow Example 17.5.

- Follow Example 17.6.

(a) Raising the temperature of water by doing work on it

The water warms as the paddle does work on it; the temperature rise is proportional to the amount of work done.

(b) Raising the temperature of water by direct heating

Direct heating can produce the same temperature change as doing work on the water.

We can define a *unit* of quantity of heat based on temperature changes of some specific material. The calorie (abbreviated cal) is defined as *the amount of heat required to raise the temperature of 1 gram of water from 14.5°C to 15.5°C*. The kilocalorie (kcal), equal to 1000 cal, is also used; a food-value calorie is actually a kilocalorie (Fig. 17.16). A corresponding unit of heat using Fahrenheit degrees and British units is the **British thermal unit**, or Btu. One Btu is the quantity of heat required to raise the temperature of 1 pound (weight) of water 1 F° from 63°F to 64°F.

Because heat is energy in transit, there must be a definite relationship between these units and the familiar mechanical energy units such as the joule. Experiments similar in concept to Joule's have shown that

$$1 \text{ cal} = 4.186 \text{ J}$$

$$1 \text{ kcal} = 1000 \text{ cal} = 4186 \text{ J}$$

$$1 \text{ Btu} = 778 \text{ ft} \cdot \text{lb} = 252 \text{ cal} = 1055 \text{ J}$$

The calorie is not a fundamental SI unit. The International Committee on Weights and Measures recommends using the joule as the basic unit of energy in all forms, including heat. We will follow that recommendation in this book.

$$Q = mc \, \Delta T \quad \text{(heat required for temperature change } \Delta T \text{ of mass } m) \quad \text{(17.13)}$$
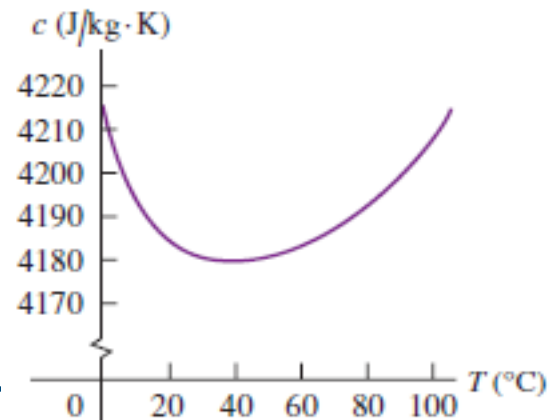
where c is a quantity, different for different materials, called the **specific heat** of the material. For an infinitesimal temperature change dT and corresponding quantity of heat dQ,

$$dQ = mc \, dT \qquad \text{(17.14)}$$

$$c = \frac{1}{m} \frac{dQ}{dT} \qquad \text{(specific heat)} \qquad \text{(17.15)}$$

During a bout with the flu an 80-kg man ran a fever of 39.0°C (102.2°F) instead of the normal body temperature of 37.0°C (98.6°F). Assuming that the human body is mostly water, how much heat is required to raise his temperature by that amount?

$$Q = mc\,\Delta T = (80\text{ kg})(4190\text{ J/kg}\cdot\text{K})(2.0\text{ K}) = 6.7 \times 10^5\text{ J}$$

17.17 Specific heat of water as a function of temperature. The value of c varies by less than 1% between 0°C and 100°C.

$$m = nM \qquad (17.16)$$

Replacing the mass m in Eq. (17.13) by the product nM, we find

$$Q = nMc\,\Delta T \qquad (17.17)$$

The product Mc is called the **molar heat capacity** (or *molar specific heat*) and is denoted by C (capitalized). With this notation we rewrite Eq. (17.17) as

$$Q = nC\,\Delta T \qquad \text{(heat required for temperature change of } n \text{ moles)} \qquad (17.18)$$

$$C = \frac{1}{n}\frac{dQ}{dT} = Mc \qquad \text{(molar heat capacity)} \qquad (17.19)$$

For example, the molar heat capacity of water is

$$C = Mc = (0.0180 \text{ kg/mol})(4190 \text{ J/kg} \cdot \text{K}) = 75.4 \text{ J/mol} \cdot \text{K}$$

Table 17.3 Approximate Specific Heats and Molar Heat Capacities (Constant Pressure)

Substance	Specific Heat, c (J/kg · K)	Molar Mass, M (kg/mol)	Molar Heat Capacity, C (J/mol · K)
Aluminum	910	0.0270	24.6
Beryllium	1970	0.00901	17.7
Copper	390	0.0635	24.8
Ethanol	2428	0.0461	111.9
Ethylene glycol	2386	0.0620	148.0
Ice (near 0°C)	2100	0.0180	37.8
Iron	470	0.0559	26.3
Lead	130	0.207	26.9
Marble ($CaCO_3$)	879	0.100	87.9
Mercury	138	0.201	27.7
Salt (NaCl)	879	0.0585	51.4
Silver	234	0.108	25.3
Water (liquid)	4190	0.0180	75.4

The last column of Table 17.3 shows something interesting. The molar heat capacities for most elemental solids are about the same: about 25 J/mol · K. This correlation, named the *rule of Dulong and Petit* (for its discoverers), forms the basis for a very important idea. The number of atoms in 1 mole is the same for all elemental substances. This means that on a *per atom* basis, about the same amount of heat is required to raise the temperature of each of these elements by a given amount, even though the *masses* of the atoms are very different. The heat required for a given temperature increase depends only on *how many* atoms the sample contains, not on the mass of an individual atom. We will see the reason the rule of Dulong and Petit works so well when we study the molecular basis of heat capacities in greater detail in Chapter 18.

c_p and C_p are specific heat at constant pressure – usually for solids.

c_v and C_v are specific heat at constant volume – usually for gas.

C_p and C_v are quite different for gases.

When heat is added to a system, what happens to its temperature? The temperature rises. Is there any situation when heat is added but there is no temperature change?

Heat is also involved in *phase changes,* such as the melting of ice or boiling of water when there is no change in temperature! What happened to the heat?

A familiar example of a phase change is the melting of ice. When we add heat to ice at and normal atmospheric pressure, the temperature of the ice *does not* increase. Instead, some of it melts to form liquid water. If we add the heat slowly, to maintain the system very close to thermal equilibrium, the temperature remains at until all the ice is melted. The effect of adding heat to this system is not to raise its temperature but to change its *phase* from solid to liquid.

More generally, to melt a mass m of material that has a heat of fusion L_f requires a quantity of heat Q given by

$$Q = mL_f$$

This process is *reversible*. To freeze liquid water to ice at 0°C, we have to *remove* heat; the magnitude is the same, but in this case, Q is negative because heat is removed rather than added. To cover both possibilities and to include other kinds of phase changes, we write

$$Q = \pm mL \qquad \text{(heat transfer in a phase change)} \qquad (17.20)$$

The plus sign (heat entering) is used when the material melts; the minus sign (heat leaving) is used when it freezes. The heat of fusion is different for different materials, and it also varies somewhat with pressure.

For any given material at any given pressure, the freezing temperature is the same as the melting temperature. At this unique temperature the liquid and solid phases (liquid water and ice, for example) can coexist in a condition called **phase equilibrium.**

Like melting, boiling is a reversible transition. When heat is removed from a gas at the boiling temperature, the gas returns to the liquid phase, or *condenses* giving up to its surroundings the same quantity of heat (heat of vaporization) that was needed to vaporize it. At a given pressure the boiling and condensation temperatures are always the same; at this temperature the liquid and gaseous phases can coexist in phase equilibrium.

A substance can sometimes change directly from the solid to the gaseous phase. This process is called *sublimation*, and the solid is said to *sublime*. The corresponding heat is called the *heat of sublimation*, L_s. Liquid carbon dioxide cannot exist at a pressure lower than about 5×10^5 Pa (about 5 atm), and "dry ice" (solid carbon dioxide) sublimes at atmospheric pressure. Sublimation of water from frozen food causes freezer burn. The reverse process, a phase change from gas to solid, occurs when frost forms on cold bodies such as refrigerator cooling coils.

Very pure water can be cooled several degrees below the freezing temperature without freezing; the resulting unstable state is described as *supercooled.* When a small ice crystal is dropped in or the water is agitated, it crystallizes within a second or less.

Supercooled water *vapor* condenses quickly into fog droplets when a disturbance, such as dust particles or ionizing radiation, is introduced. This principle is used in "seeding" clouds, which often contain supercooled water vapor, to cause condensation and rain.

Phase changes

- The *phases* (or states) of matter are solid, liquid, and gas.

- A *phase change* is a transition from one phase to another.

- The temperature does not change during a phase change (see Figure 17.19 at the right).

- The *heat of fusion*, L_f, is the heat per unit mass that is transferred in a solid-liquid phase change. The *heat of vaporization*, L_v, is the heat per unit mass transferred in a liquid-gas phase change.

- The heat transferred in a phase change is $Q = \pm mL$.

- Table 17.4 on the next slide shows some examples of heats of fusion and vaporization.
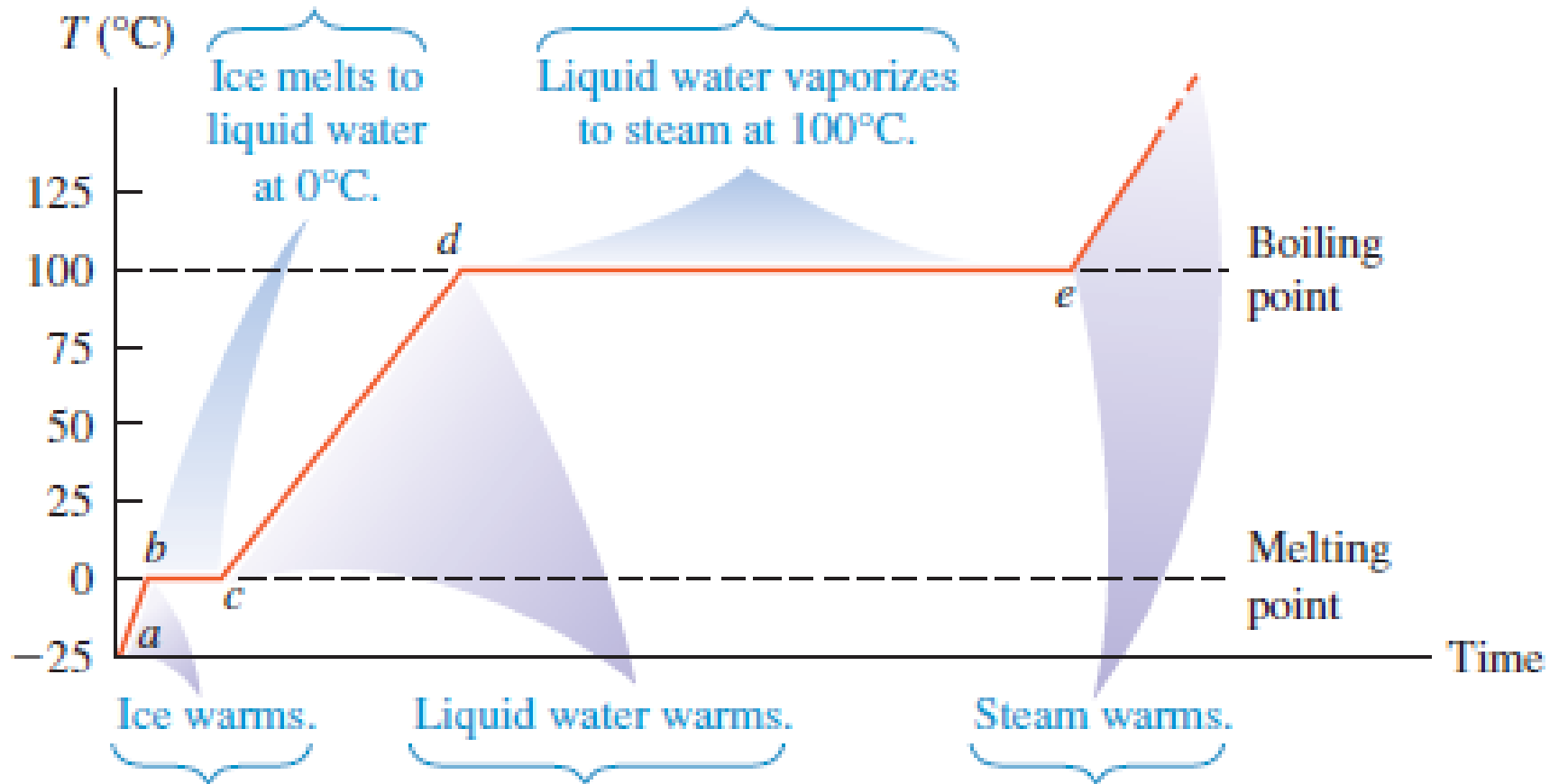
Heats of Fusion and Heats of Vaporization

Table 17.4 Heats of Fusion and Vaporization

Substance	Normal Melting Point		Heat of Fusion, L_f (J/kg)	Normal Boiling Point		Heat of Vaporization, L_v (J/kg)
	K	°C		K	°C	
Helium	*	*	*	4.216	−268.93	20.9×10^3
Hydrogen	13.84	−259.31	58.6×10^3	20.26	−252.89	452×10^3
Nitrogen	63.18	−209.97	25.5×10^3	77.34	−195.8	201×10^3
Oxygen	54.36	−218.79	13.8×10^3	90.18	−183.0	213×10^3
Ethanol	159	−114	104.2×10^3	351	78	854×10^3
Mercury	234	−39	11.8×10^3	630	357	272×10^3
Water	273.15	0.00	334×10^3	373.15	100.00	2256×10^3
Sulfur	392	119	38.1×10^3	717.75	444.60	326×10^3
Lead	600.5	327.3	24.5×10^3	2023	1750	871×10^3
Antimony	903.65	630.50	165×10^3	1713	1440	561×10^3
Silver	1233.95	960.80	88.3×10^3	2466	2193	2336×10^3
Gold	1336.15	1063.00	64.5×10^3	2933	2660	1578×10^3
Copper	1356	1083	134×10^3	1460	1187	5069×10^3

*A pressure in excess of 25 atmospheres is required to make helium solidify. At 1 atmosphere pressure, helium remains a liquid down to absolute zero.

Phase of water changes. During these periods, temperature stays constant and the phase change proceeds as heat is added: $Q = +mL$.

T (°C)

Ice melts to liquid water at 0°C.

Liquid water vaporizes to steam at 100°C.

125

100

75

50

25

0

−25

d

e

b

c

a

Boiling point

Melting point

Time

Ice warms.

Liquid water warms.

Steam warms.

Temperature of water changes. During these periods, temperature rises as heat is added: $Q = mc\,\Delta T$.

A camper pours 0.300 kg of coffee, initially in a pot at 70.0°C, into a 0.120-kg aluminum cup initially at 20.0°C. What is the equilibrium temperature? Assume that coffee has the same specific heat as water and that no heat is exchanged with the surroundings.

With subscripts C for coffee, W for water, and Al for aluminum, we have $T_{0C} = 70.0°$ and $T_{0Al} = 20.0°$; $c_W = 4190 \text{ J/kg} \cdot \text{K}$ and $c_{Al} = 910 \text{ J/kg} \cdot \text{K}$.

$$Q_C + Q_{Al} = m_C c_W \Delta T_C + m_{Al} c_{Al} \Delta T_{Al} = 0$$

$$m_C c_W (T - T_{0C}) + m_{Al} c_{Al} (T - T_{0Al}) = 0$$

Then we solve this expression for the final temperature T. A little algebra gives

$$T = \frac{m_C c_W T_{0C} + m_{Al} c_{Al} T_{0Al}}{m_C c_W + m_{Al} c_{Al}} = 66.0°C$$

$$T = \frac{0.300\,kg \times 4190\,\frac{J}{kg\,k} \times 70.0C + 0.120\,kg \times 910\,\frac{J}{kg\,k} \times 20.0\,°C}{0.300\,kg \times 4190\,\frac{J}{kg\,k} + 0.120\,kg \times 910\,\frac{J}{kg\,k}}$$

$$= 90174 / 1366.2 \doteq 66.0\,°C$$

A glass contains 0.25 kg of Omni-Cola (mostly water) initially at 25°C. How much ice, initially at −20°C, must you add to obtain a final temperature of 0°C with all the ice melted? Neglect the heat capacity of the glass.

$T = 25°C$ and $T_{0I} = -20°C$, Table 17.3 gives $c_W = 4190$ J/kg · K an $c_I = 2100$ J/kg · K, and Table 17.4 gives $L_f = 3.34 \times 10^5$ J/kg.

EXECUTE: From Eq. (17.13), the (negative) heat gained by the Omni-Cola is $Q_C = m_C c_W \Delta T_C$. The (positive) heat gained by the ice in warming is $Q_1 = m_1 c_1 \Delta T_1$. The (positive) heat required to melt the ice is $Q_2 = m_1 L_f$. We set $Q_C + Q_1 + Q_2 = 0$, insert $\Delta T_C = T - T_{0C}$ and $\Delta T_1 = T - T_{01}$, and solve for m_1:

$$m_C c_W \Delta T_C + m_1 c_1 \Delta T_1 + m_1 L_f = 0$$

$$m_C c_W (T - T_{0C}) + m_1 c_1 (T - T_{01}) + m_1 L_f = 0$$

$$m_1 [c_1 (T - T_{01}) + L_f] = -m_C c_W (T - T_{0C})$$

$$m_1 = m_C \frac{c_W (T_{0C} - T)}{c_1 (T - T_{01}) + L_f}$$

Substituting numerical values, we find that $m_1 = 0.070 \text{ kg} = 70 \text{ g}$.

$$m_1 = 4190 \frac{J}{kg} (25 \pm 0)\,c \,/\, (2100\frac{J}{kg\cdot K}(0+20c) + 3.34 \times 10^{5}) = 70g$$

The three mechanisms of heat transfer are conduction, convection, and radiation. *Conduction* occurs within a body or between two bodies in contact. *Convection* depends on motion of mass from one region of space to another. *Radiation* is heat transfer by electromagnetic radiation, such as sunshine, with no need for matter to be present in the space between bodies.

When a quantity of heat dQ is transferred through the rod in a time dt, the rate of heat flow is dQ/dt. We call this rate the **heat current**, denoted by H. That is, $H = dQ/dt$. Experiments show that the heat current is proportional to the cross-sectional area A of the rod (Fig. 17.23b) and to the temperature difference $(T_H - T_C)$ and is inversely proportional to the rod length L (Fig. 17.23c). Introducing a proportionality constant k called the **thermal conductivity** of the material, we have

$$H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L} \qquad \text{(heat current in conduction)} \qquad (17.21)$$

The quantity $(T_H - T_C)/L$ is the temperature difference *per unit length*; it is called the magnitude of the **temperature gradient**. The numerical value of k depends on the material of the rod. Materials with large k are good conductors of heat; materials with small k are poor conductors, or insulators. Equation (17.21) also gives the heat current through a slab or through *any* homogeneous body with uniform cross section A perpendicular to the direction of flow; L is the length of the heat-flow path.

The units of heat current H are units of energy per time, or power; the SI unit of heat current is the watt (1 W = 1 J/s). We can find the units of k by solving Eq. (17.21) for k; you can show that the SI units are W/m·K. Some numerical values of k are given in Table 17.5.

If the temperature varies in a nonuniform way along the length of the conducting rod, we introduce a coordinate x along the length and generalize the temperature gradient to be dT/dx. The corresponding generalization of Eq. (17.21) is

$$H = \frac{dQ}{dt} = -kA\frac{dT}{dx} \qquad (17.22)$$

The negative sign shows that heat always flows in the direction of *decreasing* temperature.

For thermal insulation in buildings, engineers use the concept of thermal resistance, denoted by R. The thermal resistance R of a slab of material with area A is defined so that the heat current H through the slab is

$$H = \frac{A(T_H - T_C)}{R} \qquad (17.23)$$

where T_H and T_C are the temperatures on the two sides of the slab. Comparing this with Eq. (17.21), we see that R is given by

$$R = \frac{L}{k} \qquad (17.24)$$

where L is the thickness of the slab. The SI unit of R is $1 \text{ m}^2 \cdot \text{K/W}$. In the units used for commercial insulating materials in the United States, H is expressed in Btu/h, A is in ft^2, and $T_H - T_C$ in $\text{F}°$. ($1 \text{ Btu/h} = 0.293 \text{ W}$.) The units of R are then $\text{ft}^2 \cdot \text{F}° \cdot \text{h/Btu}$, though values of R are usually quoted without units; a 6-inch-thick layer of fiberglass has an R value of 19 (that is, $R = 19 \text{ ft}^2 \cdot \text{F}° \cdot \text{h/Btu}$), a 2-inch-thick slab of polyurethane foam has an R value of 12, and so on. Doubling the thickness doubles the R value. Common practice in new construction in severe northern climates is to specify R values of around 30 for exterior walls and ceilings. When the insulating material is in layers, such as a plastered wall, fiberglass insulation, and wood exterior siding, the R values are additive. Do you see why? (See Problem 17.108.)

A steel bar 10.0 cm long is welded end to end to a copper bar 20.0 cm long. Each bar has a square cross section, 2.00 cm on a side. The free end of the steel bar is kept at 100°C by placing it in contact with steam, and the free end of the copper bar is kept at 0°C by placing it in contact with ice. Both bars are perfectly insulated on their sides. Find the steady-state temperature at the junction of the two bars and the total rate of heat flow through the bars.
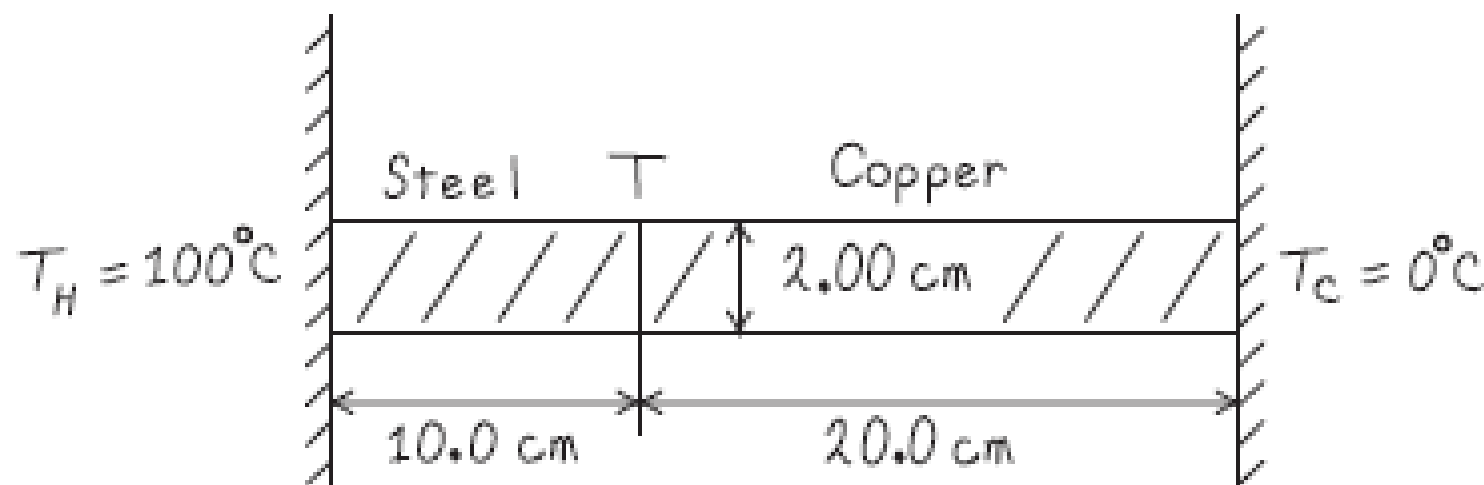
IDENTIFY and SET UP: Figure 17.25 shows the situation. The heat currents in these end-to-end bars must be the same (see Problem-Solving Strategy 17.3). We are given "hot" and "cold" temperatures $T_H = 100°C$ and $T_C = 0°C$. With subscripts S for steel and Cu for copper, we write Eq. (17.21) separately for the heat currents H_S and H_{Cu} and set the resulting expressions equal to each other.

Table 17.5 Thermal Conductivities

Substance	k (W/m · K)
Metals	
Aluminum	205.0
Brass	109.0
Copper	385.0
Lead	34.7
Mercury	8.3
Silver	406.0
Steel	50.2

EXECUTE: Setting $H_S = H_{Cu}$, we have from Eq. (17.21)

$$H_S = k_S A \frac{T_H - T}{L_S} = H_{Cu} = k_{Cu} A \frac{T - T_C}{L_{Cu}}$$

$$\frac{k_s T_H}{L_s} - \frac{k_s T}{L_s} = \frac{k_{cu} T}{L_{cu}} - \frac{k_{cu} T_C}{L_{cu}}$$

$$\therefore \left(\frac{k_{Cu}}{L_{cu}} + \frac{k_s}{L_s} \right) T = \frac{k_s}{L_s} T_H + \frac{k_{cu} T_C}{L_{cu}}$$

$$\therefore T = \left(\frac{k_s}{L_s} T_H + \frac{k_{cu}}{L_{cu}} T_C \right) / \left(\frac{k_{cu}}{L_{cu}} + \frac{k_s}{L_s} \right)$$

$L_S = 10.0 \, cm \qquad L_{cu} = 20.0 \, cm \qquad T_H = 100°C \quad T_c = 0°C$

$k_S = 50.2 \, \frac{W}{m \cdot k} \, , \qquad R_{cu} = 385.0 \, \frac{W}{m \cdot k}$

$T = ?$

$$T = \frac{\left(\dfrac{50.2 \, \frac{W}{m \cdot k}}{10.0 \, cm} \cdot 100°C + \dfrac{385.0 \, \frac{W}{m \cdot k}}{20.0 \, cm} \cdot 0°C \right)}{\left(\dfrac{50.2 \, W/m \cdot k}{10.0 \, cm} + \dfrac{385.0 \, W/m k}{20.0 \, m} \right)}$$

$= \dfrac{(502 + 0)}{24.27} = 20.68 \sim 20.7°C$

$$H_S = (50.2 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2 \frac{100°\text{C} - 20.7°\text{C}}{0.100 \text{ m}}$$

$$= 15.9 \text{ W}$$

$$H_{Cu} = (385 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2 \frac{20.7°\text{C}}{0.200 \text{ m}} = 15.9 \text{ W}$$

Convective heat transfer is a very complex process, and there is no simple equation to describe it. Here are a few experimental facts:

1. The heat current due to convection is directly proportional to the surface area. This is the reason for the large surface areas of radiators and cooling fins.

2. The viscosity of fluids slows natural convection near a stationary surface, giving a surface film that on a vertical surface typically has about the same insulating value as 1.3 cm of plywood (R value $= 0.7$). Forced convection decreases the thickness of this film, increasing the rate of heat transfer. This is the reason for the "wind-chill factor"; you get cold faster in a cold wind than in still air with the same temperature.

3. The heat current due to convection is found to be approximately proportional to the $\frac{5}{4}$ power of the temperature difference between the surface and the main body of fluid.

Radiation is the transfer of heat by electromagnetic waves such as visible light, infrared, and ultraviolet radiation. Everyone has felt the warmth of the sun's radiation and the intense heat from a charcoal grill or the glowing coals in a fireplace. Most of the heat from these very hot bodies reaches you not by conduction or convection in the intervening air but by *radiation.* This heat transfer would occur even if there were nothing but vacuum between you and the source of heat.

Every body, even at ordinary temperatures, emits energy in the form of electromagnetic radiation. Around 20^0C nearly all the energy is carried by infrared waves with wavelengths much longer than those of visible light. As the temperature rises, the wavelengths shift to shorter values. At 800^0C, a body emits enough visible radiation to appear "red-hot," although even at this temperature most of the energy is carried by infrared waves. At 3000°C, the temperature of an incandescent lamp filament, the radiation contains enough visible light that the body appears "white-hot."

The rate of energy radiation from a surface is proportional to the surface area A and to the fourth power of the absolute (Kelvin) temperature T. The rate also depends on the nature of the surface; this dependence is described by a quantity e called the **emissivity**. A dimensionless number between 0 and 1, e represents the ratio of the rate of radiation from a particular surface to the rate of radiation from an equal area of an ideal radiating surface at the same temperature. Emissivity also depends somewhat on temperature. Thus the heat current $H = dQ/dt$ due to radiation from a surface area A with emissivity e at absolute temperature T can be expressed as

$$H = Ae\sigma T^4 \qquad \text{(heat current in radiation)} \qquad (17.25)$$

where σ is a fundamental physical constant called the **Stefan–Boltzmann** constant. This relationship is called the **Stefan–Boltzmann law** in honor of its late-19th-century discoverers. The current best numerical value of σ is

$$\sigma = 5.670400(40) \times 10^{-8} \, \text{W/m}^2 \cdot \text{K}^4$$

We invite you to check unit consistency in Eq. (17.25). Emissivity (e) is often larger for dark surfaces than for light ones. The emissivity of a smooth copper surface is about 0.3, but e for a dull black surface can be close to unity.

Radiation and Absorption

While a body at absolute temperature T is radiating, its surroundings at temperature T_s are also radiating, and the body *absorbs* some of this radiation. If it is in thermal equilibrium with its surroundings, $T = T_s$ and the rates of radiation and absorption must be equal. For this to be true, the rate of absorption must be given in general by $H = Ae\sigma T_s^4$. Then the *net* rate of radiation from a body at temperature T with surroundings at temperature T_s is

$$H_{net} = Ae\sigma T^4 - Ae\sigma T_s^4 = Ae\sigma(T^4 - T_s^4) \qquad (17.26)$$

In this equation a positive value of H means a net heat flow *out of* the body. Equation (17.26) shows that for radiation, as for conduction and convection, the heat current depends on the temperature *difference* between two bodies.

A body that is a good absorber must also be a good emitter. An ideal radiator, with an emissivity of unity, is also an ideal absorber, absorbing *all* of the radiation that strikes it. Such an ideal surface is called an ideal black body or simply a **blackbody.** Conversely, an ideal *reflector,* which absorbs *no* radiation at all, is also a very ineffective radiator

This is the reason for the silver coatings on vacuum ("Thermos") bottles, invented by Sir James Dewar (1842–1923). A vacuum bottle has double glass walls. The air is pumped out of the spaces between the walls; this eliminates nearly all heat transfer by conduction and convection. The silver coating on the walls reflects most of the radiation from the contents back into the container, and the wall itself is a very poor emitter. Thus a vacuum bottle can keep coffee or soup hot for several hours. The Dewar flask, used to store very cold liquefied gases, is exactly the same in principle.
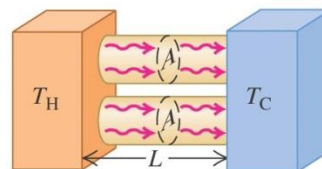
Conduction of heat

- *Conduction* occurs between bodies in contact.

- Figure 17.23 at the right illustrates steady-state heat flow.

- The *heat current* is $H = dQ/dt = kA(T_H - T_C)/L$.
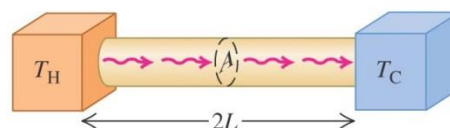
- Table 17.5 lists some thermal conductivities, k.

(a) Heat current H
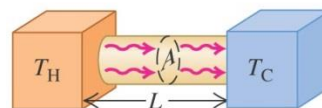
(b) Doubling the cross-sectional area of the conductor doubles the heat current (H is proportional to A).

(c) Doubling the length of the conductor halves the heat current (H is inversely proportional to L).

Table 17.5 Thermal Conductivities

| Substance	$k\,(\mathrm{W/m \cdot K})$
Metals	
Aluminum	205.0
Brass	109.0
Copper	385.0
Lead	34.7
Mercury	8.3
Silver	406.0
Steel	50.2
Solids (representative values)	
Brick, insulating	0.15
Brick, red	0.6
Concret	0.8
Cork	0.04
Felt	0.04
Fiberglass	0.04
Glass	0.8
Ice	1.6
Rock wool	0.04
Styrofoam	0.01
Wood	0.12–0.04
Gases	
Air	0.024
Argon	0.016
Helium	0.14
Hydrogen	0.14
Oxygen	0.023

Conduction into a picnic cooler

- Consider Problem-Solving Strategy 17.3.

- Follow Example 17.11 which involves a Styrofoam cooler. Use Figure 17.24 at right.
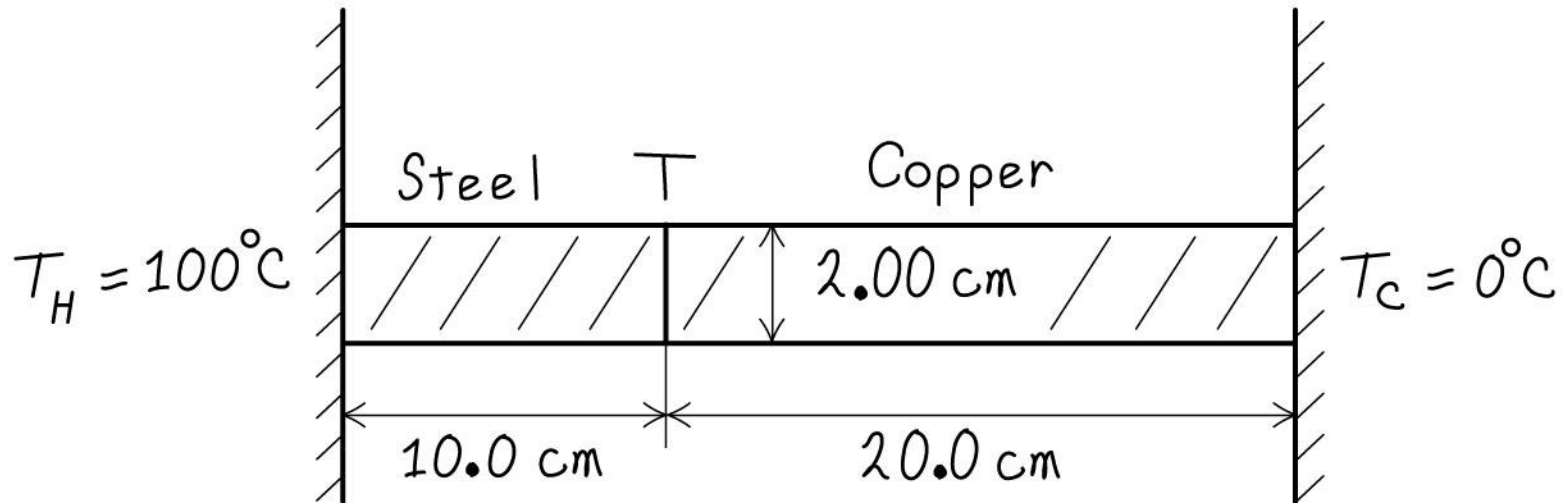
(a) A cooler at the beach
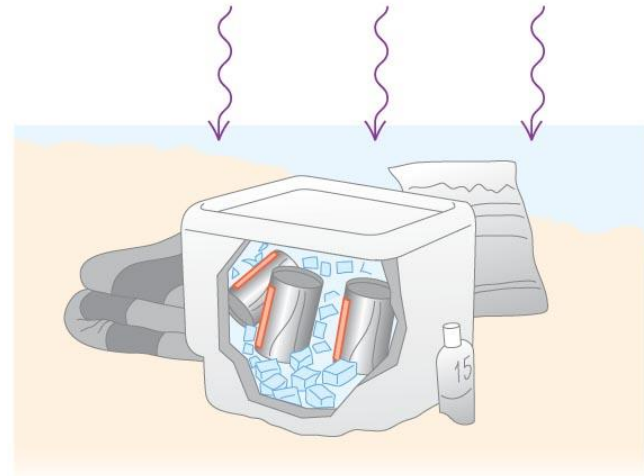
(b) Our sketch for this problem

$A = 0.80 m^2$

$T_H = 30°C$

$T_C = 0°C$

Ice

2.0 cm
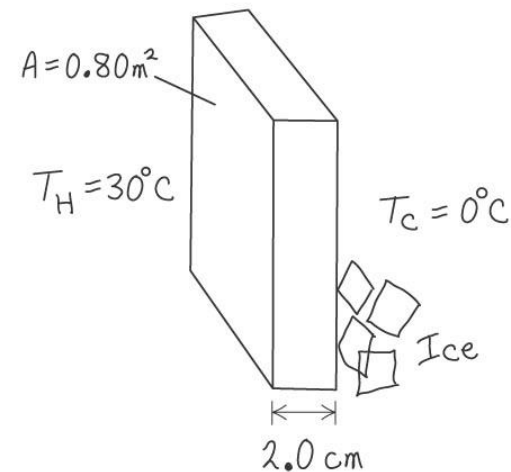
Conduction through two bars that are end-to-end

- Follow Example 17.12 in which two bars of different material are welded end-to-end. Use Figure 17.25 below.

More conduction through two bars

- Follow Example 17.13. The bars from the previous example are arranged differently.
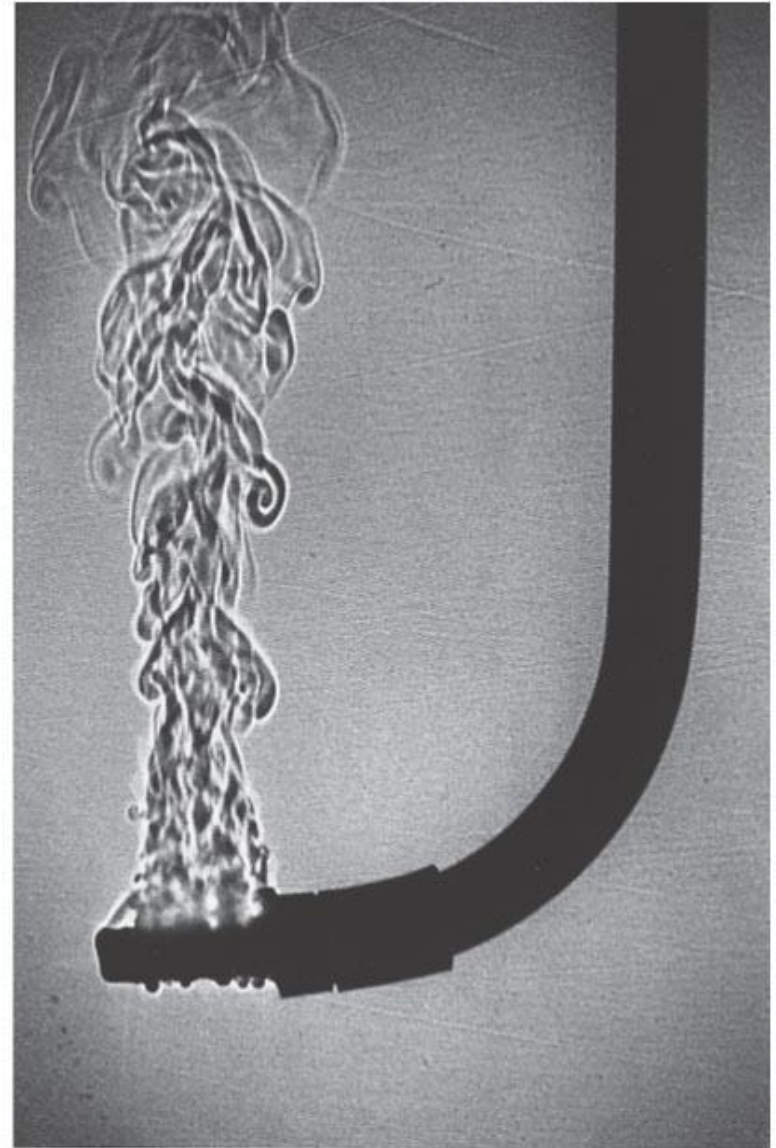
- Figure 17.26 below illustrates the problem.

Copper

$T_H = 100°C$ 2.00 cm $T_C = 0°C$

20.0 cm

Steel

$T_H = 100°C$ 2.00cm $T_C = 0°C$

10.0cm
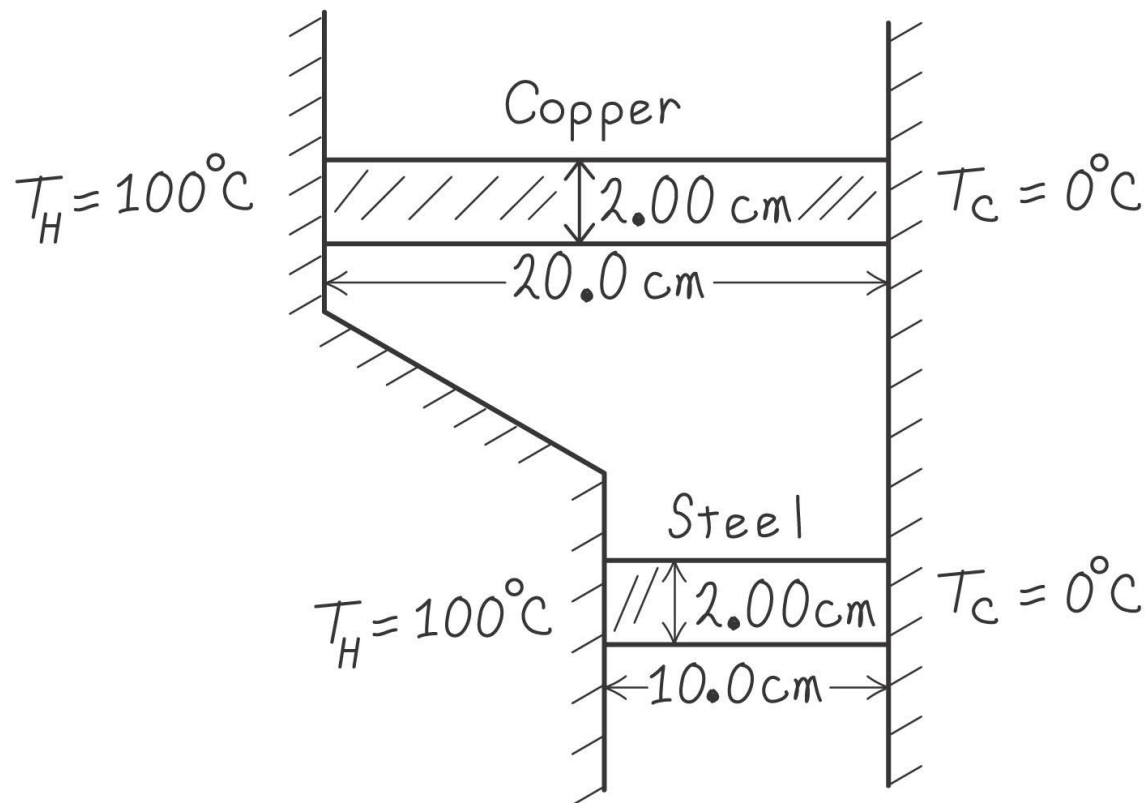
Convection of heat

- *Convection* is the transfer of heat by the mass motion of fluid.

- Figure 17.27 at the right illustrates the convection due to a heating element submerged in water.

Radiation of heat

- *Radiation* is the transfer of heat by electromagnetic waves, such as visible light or infrared.

- Figure 17.28 at the right is a false-color infrared photo in which red is the strongest emission.

- *Stefan-Boltzmann law*: The *heat current* in radiation is $H = Ae\sigma T^4$.

- Follow Example 17.14.

- Follow Example 17.15.

Radiation and climate change

- The energy radiated by the earth's surface is mostly infrared.

- CO_2 molecules in our atmosphere readily absorb some of this infrared radiation and reradiate part of it back down toward the surface. This increases the temperature of the surface. This effect is called the *greenhouse effect*.

- Figure 17.29 below shows how the increase in global temperature is correlated with the rise in CO_2 emission.

(a)

CO$_2$ concentration (parts per million)

Highest concentration in the 650,000 years before the industrial age: less than 300 parts per million

Year

(b)

Anomaly (°C) wrt 1961–90

Year

Annual average and 95% confidence range

Smoothed annual average and 95% confidence range