P7.1. Derive the equations for slope and deflection for the beam in Figure P7.1. Compare the deflection at \( B \) with the deflection at midspan.

Analysis by Double Integration

\[
\frac{w(L-x)^2}{2} = M
\]

\[
\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{w}{2EI} (L^2 - 2xL + x^2)
\]

\[
2EI \frac{d^2y}{dx^2} = -L^2 + 2wxL - wx^2
\]

\[
2EI \frac{dy}{dx} = -wLx + 2wL \frac{x^2}{8} - \frac{wx^3}{3} + C_1 = 0
\]  

(1)

At \( x = 0 \), \( \frac{dy}{dx} = 0 \) \( C_1 = 0 \)

\[
2Ely = -\frac{wL^2x^3}{2} + \frac{wLx^3}{3} - \frac{wx^4}{12} + C_2 = 0
\]

At \( x = 0 \), \( y = 0 \) \( C_2 = 0 \)

Compute \( \Delta_n \); Set \( x = L \) in Eq(2)

\[
\Delta_n = \frac{y}{B EI} = \left( \frac{wL^4}{2} + \frac{wL^3}{3} - \frac{wL^4}{12} \right)
\]

Compute \( \theta_n \); Set \( x = L \) in Eq1

\[
\theta_n = \frac{1}{2EI} \left( -\frac{wL^3}{2} + \frac{wL^3}{3} \right) = \frac{wL^3}{6EI}
\]

\[
\Delta \text{ at } \frac{L}{2} \text{ in Eq2}
\]

\[
\Delta = -\frac{17wL^4}{384EI} = -\frac{wL^4}{8E}
\]

\[
\text{Compare} \quad \frac{\Delta \text{ at } \frac{L}{2}}{\Delta \text{ at } B} = \frac{1}{22.59} = 0.35
\]

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P7.3. Derive the equations for slope and deflection for the beam in Figure P7.3. Compute the maximum deflection. Hint: Maximum deflection occurs at point of zero slope.

Compute \( R_1 : \sum M_y = 0 \)
\[
0 = M - \frac{M}{2} - R_1 L
\]
\( R_1 = \frac{M}{2L} \)

Evaluate \( Mx : \sum M \)
\[
Mx = M - R_1 x = \left[ M - \frac{M}{2L} \right] x
\]

\[
\frac{d^2 y}{dx^2} = \frac{Mx}{EI} - \left( \frac{M - Mx}{2L} \right) \frac{1}{EI} \quad (1)
\]

\[
E \frac{dy}{dx} = Mx - \frac{Mx^2}{2L} + C_1 \quad (2)
\]

\[
E y = \frac{Mx^2}{2} - \frac{Mx^3}{12L} + C_1 x + C_2 \quad (3)
\]

Substi \( y = 0 @ x = 0 \) in Eq (3)
\[
0 = 0 + 0 + 0 + C_2
\]

\( \therefore C_2 = 0 \)

Substi \( y = 0 @ x = L \) in Eq (3)
\[
0 = \frac{ML^2}{2} - \frac{ML^2}{12} + C_1 L
\]

\( C_1 = -\frac{5}{12} ML \)

\[
E \frac{dy}{dx} = Mx - \frac{Mx^2}{4L} - \frac{5}{12} ML \quad (2a)
\]

\[
E ly = \frac{Mx^2}{2} - \frac{Mx^3}{12L} - \frac{5}{12} M L x \quad (3a)
\]

Compute \( \Delta_{max} : \) Set \( \frac{dy}{dx} = 0 \) in Eq 2a to locate position \( \Delta_{max} \)

\[
0 = Mx - \frac{Mx^2}{4L} - \frac{5}{12} ML
\]

\[
x^2 - 4Lx = \frac{5L}{12} (4L^2)
\]

\[
(x - \frac{2L}{3})^2 = \frac{7}{3} L^2
\]

\( x = 0.4725L \)

Substi \( x = 0.4725L \) into (Eq.3a)

\[
\Delta_{max} = \frac{M}{EI} \left[ \frac{(0.4725L)^2}{2} - \frac{(0.4725L)^3}{12L} - \frac{5L(0.4725L)}{12} \right]
\]

\[
= -0.094ML^2
\]
P8.5. The pin-connected frame in Figure P8.3 is subjected to two vertical loads. Compute the vertical displacement of joint B. Will the frame sway horizontally? If yes, compute the horizontal displacement of joint B. The area of all bars = 5 in.², and \(E = 29,000\) kips/in.².

\[
\sum Q \cdot \delta_p = \sum F_q \frac{F_p L}{AE} \\
1^1 \cdot \delta_B = \frac{-1 (-150') (12' \times 12\%)}{5\text{ in}^2 (29000\text{ kips/in}^2)} = 0.149\text{''} \\
1^1 \cdot \delta_{\text{h}} = \frac{-2'(-150') (12' \times 12\%)}{5\text{ in}^2 (29,000\text{ kips/in}^2)} = 0.298\text{''} \\
\]