Constructing Influence Lines
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In addition to supporting fixed gravity loads (Dead Load), structures must also support gravity loads that can vary in magnitude and position (Live Loads).

To design the components of a structure, it is important to understand how to place live loads to produce the maximum response for important design quantities (e.g. support reactions, internal shear, bending moment, axial force).

The Influence Line for a response quantity is a tool to help place live loads to find the maximum response.
Construction of Influence Lines

The overhanging beam shown has a fixed support at A, a roller support at C and an internal hinge at B. Construct influence lines for:

1. The roller support at C;
2. The vertical reaction at the fixed support at A;
3. The moment reaction at the fixed support at A
1. Choose a reference coordinate;
2. Choose a sign convention for each diagram;
3. Place a unit, dimensionless load on the structure;
4. Use equilibrium analysis to find the response quantity (e.g. support reaction, internal force) at the position of the unit, dimensionless, load;
5. Move unit load to another position and repeat Step 4;
6. Plot the value of the response quantity versus the position of the unit, dimensionless, load.
The overhanging beam shown has a fixed support at A, a roller support at C and an internal hinge at B. Construct influence lines for:

1. The roller support at C;
2. The vertical reaction at the fixed support at A;
3. The moment reaction at the fixed support at A

Sign Convention for Positive Support Reactions
Place Unit Load at $x = 0$ (Point A)

Free-body Diagram

4 Unknowns – 3 Equations of Equilibrium
Need to make a cut at the hinge at B
Place Unit Load at $x = 0$ (Point A)

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium

\[ \sum M_A = 0 \rightarrow M_A = 0 \]
\[ \sum F_x = 0 \rightarrow A_x = 0 \]
\[ \sum F_y = 0 \rightarrow A_y = 1 \]
\[ \sum M_B = 0 \rightarrow C_y = 0 \]
\[ \sum F_x = 0 \rightarrow F_B = 0 \]
\[ \sum F_y = 0 \rightarrow V_B = 0 \]
Place Unit Load at $x = 5$ m

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium

\[ \sum M_A = 0 \rightarrow M_A = -5 \text{ m} \]
\[ \sum F_x = 0 \rightarrow A_x = 0 \]
\[ \sum F_y = 0 \rightarrow A_y = 1 \]
\[ \sum M_B = 0 \rightarrow C_y = 0 \]
\[ \sum F_x = 0 \rightarrow F_B = 0 \]
\[ \sum F_y = 0 \rightarrow V_B = 0 \]
Place Unit Load at $x = 9^{-} m$

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium

\[ \sum M_A = 0 \rightarrow M_A = -9 \text{ m} \]

\[ \sum F_x = 0 \rightarrow A_x = 0 \]

\[ \sum F_y = 0 \rightarrow A_y = 1 \]

\[ \sum M_B = 0 \rightarrow C_y = 0 \]

\[ \sum F_x = 0 \rightarrow F_B = 0 \]

\[ \sum F_y = 0 \rightarrow V_B = 0 \]
Place Unit Load at $x = 12 \text{ m}$

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium

\[ \sum M_A = 0 \quad \Rightarrow \quad M_A = -4.5 \text{ m} \]

\[ \sum F_x = 0 \quad \Rightarrow \quad A_x = 0 \]

\[ \sum F_y = 0 \quad \Rightarrow \quad A_y = 0.5 \]

\[ \sum M_B = 0 \quad \Rightarrow \quad C_y = 0.5 \]

\[ \sum F_x = 0 \quad \Rightarrow \quad F_B = 0 \]

\[ \sum F_y = 0 \quad \Rightarrow \quad V_B = 0.5 \]
Place Unit Load at $x = 15 \text{ m}$

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium

\[ \sum M_A = 0 \Rightarrow M_A = 0 \]
\[ \sum F_x = 0 \Rightarrow A_x = 0 \]
\[ \sum F_y = 0 \Rightarrow A_y = 0 \]
\[ \sum M_B = 0 \Rightarrow C_y = 1 \]
\[ \sum F_x = 0 \Rightarrow F_B = 0 \]
\[ \sum F_y = 0 \Rightarrow V_B = 0 \]
Place Unit Load at $x = 18 \text{ m}$

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium

1. $\sum M_A = 0 \Rightarrow M_A = 4.5 \text{ m}$
2. $\sum M_B = 0 \Rightarrow C_y = 1.5$
3. $\sum F_x = 0 \Rightarrow A_x = 0$
4. $\sum F_x = 0 \Rightarrow F_B = 0$
5. $\sum F_y = 0 \Rightarrow A_y = -0.5$
6. $\sum F_y = 0 \Rightarrow V_B = -0.5$
Plot the Influence Line for $C_y$

$C_y = \begin{cases} 
0 & \text{for } x = 0 \\
0 & \text{for } x = 5 \text{ m} \\
0 & \text{for } x = 9 - 1 \text{ m} \\
0.5 & \text{for } x = 12 \text{ m} \\
1.0 & \text{for } x = 15 \text{ m} \\
1.5 & \text{for } x = 18 \text{ m} 
\end{cases}$
Plot the Influence Line for $A_y$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$A_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5 m</td>
<td>1</td>
</tr>
<tr>
<td>$9 - m$</td>
<td>1</td>
</tr>
<tr>
<td>12 m</td>
<td>0.5</td>
</tr>
<tr>
<td>15 m</td>
<td>0</td>
</tr>
<tr>
<td>18 m</td>
<td>$-0.5$</td>
</tr>
</tbody>
</table>
Plot the Influence Line for $M_A$