Analysis of a Two-Dimensional Body with a Two-Force Member

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The structure shown is pin supported at points A and D. Members ABC and CD are connected by an internal hinge at point C. For the loading shown, find the reaction forces at the pin supports at points A and D. The weight of the members is negligible.
FBD of Entire Structure

4 Unknowns
3 Equations of Equilibrium

6 kN/m

8 kN

A

B

C

D

Ax

Ay

Dx

Dy

2 m

4 m

3 m
FBDs of ABC and CD

Equal and opposite forces at C

CD is a two-force member

6 Unknowns
6 Equations of Equilibrium

6 kN/m
4 m
3 m
3 m
2 m
8 kN

Ax
Ay

Cx
Cy

Dx
Dy

4 m
FBDs of ABC and CD recognizing that CD is a two-force member

Equal and opposite forces at C

8 kN

6 kN/m

A

B

C

D

F_{CD}

A_x

A_y

3 m

3 m

2 m

4 m

3 m
\[(6 \text{ kN/m})(2 \text{ m}) = 12 \text{ kN}\]

\[F_{CD} \cos \theta = F_{CD} \left(\frac{4}{5}\right)\]

\[F_{CD} \sin \theta = F_{CD} \left(\frac{3}{5}\right)\]

3 Unknowns
3 Equations of Equilibrium
Equilibrium of ABC

\[(6 \text{ kN/m})(2 \text{ m}) = 12 \text{ kN}\]

\[F_{CD} \left(\frac{4}{5}\right)\]

\[\sum M_A = 0\]

\[F_{CD} \left(\frac{3}{5}\right)\]

\[F_{CD} = -6 \text{ kN}\]
Equilibrium of ABC

(6 kN/m)(2 m) = 12 kN

\[ \sum F_y = 0 \]

\[ A_y = 8.4 \text{ kN} \]
(6 kN/m)(2 m) = 12 kN

\[ F_{CD} \left( \frac{4}{5} \right) \]

\[ + \sum F_x = 0 \]

\[ A_x = -3.2 \text{ kN} \]
Show results on a FBDs of ABC and CD

No need for + or – signs!
Results in terms of components

6 kN/m

8 kN

3.2 kN

8.4 kN

4.8 kN

3.6 kN

3 m

3 m

2 m

4 m
FBD of Entire Structure in Equilibrium

- **A**: 3.2 kN, 2 m
- **B**: 8 kN, 3 m
- **C**: 6 kN/m, 3 m
- **D**: 4.8 kN, 4 m
- **FBD**

Labels: A, B, C, D
Dimensions: 2 m, 4 m, 3 m, 6 kN/m