Engineering Beam Theory

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Consider an Elastic Beam with General Supports, and General Loading

We seek \( y(x) \) and \( \theta(x) \) that describe the transverse deformation of the neutral axis and the slope of the tangent line to the neutral axis.

Engineering beam theory assumptions:
• Transverse deformation is small relative to beam span;
• Effect of shear deformation is small so we can use the moment-curvature relationship from pure bending.

\[
\theta(x) = \frac{dy}{dx}
\]
Recall Relationships from Pure Bending Analysis

Moment-Curvature relationship

\[ \kappa = \frac{M}{EI} \]

Neutral axis

Bending stress distribution

\[ \sigma = -\frac{M y}{I} \]

\[ \sigma_{\text{max}} = \frac{M c}{I} \]

1. Neutral axis \((\sigma = 0)\) is located at the centroid of the beam cross section;
2. Moment-Curvature relationship is basis of bending deformation theory;
3. Bending stress varies linearly over beam cross section and is maximum at the extreme fibers of the beam;
From Analytic Geometry, Recall the Local Curvature of a Function

\[ \kappa(x) = \frac{1}{\rho(x)} = \frac{d^2 y}{dx^2} \left[ 1 - \left( \frac{dy}{dx} \right)^2 \right]^{3/2} \]

For small deformations: \( \frac{dy}{dx} \ll 1 \)

\[ \kappa(x) = \frac{1}{\rho(x)} \approx \frac{d^2 y}{dx^2} \]
\[ \kappa = \frac{d^2y}{dx^2} = \frac{M}{EI} \]

In order to solve this differential equation for \( y \) and \( \theta \) we need:

- Moment equation (from statics);
- Two boundary (or continuity) conditions on \( y \) or \( \theta \);
- Information on \( E \) and \( I \).
Common Boundary Conditions for Beam Problems

**Fixed Support**
- $\theta(a) = 0$
- $y(a) = 0$

**Pin Support**
- $\theta(a) = ?$
- $y(a) = 0$

**Roller Support**
- $\theta(a) = ?$
- $y(a) = 0$
Common Continuity Conditions for Beam Problems

Internal Support

\( \theta_L(a) = \theta_R(a) \)
\( y_L(a) = y_R(a) = 0 \)

Internal Hinge

\( \theta_L(a) = ? \)
\( \theta_R(a) = ? \)
\( y_L(a) = y_R(a) \)