Analysis of Statically Indeterminate Structures Using the Force Method

Steven Vukazich
San Jose State University
At the beginning of the course, we learned that a **stable structure** that contains **more unknowns than independent equations of equilibrium** is **Statically Indeterminate**.

### Advantages
- Redundancy (several members must fail for the structure to become unstable);
- Often maximum stresses is certain members are reduced;
- Usually deflections are reduced.

### Disadvantages
- Connections are often more expensive;
- Finding forces and deflections using hand analysis is much more complicated.
Steps in Solving an Indeterminate Structure using the Force Method

1. Determine degree of Indeterminacy
   Let \( n = \text{degree of indeterminacy} \)
   (i.e. the structure is indeterminate to the \( n \)th degree)

2. Define Primary Structure
   and the \( n \) Redundants

3. Define the Primary Problem

4. Solve for the \( n \) Relevant Deflections in Primary Problem

5. Define the \( n \) Redundant Problems

6. Solve for the \( n \) Relevant Deflections in each Redundant Problem

7. Write the \( n \) Compatibility Equations at Relevant Points

8. Solve the \( n \) Compatibility Equations to find the \( n \) Redundants

9. Use the Equations of Equilibrium to solve for the remaining unknowns

10. Construct Internal Force Diagrams (if necessary)

   - Chapters 3, 4, 5 then 7 or 8

   - Chapters 3, 4, 5 then 7 or 8

   - Chapters 3, 4, 5
Consider the beam

**Force Method of Analysis**

Beam is stable

\[ X = 5 \]

\[ 3n = 3(1) = 3 \]

Statically Indeterminate to the 2nd degree
Define Primary Structure and Redundants

- Remove all applied loads from the actual structure;
- Remove support reactions or internal forces to define a primary structure;
- Removed reactions or internal forces are called redundants;
- Same number of redundants as degree of indeterminacy;
- Primary structure must be stable and statically determinate;
- Primary structure is not unique – there are several choices.
Define and Solve the Primary Problem

- Apply all loads on actual structure to the primary structure;
- Define a reference coordinate system;
- Calculate relevant deflections at points where redundants were removed.
Define and Solve the Redundant Problems

- There are the same number of redundant problems as degrees of indeterminacy;
- Define a reference coordinate system;
- Apply only one redundant to the primary structure;
- Write the redundant deflection in terms of the flexibility coefficient and the redundant for each redundant problem.
- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;

\[ \Delta_{CC} = C_y \delta_{CC} \]
\[ \Delta_{DC} = C_y \delta_{DC} \]
Define and Solve the Redundant Problems

\[ \Delta_{CD} = D_y \delta_{CD} \]
\[ \Delta_{DD} = D_y \delta_{DD} \]
Compatibility Equations

Compatibility at Point C

\[ \Delta_C + \Delta_{CC} + \Delta_{CD} = 0 \]

Compatibility at Point D

\[ \Delta_D + \Delta_{DC} + \Delta_{DD} = 0 \]

Compatibility Equations in terms of Redundants and Flexibility Coefficients

\[ \Delta_C + C_y \delta_{CC} + D_y \delta_{CD} = 0 \]
\[ \Delta_D + C_y \delta_{DC} + D_y \delta_{DD} = 0 \]

Solve for \( C_y \) and \( D_y \)
The Force Method is Based on the Principle of Superposition

Indeterminate Problem

Primary Problem

Redundant Problem 1

Redundant Problem 2
For the indeterminate beam subject to the point load, \( P \), find the support reactions at A and C. \( EI \) is constant.

Beam is stable
\[ X = 4 \]
\[ 3n = 3(1) = 3 \]

Statically Indeterminate to the 1\(^{st}\) degree
Define Primary Structure and Redundant

- Remove all applied loads from the actual structure;
- Remove support reactions or internal forces to define a primary structure;
- Removed reactions or internal forces are called redundants;
- Same number of redundants as degree of indeterminacy;
- Primary structure must be stable and statically determinate;
- Primary structure is not unique – there are several choices.

![Diagram of Primary Structure and Redundant](image-url)
Define and Solve the Primary Problem

- Apply all loads on actual structure to the primary structure;
- Define a reference coordinate system;
- Calculate relevant deflections at points where redundants were removed.

\[
\theta_A = -\frac{PL^2}{16EI}
\]

From Tabulated Solutions

Counter-clockwise rotations positive
Define and Solve the Redundant Problem

- There are the same number of redundant problems as degrees of indeterminacy;
- Define a reference coordinate system;
- Apply only one redundant to the primary structure;
- Write the redundant deflection in terms of the flexibility coefficient and the redundant for each redundant problem.
- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;

\[ \theta_{AA} = M_A \alpha_{AA} \]

From Tabulated Solutions

\[ \alpha_{AA} = -\frac{L}{3EI} \]
Compatibility Equation at Point A

\[ \theta_A + \theta_{AA} = 0 \]

Compatibility Equation in terms of Redundant and Flexibility Coefficient

\[ \theta_A + M_A \alpha_{AA} = 0 \]

\[ -\frac{PL^2}{16EI} + M_A \left(-\frac{L}{3EI}\right) = 0 \]

Solve for \( M_A \)

\[ M_A = \frac{PL^2}{16EI} \left(-\frac{3EI}{L}\right) \]

\[ M_A = -\frac{3}{16} PL \]
Can now use equilibrium equations to find the remaining three unknowns.

\[
M_A = -\frac{3}{16}PL
\]
Can now use equilibrium equations to find the remaining three unknowns.

\[ M = 0 \]
\[ F = 0 \]
\[ F' = 0 \]
\[ A_x = 0 \]
\[ C_y = \frac{5}{16} P \]
\[ A_y = \frac{11}{16} P \]
Draw V and M Diagrams of the Beam

\[ V = \begin{cases} \frac{3}{16}PL, & x < \frac{L}{2} \\ \frac{11}{16}P, & \frac{L}{2} \leq x \leq \frac{3L}{4} \\ \frac{11}{16}P, & x > \frac{3L}{4} \end{cases} \]

\[ M = \begin{cases} \frac{5}{32}PL, & x < \frac{L}{2} \\ -\frac{3}{16}PL + \frac{11}{32}PL, & \frac{L}{2} \leq x \leq \frac{3L}{4} \\ -\frac{5}{16}P, & x > \frac{3L}{4} \end{cases} \]

\[ M_B - M_A = \left(\frac{11}{16}P\right)\left(\frac{L}{2}\right) \]
Superposition of Primary and Redundant Problems

Indeterminate Problem

Primary Problem

Redundant Problem

\[ \frac{3}{16} PL \]

\[ \frac{11}{16} P \]

\[ \frac{11}{16} P \]

\[ V \]

\[ M \]

\[ - \frac{3}{16} PL \]