Statically Indeterminate Frame Example

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Steps in Solving an Indeterminate Structure using the Force Method

1. Determine degree of Indeterminacy
   Let \( n \) = degree of indeterminacy
   (i.e. the structure is indeterminate to the nth degree)

2. Define Primary Structure and the \( n \) Redundants

3. Define the Primary Problem
4. Solve for the \( n \) Relevant Deflections in Primary Problem

5. Define the \( n \) Redundant Problems
6. Solve for the \( n \) Relevant Deflections in each Redundant Problem

7. Write the \( n \) Compatibility Equations at Relevant Points
8. Solve the \( n \) Compatibility Equations to find the \( n \) Redundants

9. Use the Equations of Equilibrium to solve for the remaining unknowns
10. Construct Internal Force Diagrams (if necessary)
Example Problem

For the indeterminate frame subjected to the point loads shown, find the support reactions and draw the bending moment diagram for the frame. $EI$ is the same for both the horizontal and vertical members.
FBD of the Frame

Frame is stable

$X = 4$

$3n = 3(1) = 3$

Statically Indeterminate to the 1st degree
Define Primary Structure and Redundant

- Remove all applied loads from the actual structure;
- Remove support reactions or internal forces to define a primary structure;
- Removed reactions or internal forces are called redundants;
- Same number of redundants as degree of indeterminacy;
- Primary structure must be stable and statically determinate;
- Primary structure is not unique – there are several choices.
Define and Solve the Primary Problem

- Apply all loads on actual structure to the primary structure;
- Define a reference coordinate system;
- Calculate relevant deflections at points where redundants were removed.

10 ft
30 k

Need to find $\Delta_D$
Use the Principle of Virtual Work
Solve the Primary Problem

Real System

Need to construct the $M_p$ diagram
Solve the Primary Problem

Virtual System to measure $\Delta_D$

Need to construct the $M_Q$ diagram
\[ M = 0 + F_x + F_y = 0 \]

\[ M_A = 250 \text{ k-ft} \]

\[ A_x = -10 \text{ k} \]

\[ A_y = 30 \text{ k} \]
\[ M_p \text{ Diagram for the Primary Problem} \]

Moment diagram is drawn on the compression side of the member.
FBD of the Virtual System

\[ \sum M_A = 0 \]

\[ M_A = 10 \text{ ft} \]

\[ \sum F_x = 0 \]

\[ A_x = 0 \]

\[ \sum F_y = 0 \]

\[ A_y = -1 \]
Diagram for the Primary Problem

Moment diagram is drawn on the compression side of the member
Solve the Primary Problem

\[ 1 \cdot \Delta_D = \frac{1}{EI} \int_0^L M_Q M_P \, dx \]

\[ \Delta_D = - \frac{23,125 \text{ k-ft}^3}{EI} \]

- \( \frac{1}{2} \) \( M_1 (M_3 + M_4) L \)
- \( \frac{1}{6} (M_1 + 2M_2) M_3 L \)
- \( \frac{1}{2} \) \( (10 \text{ ft}) (250 \text{ k-ft} + 150 \text{ k-ft})(10 \text{ ft}) \)
- \( \frac{1}{6} \) \( [5 \text{ ft} + 2(10 \text{ ft})](150 \text{ k-ft})(5 \text{ ft}) \)

-20,000 k-ft³

-3,125 k-ft³
Define and Solve the Redundant Problem

- Write the redundant deflection in terms of the flexibility coefficient and the redundant for each redundant problem.

- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;

Redundant Problem

\[ \Delta_D = D_y \delta_{DD} \]
Define and Solve the Redundant Problem

- Write the redundant deflection in terms of the flexibility coefficient and the redundant for each redundant problem.
- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;

**Flexibility Coefficient**

\[\Delta_{DD} = D_y \delta_{DD}\]

Need to find \(\delta_{DD}\)

Use the Principle of Virtual Work
Solve the Flexibility Coefficient Problem

Real System

Need to construct the $M_p$ diagram

$\delta_{DD}$
Solve the Flexibility Coefficient Problem

Virtual System to measure $\delta_{DD}$

Note that for the flexibility coefficient problem the real and virtual systems are identical.

Need to construct the $M_Q$ diagram
\[ M = 0 + F + M = 0 + F, \]
\[ M_A = 10 \text{ ft} \]
\[ A_x = 0 \]
\[ A_y = -1 \]
$M_p$ Diagram for the Flexibility Coefficient Problem

Moment diagram is drawn on the compression side of the member.
Moment diagram is drawn on the compression side of the member.
Solve the Flexibility Coefficient Problem

\[ 1 \cdot \delta_{DD} = \frac{1}{EI} \int_0^L M_Q M_P \, dx \]

\[ \delta_{DD} = \frac{1333 \text{ ft}^3}{EI} \]

\[ \Delta_{DD} = D_y \delta_{DD} \]

\[ \Delta_{DD} = D_y \left( \frac{1333 \text{ ft}^3}{EI} \right) \]

\[ (10 \text{ ft})(10 \text{ ft})(10 \text{ ft}) \]

\[ 1000 \text{ ft}^3 \]

\[ \left( \frac{1}{3} \right)(10 \text{ ft})(10 \text{ ft})(10 \text{ ft}) \]

\[ 333.333 \text{ ft}^3 \]
Compatibility Equation at Point D

\[ \Delta_D + \Delta_{DD} = 0 \]

Compatibility Equation in terms of Redundant and Flexibility Coefficient

\[ \Delta_D + D_y \delta_{DD} = 0 \]

\[ -\frac{23,125 \text{k-ft}^3}{EI} + D_y \left( \frac{1333 \text{ ft}^3}{EI} \right) = 0 \]

Solve for \( D_y \)

\[ D_y = \frac{23,125 \text{k-ft}^3}{EI} \left( \frac{EI}{1333 \text{ ft}^3} \right) \]

\[ D_y = 17.34 \text{ k} \]
$D_y = 17.34 \text{ k}$

$M_A = 76.6 \text{ k-ft}$

$A_x = -10 \text{ k}$

$A_y = 12.66 \text{ k}$
Moment Diagram for the Frame

Moment diagram is drawn on the compression side of the member.
Moment Diagrams for the Primary and Redundant Problems

Moment diagram is drawn on the compression side of the member.
Choose sign convention for internal forces for both horizontal and vertical members

For horizontal member BDE

For vertical member ABC