Moment of a Force About a Point
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Which application of the force $F$ would provide the most rotation to loosen the nut at point $O$?

Proof of the correct answer lies in the concept of the moment of a force about a point.

Position A

Position B

Position C
Moment of a Force $F$ about a Point $O$

**Magnitude of $M_O$** is the area of the parallelogram defined by $r$ and $F$.

**Direction of $M_O$** is perpendicular to the plane defined by $r$ and $F$.

**Sense of $M_O$** is defined by the right-hand rule.

$r$ is a position vector that must satisfy:
- Tail of $r$ is at point $O$;
- Tip can be on any point on the line-of-action of $F$.

$$M_O = r \times F$$

$$M_O = rF \sin \theta$$
Moment of a Force $F$ about Point $O$

$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

$\mathbf{M}_O = \mathbf{r} \mathbf{F} \sin \theta$

$\mathbf{M}_O = r_1 \times \mathbf{F} = r_2 \times \mathbf{F} = r_3 \times \mathbf{F}$

Note that $\mathbf{r}_2$ is the position vector perpendicular to the line-of-action of $\mathbf{F}$. The length of this perpendicular position vector is usually denoted as $d$

$\mathbf{M}_O = \mathbf{r}_2 \mathbf{F} \sin 90^\circ = d \mathbf{F}$
Let’s Examine Our Initial Question Applying the Concept of Moment of a Force About a Point

\[ r \theta = 30^\circ \]
\[ \sin 30^\circ = 0.5 \]

\[ \theta_A \approx 180^\circ \]
\[ \sin 180^\circ = 0 \]

\[ M^A_O \approx rF(0) \approx 0 \]

\[ \theta_B \approx 90^\circ \]
\[ \sin 90^\circ = 1.0 \]

\[ M^B_O \approx rF(1.0) \approx rF \]

\[ \theta_C \approx 30^\circ \]
\[ \sin 30^\circ = 0.5 \]

\[ M^C_O \approx rF(0.5) \approx 0.5rF \]
Moment of a Force about a Point for Planar Problems

\[ M_O = r \times F \]

\[ M_O = rF \sin \theta \]

\[ M_O = dF \]

\( r \) is a position vector that must satisfy:
- Tail of \( r \) is at point \( O \);
- Tip can be on any point on the line-of-action of \( F \)

The direction of \( M_O \) will always be in the \( z \) direction

Sense of \( M_O \) is defined by the right-hand rule
The direction of $M_o$ will always be in the $z$ direction for a planar problem.

The sense of $M_o$ is defined by the right-hand rule:
- Counter-clockwise (positive $z$ direction)
- Clockwise (negative $z$ direction)
Varignon’s Theorem

The moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point O

\[ M_0 = r \times F_1 + r \times F_2 + r \times F_3 = M_0 = r \times (F_1 + F_2 + F_3) = r \times R \]
Moment of a Force in Cartesian Vector Form about a Point

\[ \mathbf{M}_0 = \mathbf{r} \times \mathbf{F} \]

\[ \mathbf{M}_0 = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times \mathbf{F}_x \mathbf{\hat{i}} + \mathbf{r} \times \mathbf{F}_y \mathbf{\hat{j}} + \mathbf{r} \times \mathbf{F}_z \mathbf{\hat{k}} \]
Moment of a Force about a Point when the Position Vector and Force Vector are in Cartesian Vector Form

\[ M_o = r \times F \]

\[ r = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \]
\[ F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \]

\[ M_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \]

Almost always the best way to calculate the moment of a force about a point for three-dimensional problems

\[ M_o = \left( r_y F_z - r_z F_y \right) \hat{i} + \left( r_z F_x - r_x F_z \right) \hat{j} + \left( r_x F_y - r_y F_x \right) \hat{k} \]
Moment of a Force about a Point for Planar Problems

Calculate the moment of each component of $F$ using the perpendicular distance from point $O$.

Add the moment of each component (counter-clockwise rotation is positive and clockwise rotation is negative) to find the moment of the force $F$ about point $O$.

$$M_O = +F_x r_y - F_y r_x$$