Force-Couple System
Steven Vukazich
San Jose State University
In order to analyze structures subjected to general force systems that can cause rotation, we introduced the concept of the moment of a force about a point.
We then added the related concept of the moment of a couple.

We are now ready to understand one of the fundamental concepts for the analysis of bodies subjected to general force systems – the force-couple system.
Consider a force acting on a Planar Body at Point A
At Point $O$, add Two Forces With the Same Magnitude, Same Line of Action, and Opposite Sense

The net effect of the forces at point $O$ on the body is zero

The two forces outlined in yellow form a couple

\[ M = r \times F \]

\[ M = dF \]
The force-couple system at point $O$ has the same effect on the body as the force applied at $A$!

\[ M = r \times F \]

\[ M = dF \]
Note that the couple in the force-couple system is equal to the moment of the force $F$ about point $O$. 

$$F_A = M$$
Any Force System Acting on a Rigid Body can be Replaced by a Resultant Force-Couple System at Any Point

\[ \mathbf{R} = \mathbf{r}_i \times \mathbf{F}_i \]

\[ \mathbf{M}_O^R = \sum (\mathbf{r}_i \times \mathbf{F}_i) \]

\[ \mathbf{R} = \sum \mathbf{F}_i = \left( \sum F_x \right) \hat{i} + \left( \sum F_y \right) \hat{j} \]

Planar Force System

Note that the resultant force is always the same but the resultant couple depends on the location of the point
Any Force System Acting on a Rigid Body can be Replaced by a Resultant Force-Couple System at Any Point

General Three-Dimensional Force System

\[ R = \sum F_i = \left( \sum F_x \right) \hat{i} + \left( \sum F_y \right) \hat{j} + \left( \sum F_z \right) \hat{k} \]

\[ M_O^R = \sum (r_i \times F_i) \]