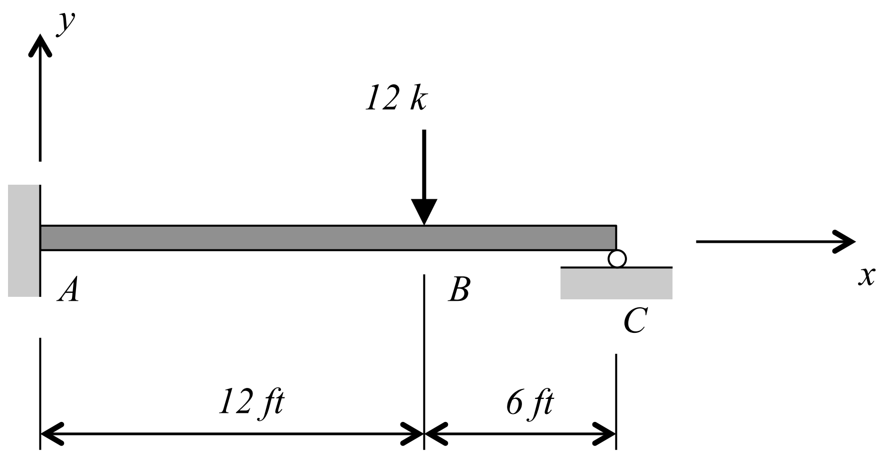
# CE 160 Indeterminate Beam Lab Problem




The beam analyzed by the direct stiffness method in Lab #11 is statically indeterminate to the first degree. Recall that the beam has the following properties:

| Section | I | E |
|---|---|---|
| W8x10 | 30.8 $in^4$ | 29000 ksi |

Using the force method of analysis, define the released structure to be a simply supported beam with a pin at A and roller and C. This released structure also defines the redundant to be the moment reaction at the fixed support at A.

1. Set up the released problem and find the slope at A for the released problem. The tabulated solutions attached to the last page of this note package will be useful to you. Write your answer below:

$$\theta_{A0} =$$

2. Set up the redundant problem and find the flexibility coefficient at point A by applying the redundant at a unit value at A. Use the attached tabulated solutions and write your answer below:

$$\alpha_{AA} =$$

3. Enforce compatibility at point A and solve for the bending moment reaction at A in the indeterminate problem. Write your solution below:

$$\theta_{A0} + M_A \alpha_{AA} = 0$$

$$M_A =$$

4. Use equilibrium to find the two force reactions at A and C and write your results below:

$$A_y =$$

$$C_y =$$

5. From your results in Steps 3 and 4, draw the loading, shear, and bending moment diagrams for the indeterminate beam below:

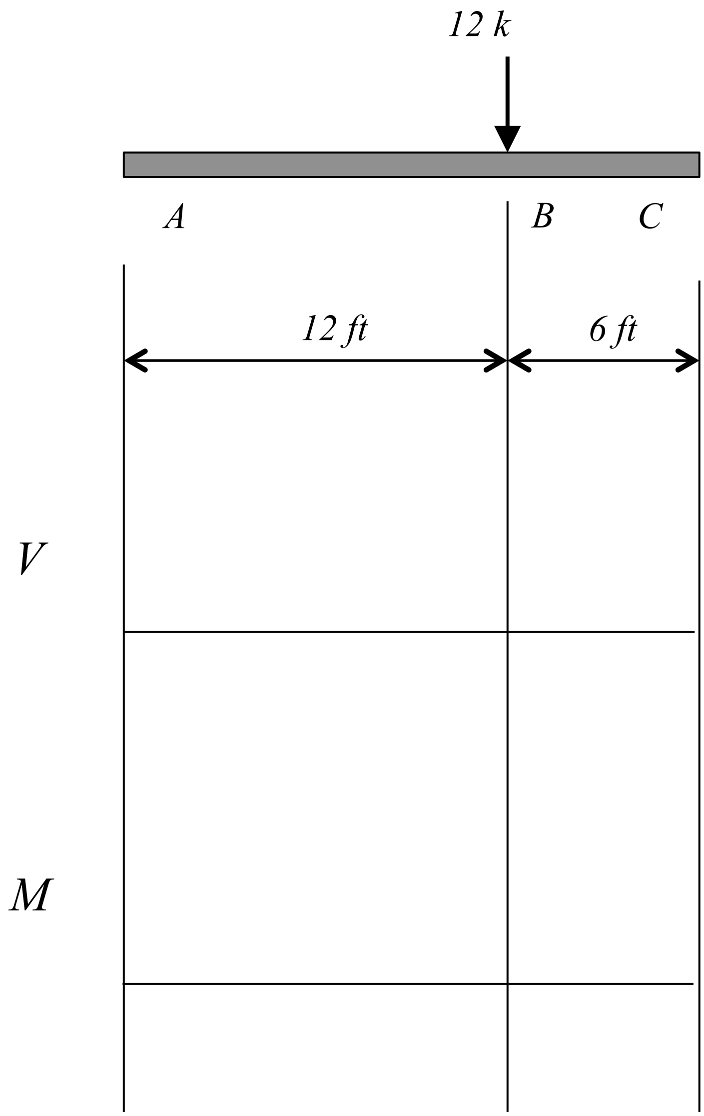
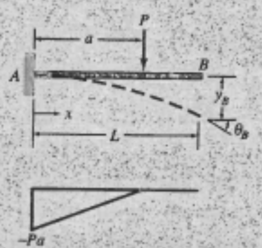
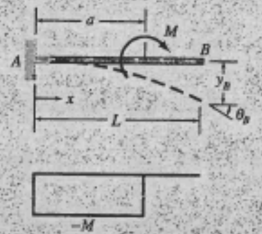
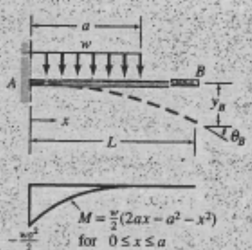
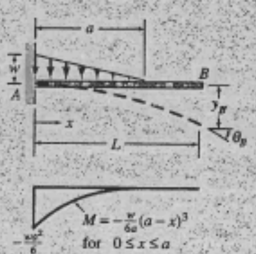
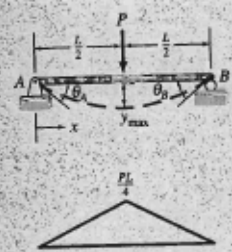
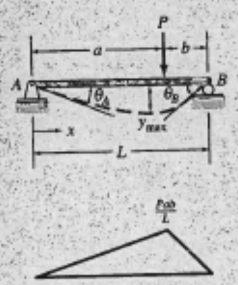
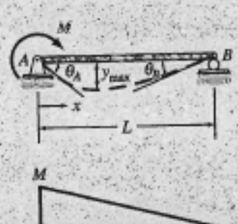


Table to aid in finding $\theta_{A0}$ and $\alpha_{AA}$ in Steps 1 and 2.

## BENDING MOMENTS, SLOPES, AND DEFLECTIONS OF BEAMS UNDER VARIOUS LOADING CONDITIONS

| Beam, Loading, and Bending Moment Diagram | Equations for Slope and Deflection |
|---|---|
|  | $0 \le x \le a$: $\theta = \frac{P}{2EI}(x^2 - 2ax)$; $y = \frac{P}{6EI}(x^3 - 3ax^2)$ <br> $a \le x \le L$: $\theta = -\frac{Pa^2}{2EI}$; $y = \frac{Pa^2}{6EI}(a - 3x)$ <br> $\theta_B = -\frac{Pa^2}{2EI}$; $y_B = -\frac{Pa^2}{6EI}(3L - a)$ |
|  | $0 \le x \le a$: $\theta = -\frac{Mx}{EI}$; $y = -\frac{Mx^2}{2EI}$ <br> $a \le x \le L$: $\theta = -\frac{Ma}{EI}$; $y = \frac{Ma}{2EI}(a - 2x)$ <br> $\theta_B = -\frac{Ma}{EI}$; $y_B = -\frac{Ma}{2EI}(2L - a)$ |
|  | $0 \le x \le a$: $\theta = \frac{w}{6EI}(3ax^2 - 3a^2x - x^3)$; $y = \frac{w}{24EI}(4ax^3 - 6a^2x^2 - x^4)$ <br> $a \le x \le L$: $\theta = -\frac{wa^3}{6EI}$; $y = \frac{wa^3}{24EI}(a - 4x)$ <br> $\theta_B = -\frac{wa^3}{6EI}$; $y_B = -\frac{wa^3}{24EI}(4L - a)$ |
|  | $0 \le x \le a$: $\theta = \frac{w}{24EIa}(x^4 - 4ax^3 + 6a^2x^2 - 4a^3x)$; $y = \frac{w}{120EIa}(x^5 - 5ax^4 + 10a^2x^3 - 10a^3x^2)$ <br> $a \le x \le L$: $\theta = -\frac{wa^3}{24EI}$; $y = \frac{wa^3}{120EI}(-5x + a)$ <br> $\theta_B = -\frac{wa^3}{24EI}$; $y_B = -\frac{wa^3}{120EI}(5L - a)$ |

| Beam, Loading, and Bending Moment Diagram | Equations for Slope and Deflection |
|---|---|
|   | $0 \le x \le \frac{L}{2}$ :<br>$\theta = \frac{P}{16EI}(4x^2 - L^2)$<br>$y = \frac{P}{48EI}(4x^3 - 3L^2x)$<br>$\theta_A = -\frac{PL^2}{16EI}; \ \theta_B = \frac{PL^2}{16EI}$<br>$y_{max} = -\frac{PL^3}{48EI}$ |
|   | $0 \le x \le a$ :<br>$\theta = \frac{Pb}{6EIL}(3x^2 + b^2 - L^2)$<br>$y = \frac{Pb}{6EIL}(x^3 + b^2x - L^2x)$<br><br>$a \le x \le L$ :<br>$\theta = \frac{Pa}{6EIL}[L^2 - a^2 - 3(L - x)^2]$<br>$y = \frac{Pa(L-x)}{6EIL}(x^2 + a^2 - 2Lx)$<br>$\theta_A = -\frac{Pb}{6EIL}(L^2 - b^2)$<br>$\theta_B = \frac{Pa}{6EIL}(L^2 - a^2)$<br><br>For $a \ge b$ :<br>$y_{max} = -\frac{Pb}{9\sqrt{3}EIL}(L^2 - b^2)^{3/2}$<br>at $x = \left(\frac{L^2 - b^2}{3}\right)^{1/2}$ |
|   | $\theta = -\frac{M}{6EIL}(3x^2 - 6Lx + 2L^2)$<br>$y = -\frac{M}{6EIL}(x^3 - 3Lx^2 + 2L^2x)$<br>$\theta_A = -\frac{ML}{3EI}; \ \theta_B = \frac{ML}{6EI}$<br>$y_{max} = -\frac{ML^2}{9\sqrt{3}EI}$<br>at $x = L\left(1 - \frac{1}{\sqrt{3}}\right)$ |