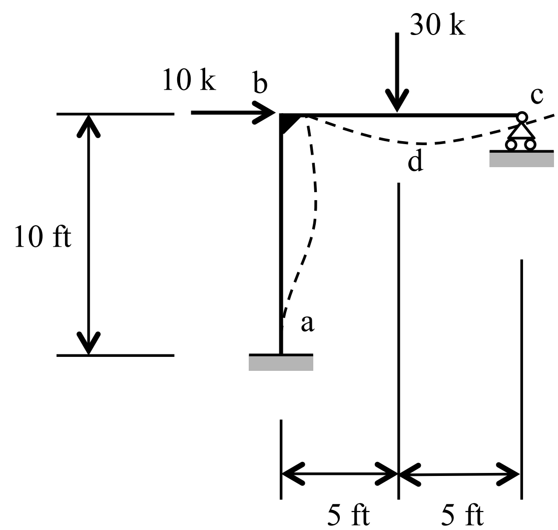
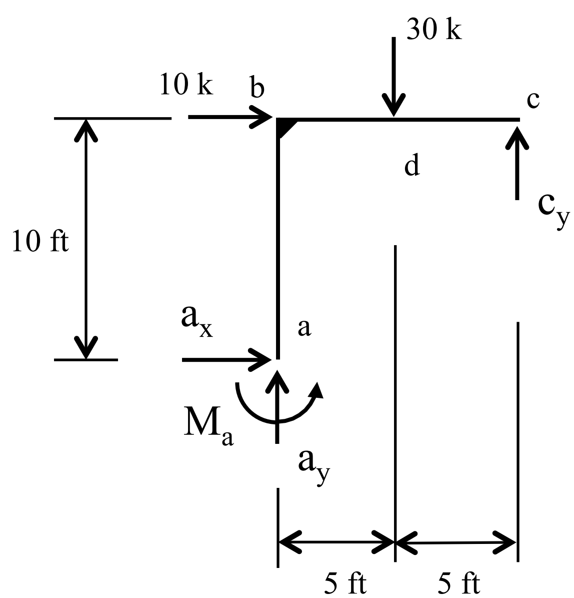
CE 160 Notes: Indeterminate Frame Example

For the indeterminate frame find the bending moment diagram. EI is the same for both the horizontal and vertical members.



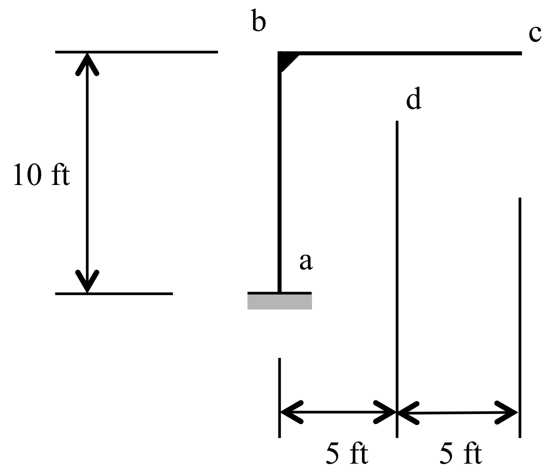
F.B.D



4 unknowns, 3 equations of equilibrium

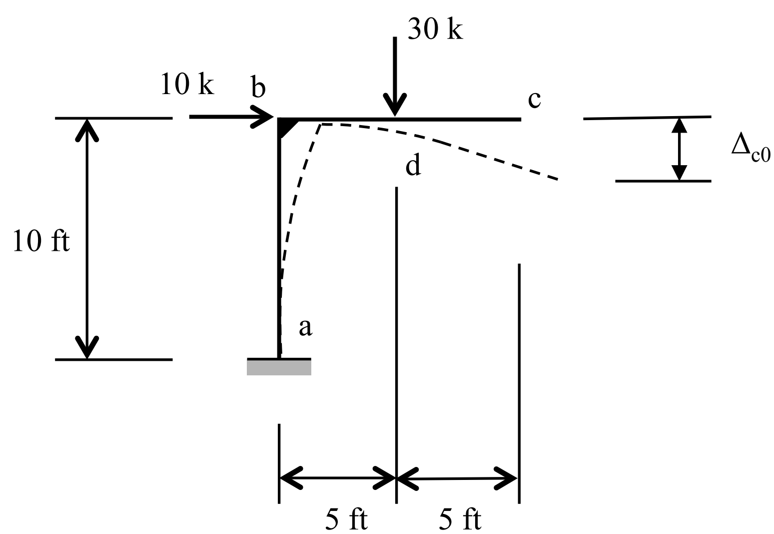
Indeterminate to the first degree

Define c_y as the redundant (Text uses the notation $X_c = c_y$), so the released structure is:

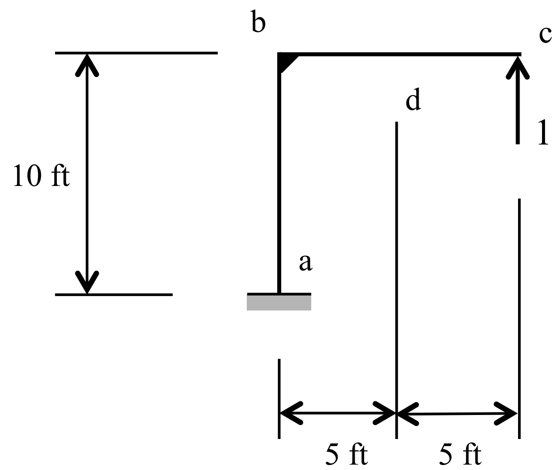


Released Problem - Find Δ_{C0}

Use the method of Virtual Work to find Δ_{c0}

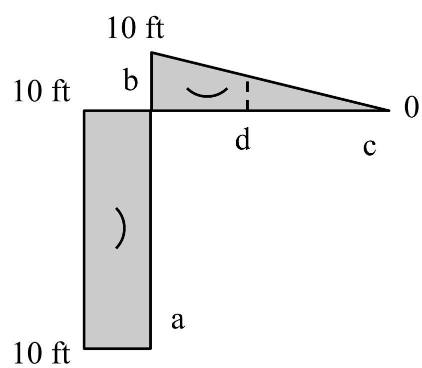


Virtual System to measure Δ_{c0}

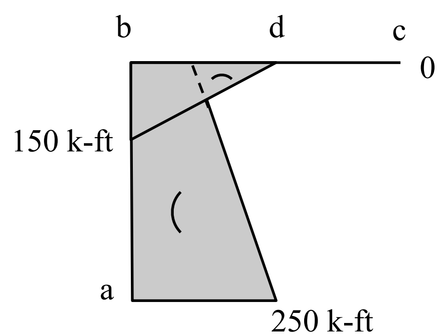


Moment Diagrams

Virtual System (M_Q)

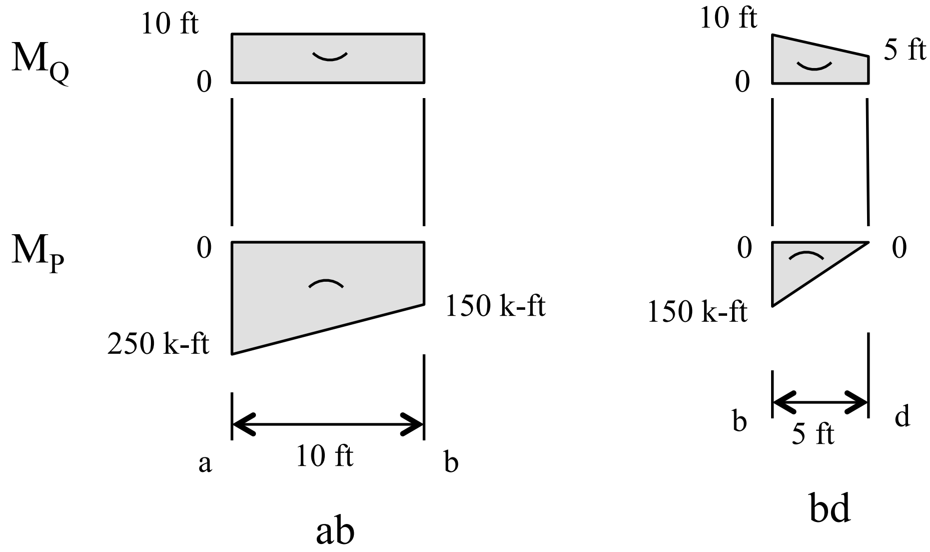


Real System (M_P)



Use Table 4 to evaluate virtual work product integrals:

$$1 \cdot \Delta_{c0} = \frac{1}{EI}\int_0^{x=L} M_Q M_P \, dx$$



Note that results are negative due to dissimilar curvatures in M_P and M_Q

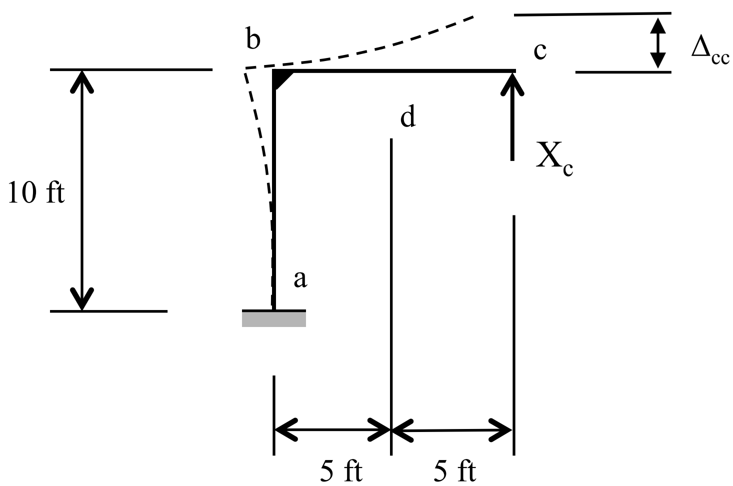
$$-\frac{1}{2}M_1(M_3+M_4)L \qquad\qquad -\frac{1}{6}(M_1+2M_2)M_3L$$

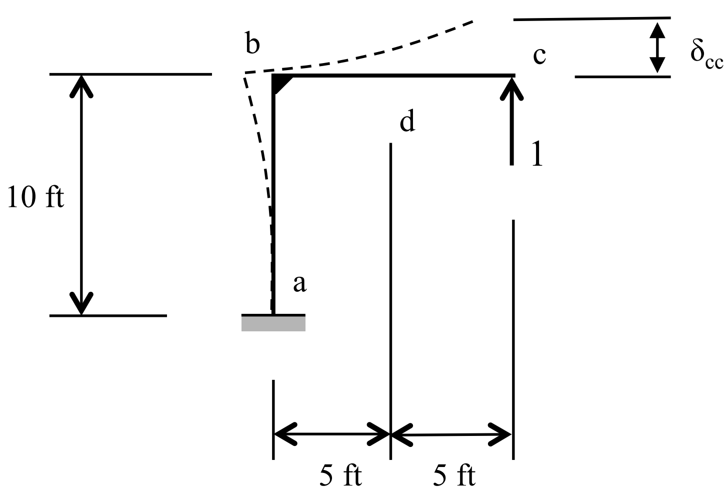
$$-\left(\frac{1}{2}\right)(10\text{ ft})(250\text{ k-ft}+150\text{ k-ft})(10\text{ ft}) \qquad -\left(\frac{1}{6}\right)[5\text{ ft}+2(10\text{ ft})](150\text{ k-ft})(5\text{ ft})$$

= -20,000 k-ft³ ***= -3,125 k-ft³***

$$\boxed{\Delta_{c0} = -\frac{23{,}125\text{ k-ft}^3}{EI}}$$

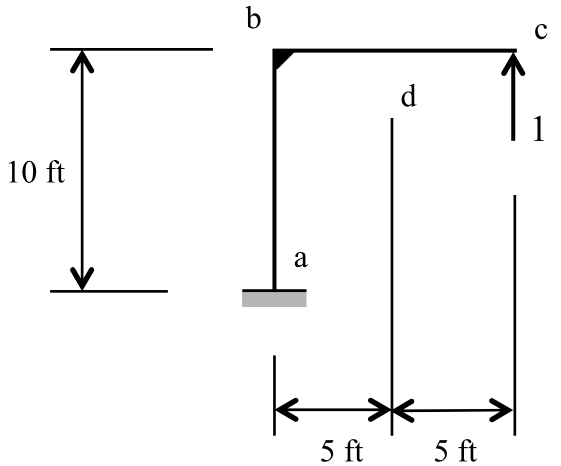
Redundant Problem





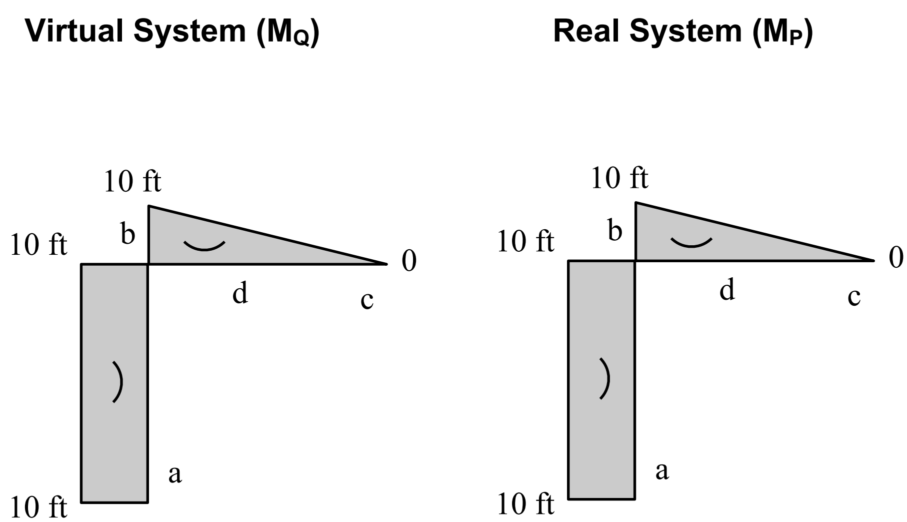
$$\Delta_{cc} = X_c \delta_{cc}$$

Virtual System to measure δ_{cc} (Same as virtual system used to measure Δ_{c0})



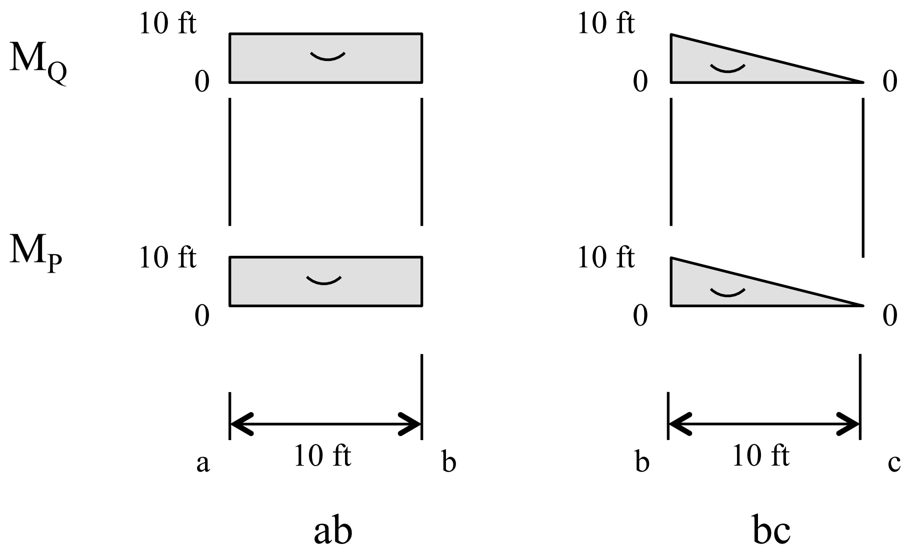
Note that for the flexibility coefficient problem to find δ_{cc}, the virtual and real systems are the same!

Moment diagrams



Use table to evaluate virtual work product integral:

$$1 \cdot \delta_{cc} = \frac{1}{EI} \int_{0}^{x=L} M_Q M_P \, dx$$



$$M_1 M_3 L$$

$$\frac{1}{3} M_1 M_3 L$$

$$(10\text{ ft})\,(10\text{ ft})(10\text{ ft})$$

$$\left(\frac{1}{3}\right)(10\text{ ft})(10\text{ ft})(10\text{ ft})$$

= 1000 ft³

= 333.33 ft³

$$\boxed{\delta_{cc} = \frac{1{,}333.33\text{ ft}^3}{EI}}$$

Compatibility Equation at Point c

$$\Delta_{c0} + X_c \delta_{cc} = \Delta_c = 0$$

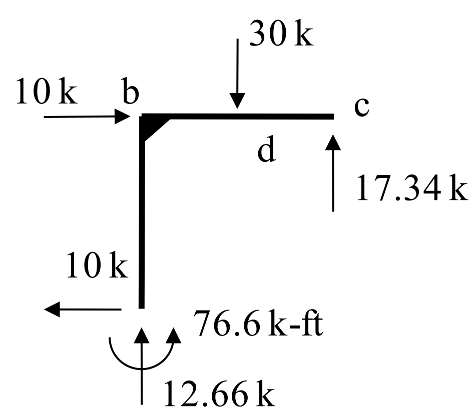
Substituting the values for Δ_{C0} and δ_{CC}

$$-\frac{23{,}125 \text{ k-ft}^3}{EI} + X_c\left(\frac{1{,}333.33 \text{ ft}^3}{EI}\right) = 0$$

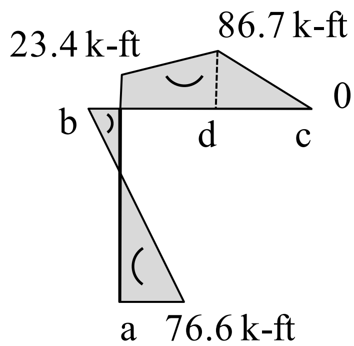
Solving for X_c (recall that $X_c = C_y$):

$$\boxed{X_c = c_y = 17.34 \text{ k}}$$

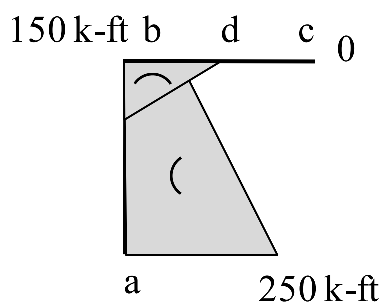
Can find other 3 reactions using the three equations of equilibrium, which yields the F.B.D. of the indeterminate frame:

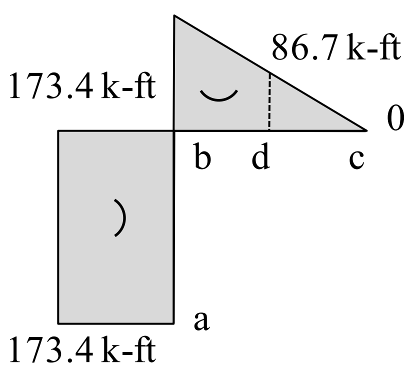


Can now construct the moment diagram for the indeterminate frame. You should be able to verify that this is the correct moment diagram.



Note the moment diagrams from the released and redundant problems are:

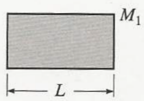
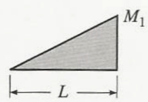
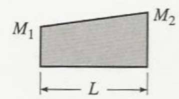
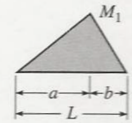
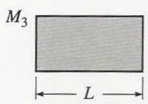
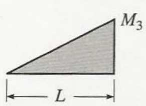
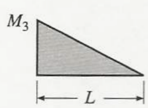
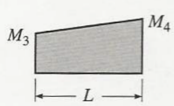
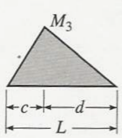
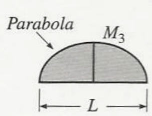
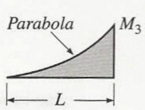




The moment diagrams from the primary and redundant problems add to give the moment diagram of the indeterminate problem.

Table 4 to evaluate product integrals to find δ_{cc} and Δ_{c0}

Table 4: Values of Product Integrals $\int_{x=0}^{x=L} M_Q M_P \, dx$

M_Q / M_P				
	M_1M_3L	$\frac{1}{2}M_1M_3L$	$\frac{1}{2}(M_1 + M_2)M_3L$	$\frac{1}{2}M_1M_3L$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{3}M_1M_3L$	$\frac{1}{6}(M_1 + 2M_2)M_3L$	$\frac{1}{6}M_1M_3(L + a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3L$	$\frac{1}{6}(2M_1 + M_2)M_3L$	$\frac{1}{6}M_1M_3(L + b)$
	$\frac{1}{2}M_1(M_3 + M_4)L$	$\frac{1}{6}M_1(M_3 + 2M_4)L$	$\frac{1}{6}M_1(2M_3 + M_4)L$ $+\frac{1}{6}M_2(M_3 + 2M_4)L$	$\frac{1}{6}M_1M_3(L + b)$ $+\frac{1}{6}M_1M_4(L + a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3(L + c)$	$\frac{1}{6}M_1M_3(L + d)$ $+\frac{1}{6}M_2M_3(L + c)$	for $c \leq a$: $\left(\frac{1}{3} - \frac{(a-c)^2}{6ad}\right)M_1M_3L$
	$\frac{2}{3}M_1M_3L$	$\frac{1}{3}M_1M_3L$	$\frac{1}{3}(M_1 + M_2)M_3L$	$\frac{1}{3}M_1M_3\left(L + \frac{ab}{L}\right)$
	$\frac{1}{3}M_1M_3L$	$\frac{1}{4}M_1M_3L$	$\frac{1}{12}(M_1 + 3M_2)M_3L$	$\frac{1}{12}M_1M_3\left(3a + \frac{a^2}{L}\right)$