The statically determinate frame from our internal force diagram example is made up of columns that are W8x48 ($I = 184 \text{ in}^4$) members and a beam that is a W10x22 ($I = 118 \text{ in}^4$). The modulus of elasticity of structural steel is 29,000 ksi, which yields the following bending stiffnesses:

- $EI_{ABC} = 5,336,000 \text{ k-in}^2$
- $EI_{BDE} = 3,422,000 \text{ k-in}^2$

Find the horizontal displacement of point C using the method of virtual work.

**Real Problem**

We found the Shear, Moment, and Axial force diagrams for this frame in a previous example earlier in the semester.
Moment Diagram of Real or P-System ($M_p$ diagram)

(You should review your notes and verify this is the correct diagram)

Virtual Problem

Virtual System to measure $\delta_C$
Free Body Diagram of Virtual System (or Q-system)

Equilibrium \( \sum M_A = 0; \sum F_y = 0; \sum F_x = 0 \) yields the following support reactions:
Virtual Moment Diagram ($M_0$ diagram)
(You should be able to verify this is the correct diagram)
Use Table 4 in the text to evaluate the virtual work product integrals that come from the principle of virtual work:

\[ 1 \cdot \delta_C = \frac{1}{EI} \int_0^L M_Q M_P \, dx \]

Integrate over three segments: AB, BC, and BDE

**Segment AB**
From Virtual Work Integral Table (see page 8):

\[ \frac{1}{3} M_1 M_3 L = \left(\frac{1}{3}\right) (10 \text{ ft})(10 \text{ k-ft})(10 \text{ ft}) \]

\[ = 333.333 \text{ k-ft}^3 \]

\[ = 576,000 \text{ k-in}^3 \]

**Segment BC**
From Virtual Work Integral Table (see page 8):

\[ \frac{1}{3} M_1 M_3 L = \left(\frac{1}{3}\right) (8 \text{ ft})(8 \text{ k-ft})(8 \text{ ft}) \]

\[ = 170,666.7 \text{ k-ft}^3 \]

\[ = 294,912 \text{ k-in}^3 \]
Segment BDE -- Split into two segments at inflection point of $M_P$ diagram (point Z)

\[
\frac{18 \text{ ft}}{x} = \frac{(12 \text{ ft})}{(5 - x)}
\]

\[x = 3 \text{ ft}\]

$M_Q$ Diagram

$M_P$ Diagram

Segment BZ

From Virtual Work Integral Table (see page 8):

\[
\frac{1}{6}(M_1 + 2M_2)M_3L = \left(\frac{1}{6}\right)[13.091 \text{ ft} + 2(18 \text{ ft})](18 \text{ k-ft})(3 \text{ ft})
\]

\[= 441.818 \text{ k-ft}^3\]

\[= 763,461.8167 \text{ k-in}^3\]
Segment ZDE

From Virtual Work Integral Table (see page 8):

Note that result is negative due to dissimilar curvatures

\[-\frac{1}{6}M_1M_3(L + c) = -\left(\frac{1}{6}\right)(13.091 \text{ ft})(12 \text{ k-ft})[8 \text{ ft} + 6 \text{ ft}]\]

\[= -366.5454 \text{ k-ft}^3\]

\[= -633,390.54 \text{ k-in}^3\]

Total for segment BDE

\[763,461.8167 \text{ k-in}^3 - 633,390.54 \text{ k-in}^3 = 130,071.28\]

Divide by EI of each segment

\[\delta_c = \frac{576,000 \text{ k-in}^3}{5,336,000 \text{ k-in}^2} + \frac{294,912 \text{ k-in}^3}{5,336,000 \text{ k-in}^2} + \frac{130,071.28 \text{ k-in}^3}{3,422,000 \text{ k-in}^2}\]

\[\delta_c = 0.1079 \text{ in} + 0.05527 \text{ in} + 0.03801 \text{ in}\]

\[\delta_c = 0.201 \text{ in}\]

Result is positive, so deflection is in the direction of the unit load – to the right
Product Integral Evaluation using Table 4 in text:

### Table 4: Values of Product Integrals $\int_{x=0}^{x=L} M_p M_p dx$

<table>
<thead>
<tr>
<th>Shape</th>
<th>$M_p$</th>
<th>$M_p M_3 L$</th>
<th>$\frac{1}{2}M_p M_3 L$</th>
<th>$\frac{1}{2}(M_1 + M_2)M_3 L$</th>
<th>$\frac{1}{2}M_p M_3 L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoid</td>
<td>$M_3$</td>
<td>$\frac{1}{2}M_1 M_3 L$</td>
<td>$\frac{1}{3}M_1 M_3 L$</td>
<td>$\frac{1}{6}(M_1 + 2M_2)M_3 L$</td>
<td>$\frac{1}{6}M_1 M_3(L + a)$</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>$M_4$</td>
<td>$\frac{1}{2}M_1(M_3 + M_4)L$</td>
<td>$\frac{1}{6}M_1(M_3 + 2M_4)L$</td>
<td>$\frac{1}{6}M_1(2M_1 + M_3)L$</td>
<td>$\frac{1}{6}M_1 M_3(L + b)$</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>$M_5$</td>
<td>$\frac{1}{2}M_1 M_3 L$</td>
<td>$\frac{1}{6}M_1 M_3(L + c)$</td>
<td>$\frac{1}{6}M_1 M_3(L + d)$</td>
<td>$\frac{1}{6}M_1 M_3(L + c)$</td>
</tr>
<tr>
<td>Parabola</td>
<td>$M_3$</td>
<td>$\frac{2}{3}M_1 M_3 L$</td>
<td>$\frac{1}{3}M_1 M_3 L$</td>
<td>$\frac{1}{3}(M_1 + M_2)M_3 L$</td>
<td>$\frac{1}{3}M_1 M_3(L + \frac{ab}{L})$</td>
</tr>
<tr>
<td>Parabola</td>
<td>$M_5$</td>
<td>$\frac{1}{3}M_1 M_3 L$</td>
<td>$\frac{1}{4}M_1 M_3 L$</td>
<td>$\frac{1}{12}(M_1 + 3M_2)M_3 L$</td>
<td>$\frac{1}{12}M_1 M_3\left(3a + \frac{a^2}{L}\right)$</td>
</tr>
</tbody>
</table>